2.6 Solving Problems Involving Sinusoidal Functions

Study Guide and Exercise Book Pages
39 to 42

Tools
- grid paper
- graphing calculator

Related Resources
- G–5 Trigonometric Graph Paper
- T–4 The TI-Nspire™ CAS Calculator
- BLM 2–3 Chapter 2 Review
- BLM 2–4 Chapter 2 Practice Test
- BLM 2–5 Chapter 2 Case Study

Key Terms
- periodic function
- periodic phenomenon
- primary data
- secondary data

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Discuss with students the difference between primary and secondary data.
- You may wish to have students reflect on what kind of data from primary or secondary sources would form a sinusoidal graph. Have students look at some of the application questions presented in the previous sections.
- Will data from primary or secondary sources always form a perfect sine or cosine function? Why or why not?
- Discuss with students the domain and range of sample real-world problems. Why do real-world problems always have restrictions of some type?

Example

- If a computer is not available, provide students with a handout of the data. Have students plot the ordered pairs on a large sheet of grid paper.
- As an extension, have students analyse the graph: sketch a horizontal axis, and find the amplitude, period, phase shift, vertical shift, and one or more appropriate equations. Have students compare their equations with those of other students. Answers may vary, especially if some students drew small, inaccurate graphs.
- Provide students with formulas to find the approximate amplitude and vertical shift: $a = \frac{h - l}{2}$ and $c = \frac{h + l}{2}$, where h represents the highest number in the table and l represents the lowest number in the table.
- It is tempting to make use of trigonometric regression on the graphing calculator. However, the regression function assumes radian measure, which is not used in this course.

Questions

- You may wish to use **G–5 Trigonometric Graph Paper** for questions in this section.
- For **question 1**, have students describe two ways to determine the amplitude of the given graph.
- As an extension to **question 1**, have students construct a sinusoidal model for this graph. Discuss the meaning of *sinusoidal model*.
- Encourage students to always add the horizontal axis to their sketches.
- Remind students that most application problems will not have degrees along the x-axis.
- Before beginning **question 2**, discuss the notation $h(t)$. This will help students determine the units on the x- and y-axes.
- For **question 2**, students may need help choosing appropriate window settings. Encourage students to check the table of values to help them.
- For **question 3**, discuss why the period is one year, or 12 months. Have students write an equation of a cosine function. This reinforces the concept that either a sine or a cosine function can be used to model a periodic process. Explain to students that although they are creating an equation for a sinusoidal model, it does not necessarily mean that they have to write a sine equation.
- As an extension to **question 4d**), you may wish to show students how to solve this question both graphically and algebraically, since there is more than one answer.

Common Errors

- Some students have trouble labelling the axes on their graphs with very large or very small numbers.

R_x Rounding values may sometimes help; working with a partner is useful as well.

Differentiated Instruction

- Use **gallery walk** to practise the concepts in this section. Provide each group of four students with a real-world table of values that can be modelled with a sinusoidal function. Have the group plot a neat graph on poster paper. The groups rotate, and add the horizontal axis, phase shift, and vertical shift to a graph. The groups rotate, and add the period and amplitude to a graph. The groups rotate, check the previous two groups' work, and add a suitable equation. Groups return to their original station and check all other groups' work.

- For **question 5**, have students research on the Internet how a motion sensor can be used to generate data that will create a sinusoidal function.
- For **question 6c**), remind students of the formula for circumference of a circle.
- For **question 7**, students may need help identifying and explaining the restrictions on the domain and range of the function. You may wish to have students graph this function. They may benefit from discussing in groups the meaning of each of the variables.
- For **question 8**, ask students why the period is such a small number or, conversely, why the k-value is such a large number. Encourage students to research and learn more about the application of a sinusoidal function that represents the movement of a piston in an automobile engine.
- **Question 12** gives students an opportunity to connect sinusoidal graphs to real-world situations. Ask students to compare their graph with a classmate's and explain any differences.
- Students may have difficulty finding secondary data on the Statistics Canada web site. For **question 13**, guide students to data tables that can be modelled by sinusoidal functions. Suggest that students begin by working in groups to brainstorm a list of real-world data that might be periodic.
- You may wish to use **BLM 2–3 Chapter 2 Review** to help students identify areas in which they need to further their understanding.
- Provide students with **BLM 2–4 Chapter 2 Practice Test** to prepare them for the chapter test.

