3.3 Adding Vectors

Study Guide and Exercise Book Pages
50 to 53

Tools
- grid paper
- ruler
- protractor
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- T3–4 How to Do Section 3.3 #2 Using *The Geometer's Sketchpad®*

Key Terms
- triangle method or head-to-tail method
- parallelogram method or tail-to-tail method
- zero vector
- commutative property
- associative property
- identity property

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Students will have difficulty understanding the **Key Concepts**. Diagrams are necessary to help explain each concept.
- Have students add each rule, with a diagram, to their chapter reference sheets.
- Before introducing vector notation, explain how the commutative property, associative property, and identity property work with algebra.

Example

- As an extension, have students do the opposite to part **a**) on grid paper. Have them place $\vec{u}$ on the head of $\vec{v}$, and then draw the resultant vector. Have students discuss their observations in pairs. Ask, "Which vector property have you just proved?" ($\vec{u} + \vec{v} = \vec{v} + \vec{u}$, the commutative property)
- As an extension to the **Example**, have students create any three vectors, $\vec{u}$, $\vec{v}$, and $\vec{w}$, and prove that $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (the associative property) and that $\vec{v} + \vec{0} = \vec{v} = \vec{0} + \vec{v}$ (the identity property).
- Before teaching the tail-to-tail method, it may be necessary to review the geometry of parallelograms with students.
- To prepare students for **questions 9** and **10**, you may wish to complete an example in which students find the magnitude and direction of a resultant vector when the two given vectors are perpendicular.
- To prepare students for **questions 11** and **12**, you may wish to complete an example in which students find the magnitude and direction of a resultant vector when the two given vectors are not perpendicular. Help students reactivate their knowledge of the sine law and the cosine law.

Questions

- Give students time to practise finding the resultant of two vectors.
- Encourage students to use both the head-to-tail (triangle) method and the tail-to-tail (parallelogram) method when completing the questions.
- For **question 1**, remind students that the resultant vector extends from the tail of the first vector to the head of the final vector. Stress that the magnitudes are not always simply added because direction must be considered.
- For **question 2**, remind students of the difference between distance and displacement. Distance, a scalar quantity, is the sum of the magnitudes of each vector in the path. Displacement, a vector quantity, is the magnitude *and* direction of the resultant vector.
- Students cannot successfully answer **question 3** unless they draw an accurate diagram on grid paper.
- Consider taking up **questions 4**, **7**, and **8** as a class to help students understand how to name a single vector equivalent to an expression.
- For **question 7c**), there is no single vector that is equivalent to the given expression. Help students write a combination of vectors that is equivalent.
- Students may be unsure how to express the direction of the resultant vector in **questions 9** and **10**. Have them revisit true bearings and quadrant bearings, and suggest that they express the direction using the method with which they are most comfortable.

Common Errors

- Some students may have difficulty translating vectors and adding two or more vectors together.

R_x You may wish to have students work with sticks and rulers, or metre sticks, to better understand the translating and adding process. Also, encourage them to use grid paper and a ruler to draw diagrams for each practice question.

Common Errors

- When drawing a vector at a 45° angle on grid paper, the vector runs diagonally through grid cells. Some students may count the cells along the ray, arriving at an incorrect solution.

R_x Discuss with students why the diagonal value of a grid cell is not the same as its horizontal or vertical value. Ask how they could use the Pythagorean relationship to prove this.

Differentiated Instruction

- There are many new concepts in this section that can be very confusing for students. Add the **Key Concepts** to the **word wall** with a large, detailed sketch.
- Use **timed retell** to summarize the concepts of this lesson.
- Some students may find it helpful to work with "real" vectors cut from paper.
- If students are having difficulty drawing representations of the vectors, encourage them to work with scale models or to work with *The Geometer's Sketchpad®*. (Consider having students install the software on their home computers and ensure that it is available on any general use computers in the school.)
- Encourage students to work together in pairs or small groups. This will give them an opportunity to practise the concepts and ideas, and communicate their understanding in the classroom, rather than working alone at home.

Ongoing Assessment

- Use the following questions to assess students' understanding:
  - On grid paper, draw and label a parallelogram PQRS. Label the intersection point of the diagonals T. Express a resultant, $\overrightarrow{QS}$, five different ways, by adding two or more single vectors together.
  - Simplify:
    $\overrightarrow{QS} + \overrightarrow{SR} + \overrightarrow{RT} + \overrightarrow{TM}$
    Draw a diagram.

- For **questions 9** and **10**, have students discuss which method they find easier, the triangle method or the parallelogram method.
- For **questions 11** and **12**, remind students to redraw the given diagram, depending on which method they have chosen to find the resultant vector. Since students learn vector addition best through scale drawings, emphasize the importance of making their diagrams as accurate as possible.
- Students draw their own diagrams in **questions 12**, **13**, and **16.** Suggest that they compare their diagrams and solutions with those of a partner.
- In **question 16**, help students understand that the wind is heading east.
- Explaining with pictures and words why **question 17** is true should be within the grasp of most students, given that they understand that the absolute value signs give the vector's magnitude. Proving this statement formally may be outside the grasp of most students.

Technology Suggestions

- In **question 2**, the difference between distance and displacement can be confusing for many students, especially those who have difficulty visualizing two-dimensional situations. Use the dynamic nature of *The Geometer's Sketchpad®* to move a destination in relation to the origin, and let students see how the displacement and distance change accordingly. Then, ask students to predict a situation in which displacement does not change but distance does. Illustrate the prediction dynamically. Finally, ask students to predict a situation in which distance does not change but displacement does. Illustrate the prediction dynamically. See **T3–4 How to Do Section 3.3 #2 Using *The Geometer's Sketchpad®*** for detailed instructions.
- In **question 3**, you can use *The Geometer's Sketchpad®* to draw the three vectors. You can then dynamically manipulate each vector separately, moving them to illustrate the vector additions needed. This is an excellent way to illustrate the commutative nature of vector addition.
- For **questions 9** to **11**, if you have chosen to have students program graphing calculators with the sine law and cosine law, they can use the programs to do the repetitive calculations involved in these questions. This will enable students to focus on the higher vector concepts. Alternatively, they can use these programs to check solutions.
- Use *The Geometer's Sketchpad®* for **questions 13** and **16** to dynamically illustrate the solution for each of these questions. Then, show the effect of small changes in the given information.
- Before proving the proposition in **question 17**, illustrate the meaning of the statement using a sketch created with *The Geometer's Sketchpad®*.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	11, 12
Reasoning and Proving	17
Reflecting	9–12, 17
Selecting Tools and Computational Strategies	9–12
Connecting	9–12, 17
Representing	3
Communicating	2, 3, 5