4.2 Solving Exponential Equations Graphically

Study Guide and Exercise Book Pages
64 to 67

Tools
- grid paper
- graphing calculator
- computer
- *Fathom*™
- *The Geometer's Sketchpad*®

Related Resources
- G–1 Grid Paper
- T–3 *Fathom*™
- T4–2 How to Do Section 4.2 #4 Using *The Geometer's Sketchpad*®
- T4–3 How to Do Section 4.2 #7 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS
- A–7 Communicating

Key Terms
- exponential function
- exponential growth
- exponential decay
- horizontal asymptote

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Have students make tables of values and graphs for $y = ab^x$, letting i) $a = 1$ and $b = 3$, ii) $a = 2$ and $b = 3$, iii) $a = 1$ and $b = 0.5$, and iv) $a = 3$ and $b = 0.5$. Compare the four graphs and help students to summarize that the value of a affects the y-intercept and the value of b determines whether the graph is exponentially growing or decaying.
- Discuss the meaning of the word *asymptote*.
- Have students add these concepts to their chapter reference sheets.
- You may wish to have students draw more graphs for exponential growth and decay by varying the b-values. Have students notice that when $b > 1$, as b increases, the slope of the graph also increases.
- Brainstorm with students examples of exponential growth and decay in real life.

Example

- Some students may have trouble remembering that $y = 8$ represents a horizontal line.
- Review with students how to clear the RAM from a graphing calculator.
  – Press **2nd, +** for the **MEMORY** menu.
  – Select **7:Reset**.
  – From the **RAM** menu, select **1:All RAM…** and then **2:Reset**.
- Review how to use the **Intersect** operation on a graphing calculator to find the point of intersection of two curves.

Questions

- When students are solving exponential equations graphically, it may be helpful for them to use a ruler to determine a y-value for a given x-value, or an x-value for a given y-value.
- Remind students about the order of operations. For expressions such as $3(2^x)$, they have to evaluate the exponent before they multiply by 3, and $3(2^x)$ is not equal to 6^x.
- In **question 1**, have students draw tables of values. Some students may only use positive x-values in their table. Have them add some negative values. After students have drawn the graphs by making a table of values and have made comparisons between the graphs, you may wish to encourage them to draw future graphs without tables of values. Help students redraw the graphs from **question 1** by just plotting main features: the asymptote at $y = 0$, the y-intercepts at $(0, 1)$, and a second point at $(1, y)$.
- For **question 2**, have students write 0.8 as a fraction. Otherwise, some students may not make the connection among the three graphs.
- For **question 3a**), have students compare the domain, range, asymptote, y-intercept, x-intercept, and y-values when $x = 1$ and $x = -1$.
- Review exponent rules learned in the previous section for **question 3b**).
- **Questions 5** and **6** help students to think backward and to visualize their answers.

Common Errors

- When sketching graphs of exponential functions, some students may show the curve touching or intersecting the x-axis.

R_x Have students practise drawing their graphs using a sharp pencil. Have them carefully get the curve to "hug," but never quite touch, the asymptote.

Differentiated Instruction

- Add the **Key Concepts** to the **word wall**.
- Construct a **Frayer model** on exponential growth and decay. Use the **Example** and **Key Concepts** from this section for definitions, characteristics, examples, and non-examples of exponential growth and decay.