Case Study

- You may wish to have students complete **BLM 2–5 Chapter 2 Case Study**, which incorporates the learning from Chapter 2.
- If possible, demonstrate the concept of frequency (pitch) using an electronic keyboard or other instrument.
- The idea of timbre can be shown by playing the same note using two different instruments.
- It may help to clarify the relationship between a fundamental frequency and its harmonics by playing different octaves on an instrument.
- For **question 1b**), students should realize that 23 400 represents the value of k. So, they need to divide 360° by this to determine the period. Encourage them to express the value as a fraction reduced to lowest terms, rather than a decimal number.
- To graph two cycles, use these window settings: Xmin = 0, Xmax = $\frac{2}{65}$, Xscl = 0.005, Ymin = −100, Ymax = 100.
- The graph in **question 3b**) should approximate square waves. Students may need to adjust the window settings.
- Students should recognize the pattern in the terms that are added to the basic equation for the fundamental frequency. The more terms that are added, the more square the graph should appear.

Technology Suggestions

- You may wish to have copies of **T−4 The TI-Nspire™ CAS Calculator** available.
- For the **Example**, you may wish to have students enter the data into lists on TI-83 Plus/TI-84 Plus. Enter the year since 1976 in list L1 and the population in millions in list L2. Draw a scatter plot and consider the period, amplitude, phase shift, and vertical translation. Students should use the key features to generate an equation rather than using a sinusoidal regression. To set the window, press **ZOOM**, **9: Zoomstat**.

Ongoing Assessment

- Ask students which questions they had difficulty with. Have them explain what was difficult about those questions.

Summative Assessment

- You may wish to use the chapter test that you can find in the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

- Alternatively, enter the data from the **Example** into a **Lists & Spreadsheet** page using TI-Nspire™ CAS. Enter the years since 1976 in the first column and the population in millions in the second column. Set the window so YMin = −1 and YMax = 3. Graph $y = \sin x$. Manipulate the base function to fit a sinusoidal curve to the data. Grab a maximum. Press the up or down arrow to vertically stretch/compress $y = \sin x$. Press the left or right arrow to horizontally stretch/compress $y = \sin x$. If you grab an inflection point, you can press the up/down arrow to translate vertically or press the left/right arrow to translate horizontally.
- You may wish to have students use TI-Nspire™ CAS or TI-83 Plus/TI-84 Plus to complete **questions 1** to **14**.
- To complete **question 2** using TI-Nspire™ CAS, open a **Graphs & Geometry** page and graph the function. Press (menu). Choose **Trace**, then **Graph Trace**. Use the arrow keys to move the cursor to a maximum point. Press (enter). The first coordinate in the ordered pair is the time since midnight. To determine the height of the tide at 9 a.m., type 9, and then press (enter).
- To complete **question 2** using TI-83 Plus/TI-84 Plus, graph the function, then press **TRACE**. Use the arrow keys to move the cursor.
- To complete **question 4d**) using TI-Nspire™ CAS, press (tab). Type 600, and then press (enter). To find the points of intersection of the horizontal line and the sinusoidal function, press (menu), choose **Points & Lines**, then choose **Intersection Point(s)**.
- To complete **question 4d**) using TI-83 Plus/TI-84 Plus, press **Y=** and enter 600 in Y2. Press **GRAPH** to see the horizontal line and the sinusoidal graph. Press **2nd**, **TRACE** to access the **CALC** menu and select **5:intersect**. Press **ENTER** to identify the first curve, press **ENTER** again to identify the second curve, and press **ENTER** a third time to accept the guess created by the calculator.
- TI-Nspire™ CAS can be used effectively to measure the period and amplitude for those students who have difficulty using mathematical operations. Using the **TRACE** feature helps determine the maximum and minimum values and the associated x-values.
- For applets demonstrating how to construct a sinusoidal function from a swinging pendulum, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	4, 6, 8, 9, 11, 12, 16
Reasoning and Proving	5, 8, 11
Reflecting	1, 3, 12
Selecting Tools and Computational Strategies	6, 11
Connecting	2, 6–11, 16
Representing	2, 4, 9–12
Communicating	1, 2, 4, 5, 7, 8, 13–15