3.5 Solving Problems Involving Vectors

Study Guide and Exercise Book Pages
57 to 59

Tools
- grid paper
- ruler
- protractor
- graphing calculator
- string, masses, and two hanging-weight scales
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 3–3 Chapter 3 Review
- BLM 3–4 Chapter 3 Practice Test
- BLM 3–5 Chapter 3 Case Study
- T3–6 How to Do Section 3.5 #5 Using *The Geometer's Sketchpad®*

Key Terms
- resultant force
- ground velocity
- heading

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Help students reactivate their knowledge of trigonometry, including the following:
  - the Pythagorean theorem
  - the primary trigonometric ratios and how to determine an angle in a right triangle
  - the cosine law to find the third side of a non-right triangle
  - the sine law to find a second angle in a non-right triangle

Example

- Review the meaning of the term *displacement* with students.
- Have students note the use of proper vector notation in the **Example**.
- After calculating the magnitude in part **a)**, some students may want to use the sine or cosine ratio to calculate the angle. Discuss with students that this would be an option, but that it is important to use the exact value for $|\vec{R}|$, $\sqrt{13}$, in their calculation.
- Revisit with students how to express a quadrant bearing as a true bearing.
- To prepare students for the practice questions, consider completing another example that requires the cosine law and the sine law.
- You may want to consider working through a second example that does not involve a right triangle. For example, consider having students work through the following: Mary is walking to the library. She travels 3 km in the direction N65°E, and then she travels 2 km north. Determine the magnitude and direction of the resultant displacement vector.
- If you present the second example, consider having students work through it in pairs. Watch for the following difficulties:
  - Some students may have difficulty finding the angle between the two vectors.
  - Some students will find it challenging to determine the direction of the resultant vector.
  - Students typically have difficulty expressing the direction of the resultant vector as a quadrant bearing.
  - When finding a second angle in their triangle with the sine law, remind students to preserve accuracy by not rounding the magnitude of the resultant vector.
  - Have partners write their concluding statement. Have they expressed their answer clearly with a correct bearing?

Questions

- Most of the application problems in this section do not provide diagrams. Encourage students to draw diagrams and vector diagrams even when the question does not ask for them. Suggest that students watch out for clues in the problem and highlight key words. Tell them to watch out for direction words in particular, such as *to*, *from*, *on,* and *at*.
- It may be necessary to do a review on quadrant bearings for students who are using a directional diagram.
- Have students refer to a directional diagram to double-check their diagrams in **question 1**.

Common Errors

- Students often have difficulty with bearing problems, due to the complex nature of the drawings.

R_x Have students draw a small diagram for each instance of a bearing, heading, or resultant that is mentioned. Ensure the arrows are drawn based on the clue words *to*, *from*, *on,* and *at*. Students can then combine the clues to create a large appropriate vector diagram. They may also wish to faintly draw the axes at each vertex of their vector diagram to help them find the direction of the resultant.

Differentiated Instruction

- Use **think aloud** when teaching this section.
- Use **think-pair-share** to reinforce the concept of bearings.
- Concrete and kinesthetic learners might benefit from creating a scenario similar to the one in **question 16**. They could use a mass, a string, and two scales (such as scales from the physics lab) to mimic the situation in this problem.

