2.5 Representing Sinusoidal Functions

Study Guide and Exercise Book Pages
36 to 38

Tools
- grid paper
- graphing calculator

Related Resources
- G–5 Trigonometric Graph Paper
- T–4 The TI-Nspire™ CAS Calculator

Teaching Suggestions

Key Concepts

- The graph of $y = \sin x$ is the graph of $y = \cos x$ translated $90° + 180n°$, where n is an integer.
- Students may have difficulty deciding whether to add 90° to or subtract 90° from the sine function in order to rewrite it as a cosine function. Have students draw graphs to help them visualize the correct translation.
- Ensure students understand that there are many correct ways to express a sine function as a cosine function.
- Encourage students to use technology to understand that sinusoidal functions with different equations may represent the same graph.

Example

- The **Example** provides students with the opportunity to think backward.
- You may wish to have students sketch the final graph and check if the criteria in the question are met.
- To give more practice, provide students with graphs of transformed sine and cosine functions. Have them write one sine function and one cosine function for each diagram. As an extension, have students write another sine function and cosine function for each diagram. Discuss how many possible answers there are.
- To practise problem solving as a class, create a problem similar to **question 6**, and brainstorm how best to find a solution and clearly communicate the solution.

Questions

- You may wish to use **G–5 Trigonometric Graph Paper** for questions in this section.
- For **question 2a**), have students write another equation with the given features.
- For **question 2b**), review the key features that students are to list.
- Encourage students to show complete solutions when they are determining the equations of sine or cosine functions. Refer students to the **Example** if they are unsure how to communicate clearly. Remind students that a labelled sketch is an effective way to communicate.
- Encourage students to sketch graphs for **questions 3** to **7** before attempting to write the equation.
- For **question 5b**), ask students to explain which key features remain the same and which features change.
- For **questions 6** and **7**, it may be easier for students to refer to a sketch of the function to determine the cosine equation than to transform the sine function.
- As an extension, you may want to ask students to represent the functions in **questions 6** and **7** as reflected sine and cosine functions.
- Some students may have trouble using given information to write a sine or cosine function. Show students a complete solution, or have them work in groups to create a complete solution and then present their solution to their classmates.

Common Errors
- Students may have difficulty writing an equation for a sinusoidal function given information about the graph.

R_x Have students sketch a graph before calculating the function's attributes.

Differentiated Instruction
- Start the lesson with **blast off**. Ask students to record five important attributes that a transformed sinusoidal function can have (reflection, amplitude, phase shift, period, vertical shift). Each attribute can then be discussed in more detail.
- Use **think aloud** and have one group lead the class through **question 2**, and another group lead the class through **question 4**.

- Encourage student to work out **questions 9** and **10** in two ways: draw a sketch to find the cosine function, or calculate one quarter of the period and add it to the *d*-value.
- For **question 11**, have students write two other equations that represent the same graph.
- For **question 17**, ask students if all the given information was required to answer the question.
- For **question 18**, ask students how many different equations are possible for this function.
- For **question 19**, ensure students realize that the units for the *x*-axis are hours, not degrees. Have students create a visual model to represent the water depth.
- For **question 20**, since the *a*-value may be negative, the answer should include an absolute value sign.
- Have students compare their answers to **questions 20** and **21**. How are they the same? How are they different?
- Encourage students to research sinusoidal functions on the Internet. What types of data will produce a sinusoidal function when graphed?

Technology Suggestions

- You may wish to have copies of **T–4 The TI-Nspire™ CAS Calculator** available.
- For **question 6**, encourage students to graph the sine and cosine functions using TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus to check that they both have period 180° and a maximum at $\left(0, \frac{5}{2}\right)$.
- If TI-Nspire™ CAS is used for **questions 5** to **7**, students can use the **Geometry Tools** to measure the length representing the period. They can put two points on the graph representing their equation by using the **Points & Lines** tool at the start and end of the first period, and measure the length *b* using the **Measurement** tool. They could move one of the points to a maximum to confirm that the maximum value is correct.
- For **questions 9** to **12**, students can enter the equation they developed and work in reverse to determine if the key features are correct.

Ongoing Assessment

- You may wish to have students complete the following question to assess their understanding of the material in this section: Fatima is in the car of a Ferris wheel. The wheel has a diameter of 24 m. The centre of the wheel is 14 m off the ground. Fatima completes one revolution of the Ferris wheel in 60 s. Draw a sketch of Fatima's height, in metres, off the ground versus time, in seconds, for the first 2 min of the ride. Model your graph with a sinusoidal equation.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	19
Reasoning and Proving	n/a
Reflecting	1
Selecting Tools and Computational Strategies	2–7
Connecting	6, 7, 18
Representing	9, 10, 13, 14
Communicating	1–3, 6, 12