- To answer **question 7**, it will be important for students to be familiar with the key points of an exponential function. For example, if the y-intercept is 1 and the graph is increasing, then $a = 1$ and $b = y$-value when $x = 1$ in the equation $y = ab^x$.
- To answer **question 8**, students must be familiar with the key points of an exponential function. For example, if the y-intercept is 1 and the graph is decreasing, then $a = 1$ and $b = \frac{1}{y\text{-value}}$ when $x = -1$ (or $b = y$-value when $x = 1$) in the equation $y = ab^x$.
- **Questions 7** and **8** may be used to assess students' understanding of increasing and decreasing exponential functions.
- For **question 9**, refer students to the **Example**. If they are using technology to answer **9b)**, have students adjust their window so that they can see the line $y = \frac{1}{16}$ clearly on their screen.
- **Question 10a)** highlights the importance of sketching graphs that are as accurate as possible. You may wish to have students compete in drawing as accurate a graph as possible and approximating the solution to the best of their ability. Whoever is closest to the exact answer receives a prize.
- To answer **questions 12b)** and **c)**, have students substitute $n = 0$. **Question 12e)** is a valuable discussion question that will help students prepare for similar application questions.
- For **question 13**, have students create a table of values for how many people know the fact on each day. When graphing the equation using technology, it will be helpful for students to determine the window by checking the x-values and the y-values in the tables of values.
- Students typically have more trouble with writing equations for decreasing exponential functions, such as in **question 14**. Some students will write an equation for the car as its value decreases *to* 20% each year. Discuss with students the difference between decreasing *by* 20% and decreasing *to* 20%. Also, is it logical that a car decreases *to* 20% of its value every year?
- In **question 14d)**, make students aware that the question asks for the amount the car depreciates during the fifth year, not during the first five years.
- For **question 15**, students may need guidance as they write the intervals of increase/decrease and as they are asked to draw a function that has been reflected in the x-axis.
- Challenge students by encouraging them to explain in detail the graphs of the exponential functions in **question 15**.

Technology Suggestions

- You can use *The Geometer's Sketchpad*® as a possible Method 3 for the **Example**. Plot the functions $y = 2^x$ and $y = 8$. Plot a point on the first function. Measure the coordinates of the point. Move the point until it reaches the intersection. If students have been given *The Geometer's Sketchpad*® (GSP) for home use, the homework questions can be expanded to include this method of solution. Go to **www.mcgrawhill.ca/books/mct12** and follow the links to this GSP sketch.
- **Question 3** is a good candidate for a solution using *The Geometer`s Sketchpad*®.

- **Question 4** mentions the use of sliders. Use **T–3 *Fathom*™** if needed to support this activity. Sliders are also available in *The Geometer`s Sketchpad*®. Click the **Custom Tools** button, and choose **Sliders.** Then, choose an appropriate slider format. You can increase the sensitivity of a slider by increasing its length. See **T4–2 How to Do Section 4.2 #4 Using *The Geometer's Sketchpad*®** for detailed instructions. Go to **www.mcgrawhill.ca/books/mct12** and follow the links to this GSP sketch.
- For **question 7**, exponential regression on a graphing calculator can be used to generate the exponential function that goes through the three points. Enter the coordinates of the points in the Lists. Press **STAT**. From the **CALC** menu, select **0:ExpReg**. See **T4–3 How to Do Section 4.2 #7 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS** for detailed instructions. Alternatively, you can use two sliders in *The Geometer`s Sketchpad*® to model a general exponential function. Plot the three points given, and adjust the sliders to match the function to the points.
- **Question 8** can also be completed using exponential regression as in **question 7** or sliders for a dynamic geometry solution.
- You can use *The Geometer`s Sketchpad*® to solve and/or check any of **questions 10** to **16**. Consider specifically assigning some to be solved dynamically.
- Students may make an error typing the function from **question 16** into the graphing calculator. Tell students that they must put brackets around the exponent.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Ongoing Assessment

- Check that students understand the key features of increasing and decreasing exponential graphs. Questions such as **1** and **2** may be useful for assessing students' understanding. You may wish to use **A–7 Communicating** to assist you.

Process Expectation	Selected Questions
Problem Solving	12–15, 17
Reasoning and Proving	17
Reflecting	1, 2, 4, 12, 16, 17
Selecting Tools and Computational Strategies	10
Connecting	7, 8, 12, 16, 17
Representing	3, 16
Communicating	1–3, 5, 6