- In **question 4**, ask students what relationship the word *displacement* has with the question, "How far has the car travelled?" This will help them calculate the length of the two vectors.
- Review true bearings for **question 5**. Note that the wind is blowing from the direction 180° (south). Have students recognize that they want to draw the wind blowing to 0° (north) before adding their vectors. Before doing this question, ask students what the difference is between *speed* and *velocity* (speed is a scalar quantity and velocity is a vector quantity).
- For **questions 4**, **6**, and **8**, it may be necessary to revisit with students how to determine distance when given velocity and time.
- Some students may require help with the diagram in **question 7**. It may be useful to model the scenario with sticks, rulers, or pipe cleaners.
- For **question 8**, discuss the concept of resolving vectors with students. Ensure that they convert the 6 min to hours before attempting to find the displacement.
- In **questions 9** and **10**, some students may not recognize that they have to rearrange the vectors in their diagrams in order to add them together.
- It may be helpful for some students to recreate the situation in certain questions, such as **question 10**, in which Wyatt and Jenna are pulling on two ropes attached to a trailer.
- In **questions 11** to **15**, students are required to calculate the direction for a resultant vector in a non-right triangle. These types of problems are typically very challenging for students because they involve many steps. Ensure that students are drawing large, neat diagrams for each situation, so that it is clear to them what they need to calculate as their next step.
- To highlight the importance of drawing a good diagram, have students find the resultant direction in **question 14**. If a scale diagram is not drawn, students may be unsure whether the resultant direction is west of north or east of north.
- You may wish to use **BLM 3–3 Chapter 3 Review** to help students identify areas in which they need to further their understanding.
- Provide students with **BLM 3–4 Chapter 3 Practice Test** to prepare them for the chapter test.

Case Study

- You may wish to have students complete **BLM 3–5 Chapter 3 Case Study**, which incorporates learning from Chapter 3.
- Use PowerPoint to create examples of the effect described in the case study, or find web sites that use a similar effect.
- It may be helpful to show how perspective is used in paintings to create a three-dimensional impression of distance.
- If students need clarification for **question 2**, ask them what they would observe if the two vertical lines were the same size.
- Students should be able to determine that the two vectors are not equivalent by simply looking at the diagram. If they are having difficulty, ask what they notice when they compare the angle between the horizontal and $\vec{b}$, and the angle between the horizontal and $\vec{t}$ (it is greater, which should indicate that the vectors are not equal).
- The problem posed in **question 4** is relatively easy to solve using the Pythagorean theorem. Students may need some guidance on using the dimensions of the screen to locate the position of the central vertical line.

Differentiated Instruction

- Provide students with opportunities to work in a school computer lab and to use *The Geometer's Sketchpad*® to help them visualize the situations in the questions. You may need to spend some time with students in the computer lab, helping them use *The Geometer's Sketchpad*® to represent vector situations.

Technology Suggestions

- After presenting the solution for the **Example** using pencil and paper, use *The Geometer's Sketchpad*® to illustrate an alternative method. This offers reinforcement to the pencil-and-paper method, as well as supporting visual learners.
- Students could program their graphing calculator to avoid the repetitive calculations involved in **question 1**. These programs could also be used to check solutions. Students could also use *The Geometer's Sketchpad*® to illustrate and check solutions for this question.
- For **questions 2** to **5**, use a prepared sketch for vector addition to quickly illustrate and check solutions. Change the scale of the vector diagram by dragging the unit point or the numbers along the axis. Drag the heads of the vectors to match the sketch in a given question.
- An interesting extension for **question 5** is to ask what heading the aircraft needs to be on so that the resultant falls along a heading of 90°. This can be quickly illustrated and solved using *The Geometer's Sketchpad*®. See **T3–6 How to Do Section 3.5 #5 Using *The Geometer's Sketchpad*®**.
- **Questions 6** to **14** can be quickly illustrated and checked using *The Geometer's Sketchpad*®. This need not be done for each question, but it would be useful to select one or two examples for technology solutions.
- **Question 15** lends itself well to a solution using *The Geometer's Sketchpad*®. Once the sketch has been constructed, it is easy to drag each vector until the desired conditions are achieved.

Ongoing Assessment

- Use the following question to assess students' understanding of this section:
  - An airplane is flying with an airspeed of 450 km/h on a heading of 90°. The ground velocity of the airplane is 455 km/h at a heading of 95°. What is the speed of the wind? From what direction is the wind blowing?

Summative Assessment

- You may wish to use the chapter test that you can find in the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	3–16
Reasoning and Proving	n/a
Reflecting	11–16
Selecting Tools and Computational Strategies	16
Connecting	9, 11–15
Representing	3–15
Communicating	n/a