4.1 The Exponent Laws

Study Guide and Exercise Book Pages

60 to 63

Tools

- scientific calculator
- computer algebra system

Related Resources

- G–2 Placemat
- A–4 Selecting Tools and Computational Strategies
- BLM 4–1 Chapter 4 Prerequisite Skills
- BLM 4–2 Chapter 4 Self-Assessment Checklist
- T4–1 How to Do Section 4.1 Example Using TI-Nspire™ CAS

Key Terms

- power
- base
- exponent
- factor

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

- Before students begin the chapter, you may wish to have them complete **BLM 4–1 Chapter 4 Prerequisite Skills** to activate their prior skills.
- Give students **BLM 4–2 Chapter 4 Self-Assessment Checklist** to keep track of their skills and knowledge. Have students return to it throughout the chapter.
- For further teacher support for this chapter, go to the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Key Concepts

- Have students create a reference sheet that they can refer to throughout this chapter. They can begin by adding the exponent rules found in the **Key Concepts**. You may wish to have them add an example of how each rule is applied.
- Students will need to be reminded about the difference between integers, rational numbers, and real numbers.
- Have students prove $a^m \times a^n = a^{m+n}$ and $(a^m)^n = a^{m \times n}$ by substituting numbers and then writing each side in expanded form.
- Have students prove $a^m \div a^n = a^{m-n}$ by substituting numbers into the left-hand side, writing it in expanded form, and cancelling factors from the numerator and denominator.
- $a^0 = 1$ can easily be shown to be true by a succession of steps. Start with an example such as $3^3 = 27$. Divide each side by 3 to get $3^2 = 9$. Divide each side by 3 to get $3^1 = 3$. Divide each side by 3 to get $3^0 = 1$. Discuss why this rule is not true when $a = 0$.
- The previous example can be extended to demonstrate how negative exponents work. Divide each side by 3 to get $3^{-1} = \frac{1}{3}$. Divide each side by 3 to get $3^{-2} = \frac{1}{9}$, etc. Discuss why this rule does not work when $a = 0$.
- Students will have trouble conceptualizing $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$. Clarify that a square root sign means "an exponent of $\frac{1}{2}$." You may wish to complete **question 14** to introduce this concept.
- Review with students the concept of a cube root, fourth root, and so on before introducing the notation.
- Complete examples for the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$. When simplifying an example such as $32^{\frac{3}{5}}$, show students that it is much simpler to take the fifth root before cubing the answer than to cube the answer and take the fifth root. Demonstrate to students that examples such as these can be done in their head rather than on a calculator.
- You may wish to teach the rules found in the **Key Concepts** over two or more lessons.

Example

- Have students write down which exponent rule was used to evaluate each step in part **b**) of the **Example**.
- Ask students why solution **b**) may be preferable.

Common Errors

- Many students have trouble simplifying an expression in which the exponent rules are combined, even though they understand each rule separately.

R_x Have students list the exponent rules and number each rule. As they work through simplifying more complex expressions, have them state which rule they are applying at each step, and why.

Differentiated Instruction

- Use a **placemat** divided into six or more parts to summarize the various rules from this section. Have students make up two examples that use each rule.
- Construct a **word wall** of terms and rules relevant to this chapter. Include words from the Key Terms section and the exponent laws from the **K**ey **C**oncepts.

