3.2 Components of Vectors

Study Guide and Exercise Book Pages
47 to 49

Tools
- grid paper
- ruler
- protractor
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad*® 4
- T3–3 How to Do Section 3.2 #3a) and b) Using *The Geometer's Sketchpad*®

Key Terms
- perpendicular (or rectangular) component
- resultant
- horizontal component
- vertical component
- directed line segment
- tension
- momentum

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Suggest that students add new terminology to their chapter reference sheets.
- Clarify with students what *resolve* means.
- Consider reviewing how to label the sides of a right triangle. Encourage students to circle the angle they will use to name the sides, and then label the hypotenuse, the opposite side, and the adjacent side.
- Some students may need a review of the primary trigonometric ratios. A review of the sine ratio, $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, and the cosine ratio, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, may be particularly helpful.
- When referring to the sides in the triangle, suggest that students refer to the sides as being across from the right angle or across from the designated angle.
- Help students recognize that the horizontal component is beside, or adjacent to, θ, and the vertical component is opposite, or across from, θ.
- Although addition of vectors is not taught until the next section, have students observe how the vectors in the diagram have been arranged, and have them draw their diagrams for the practice questions with the vectors pointing in the correct directions.

Example

- Encourage students to draw neat and fully labelled diagrams of the horizontal and vertical components of forces. Have them include all arrowheads, units, and proper vector notation.
- When students are using a calculator to determine sin 35°, remind them to set the calculator to degree mode.
- Highlight the fact that the absolute value signs are important to achieve a correct solution.
- Have students recall that the symbol $\doteq$ means *approximately equal to*.
- As an extension, it may be useful to work through an example, as a class, in which the value of θ is not obvious, such as **question 1b**) in the practice questions.
- As an extension, it may be useful to work through with the class an example where a right triangle can be created by joining the vector to the vertical line, such as **question 2** in the practice questions.

Questions

- Prior to working on the practice questions, have students work in pairs to define other words used in the questions, such as *airspeed, horizontal groundspeed, displacement vector*, and *momentum*.
- Students may have trouble with **question 1b**). Discuss how to find θ when it is not provided in the diagram. They may also incorrectly think that the vertical component in this question is negative because the vector is pointing downward. Ask students to explain why there are absolute value signs in the formula to calculate a vector quantity.

COMMON ERRORS

- When a vector is described as being located clockwise or counterclockwise from the vertical, some students may create a right triangle by joining the vector to the vertical. They then may use the wrong trigonometric function to determine the two rectangular vector components.

R_x You may suggest that students always join the vector to the horizontal. Doing so creates consistency in the designated angle (the angle between the vector and the horizontal) and the trigonometric functions that are used to calculate the rectangular components: $|\vec{x}| = |\vec{v}| \cos \theta$ and $|\vec{y}| = |\vec{v}| \sin \theta$.

- Some students may confuse clockwise with counterclockwise, and vertical with horizontal.

R_x Consider having students create a diagram, labelling the horizontal and vertical axes. Then, have them draw two arrows, one pointing clockwise and the other counterclockwise, and label these as well.

DIFFERENTIATED INSTRUCTION

- Have students add the Key Terms to the **word wall**.
- Suggest students create a journal entry that explains components of vectors to a classmate who missed the lesson.
- There are many new terms in this section with which students will not be familiar. Have students work in pairs or small groups so that they learn to communicate effectively using these new terms.

- Students may have difficulty explaining **question 4f**). It may be useful to summarize this question by rewriting the two formulas in the **Key Concepts:**

  −That $|\vec{x}|=|\vec{v}| \cos \theta$ is equivalent to $|\vec{x}|=|\vec{v}| \sin (90° - \theta)$

  −That $|\vec{y}|=|\vec{v}| \sin \theta$ is equivalent to $|\vec{y}| =|\vec{v}| \cos (90° - \theta)$
- Have students draw a properly labelled diagram for **question 5**.
- For **question 10**, students may have trouble visualizing the speed of the aircraft's shadow on the ground. Help students find the rate of climb so that they can find the height of the aircraft after two minutes.
- For **question 11**, help students relate east to horizontal and south to vertical. Then, ask them how they might use the Pythagorean theorem to determine magnitude. Finally, once students have determined the unknown angle, they may need help to express the angle as a true bearing or a quadrant bearing.
- To challenge students, make up another question like **question 11**, but provide the rate, the direction, and the amount of time, and then ask students to determine the magnitude.
- For **question 15**, discuss the term *momentum* with the class and the units used.
- For **question 16**, students may need more explanation in order to learn how to represent the situation to keep the box at rest.
- Most students will find **question 17** challenging because they find ratios to be an abstract concept. Consider having them work in pairs. You may also want to help them recall how to determine an angle from the sine ratio or cosine ratio.

Technology Suggestions

- For the **Example**, use *The Geometer's Sketchpad*® and the **Arrowhead (Closed)** custom tool to illustrate a vector and its components. Turn on the grid and draw the vector with its tail at the origin. Then, measure lengths and angles to complete the illustration. Alternatively, you can draw the vector from the origin and measure the coordinates of the head. Then, use the **Plot Points…** feature from the **Graph** menu to plot the components. Complete the diagram by drawing the components as vectors.
- You can program a graphing calculator such as the TI-83 Plus/TI-84 Plus or TI-Nspire™ CAS to accept length and angle as inputs, and to output vertical and horizontal components.
- In **questions 1** to **3**, use *The Geometer's Sketchpad*® to prepare a generic vector, starting from the origin. From the **Measure** menu, select **Coordinates** to determine the lengths of the components. Measure the length of the vector using **Coordinate Distance** and the angle the vector makes with one of the axes. You can then drag the head of the vector to simulate different problems. If necessary, you can change the scale using the unit point, or by dragging the axis numbers. See **T3–3 How to Do Section 3.2 #3a) and b) Using *The Geometer's Sketchpad*®** for detailed directions.
- After working through the solution for **question 4** using pencil and paper, review the solution quickly and dynamically in *The Geometer's Sketchpad*®. Calculations can be made and displayed in seconds. If students are familiar with *The Geometer's Sketchpad*®, you can turn this task over to class members, each taking a turn at illustrating a point. Ask for volunteers to avoid undue anxiety, but ensure that everyone in the class eventually has a turn at demonstrating the use of *The Geometer's Sketchpad*®.
- The solutions for **questions 5** to **8** can be illustrated using *The Geometer's Sketchpad*®. Alternatively, you can have students program component calculations in a graphing calculator, which they can then use to check their answers quickly and accurately.

- **Question 9** is a "what if" question. Questions such as this benefit enormously from the dynamic geometry capabilities of *The Geometer's Sketchpad*®. After soliciting student predictions, use *The Geometer's Sketchpad*® to quickly and dynamically illustrate what happens when the length of the rope is changed.
- To accurately plot a point using *The Geometer's Sketchpad*®, you can use the **Transform** menu. Select a **Polar** translation vector, and type in the desired angle as accurately as desired. This method is particularly useful when an angle is given to the tenth of a degree.
- You can use the **Calculate** functions from the **Measure** menu of *The Geometer's Sketchpad*® to solve momentum problems, such as **question 15** (or any other problems involving a calculation).

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	6, 10–12, 15–17
Reasoning and Proving	6
Reflecting	4, 9, 11, 17
Selecting Tools and Computational Strategies	1, 17
Connecting	1, 12, 13, 16
Representing	11, 12
Communicating	2, 4

Ongoing Assessment

- Determine the horizontal and vertical components of a force with magnitude 560 N, at an angle of 21° counterclockwise from the vertical.