Questions

- Students may have trouble when multiplying and dividing algebraic expressions in exponent form with negative signs. Review integer questions such as $-7 - (-3) = -7 + 3 = -4$.
- Some students may need a review of the rule for the multiplication of positive and negative numbers when they are dealing with questions that have powers such as $(3^2)^{-3}$.
- Remind students that they should not multiply the bases when multiplying algebraic expressions in exponent form.
- Remind students that they should not divide the bases when dividing algebraic expressions in exponent form.
- Show the difference between $2^4 = 16$, $(-2)^4 = 16$, and $-2^4 = -16$.
- Compare $64^{\frac{1}{3}} = 4$, $-(64)^{\frac{1}{3}} = -4$, and $(-64)^{\frac{1}{3}} = -4$.
- Compare $64^{\frac{1}{2}} = 8$, $-(64)^{\frac{1}{2}} = -4$, and $(-64)^{\frac{1}{2}} =$ undefined. Note that *undefined* means it has no answer in the real number system.
- If students are having difficulty understanding the exponent laws, encourage them to write complete solutions and show all of the steps.
- If students are having difficulty understanding negative exponents, review the multiplying pattern and the dividing pattern in the tables of values for exponential functions such as $y = 2^x$.
- Remind students that numbers such as 7 and 8 can be rewritten in exponent form as 7^1 or 8^1, and that variables such as x and y can also be rewritten in exponent form as x^1 and y^1.
- Encourage students to investigate the relationship between the exact values that they will get when evaluating exponents without calculators, such as $3^{-3} = \frac{1}{27}$, and the approximate values that they will get for answers when they use a calculator to evaluate the same exponent, such as $3^{-3} = 0.037$.
- For **questions 1** and **2**, have students communicate which rule(s) they used to simplify the question.
- In **questions 2g**) and **h**), some students will write reciprocals as $\frac{1}{\text{quantity}}$. To eliminate the negative in the exponent, encourage students to simply take the reciprocal by writing the fraction upside down.
- For **question 5**, review adding, subtracting, and multiplying fractions.
- In **question 11**, students have to make connections to solve the equation. They must recognize that the fifth root is the opposite of the fifth power.

Technology Suggestions

- Scientific calculators can vary considerably in the format of the keys and the sequence of keystrokes needed to perform a given calculation. Students must take the time to become familiar with their own calculators, and be confident of entering keystrokes in the order appropriate for their particular models. Consider coaching students to estimate expected answers using mental math, at least for the simpler expressions.
- For the **Example**, review how to enter a complicated expression into a scientific and/or graphing calculator. Enter the entire expression before pressing the = key. Some students like to evaluate partial calculations, and then re-enter a rounded version into the calculator. This results in a cumulative error, as well as taking up a lot of time.

- Consider an alternative solution to the **Example** using a computer algebra system (CAS) such as the one on the TI-Nspire™ CAS calculator. Enter the expression $\frac{a^5b^3c^4}{\sqrt{a^4b^2}}$ as a function $f1(a, b, c)$. Then, evaluate the function for $a = 3$, $b = 4$, and $c = -2$. Use the **Expand** function to simplify the expression, and store it as function $f2$. Finally, evaluate $f2$ using the values given in the question. See **T4–1 How to Do Section 4.1 Example Using TI-Nspire™ CAS** for instructions. Ensure that you have the CAS version of TI-Nspire™. The calculator is also sold in a non-CAS (blue) version. Point this out to students, especially those considering the purchase of a TI-Nspire™.
- For **question 2**, students can check answers using a CAS.
- For **question 4**, some students may not be familiar with the "y^x notation" or the "^ notation" for evaluating exponents using a calculator. Review these functions with students.
- For **question 4**, ensure that students enter the entire expression before pressing the = key. Coach students to recognize when they need brackets to force a particular order of operations.
- For **question 14**, the **Lists** on a graphing calculator is an alternative method for the tables in parts **a**) and **d**). Enter the numbers in list **L1**. Then, enter the formulas at the top of lists **L2** and **L3**.
- For **question 15**, you can use the **Solve** function in a CAS to solve the formula for the volume of a cone for the radius. This is one of the more powerful features of a CAS. Define the result as a function, and use it to evaluate the function for any desired values.
- For **question 16**, you can use the **Solve** function in a CAS to solve the formula for the pressure-volume relation in a gas. Define the result as a function, and use it to evaluate the function for any desired values.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	16
Reasoning and Proving	9
Reflecting	9
Selecting Tools and Computational Strategies	11
Connecting	7, 10, 11
Representing	10, 12, 15
Communicating	3, 7–9, 13, 14

ONGOING ASSESSMENT

- Check that students understand how to apply each of the exponent rules by assessing with a question such as the following: Simplify the following. For each step, explain the rule being applied: $\left(x^{-\frac{1}{2}}\right)^3\left(x^{\frac{1}{3}}\right)^2$. You may wish to use **A–4 Selecting Tools and Computational Strategies** to assist you.