4.3 Solving Exponential Equations Numerically

Study Guide and Exercise Book Pages
68 to 70

Tools
- grid paper
- scientific calculator
- graphing calculator
- computer with *The Geometer's Sketchpad®*

Related Resources
- G–1 Grid Paper
- T4–4 How to Do Section 4.3 #6 Using *The Geometer's Sketchpad®*
- A–7 Communicating

Key Terms
- exponential equation

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Review with students the difference between an exponential expression and an exponential equation. Stress that equations (usually) have solutions and expressions can only be simplified.
- Review with students the difference between exact answers and approximate answers; for example, $3^{-3} = \frac{1}{27}$, but the approximate value that they will get from a scientific calculator is 0.037. Since students tend to want to use a calculator for every step of a question, it may be useful to tell students that "find an exact answer" means that they should not use a calculator.
- Have students add an example of their own, similar to the given **Example**, to their chapter reference sheets.

Example

- You may need to review with students how to solve equations in one variable.
- Have students notice that the y-value from the point of intersection on the calculator is equivalent to the $\frac{1}{8}$ answer in part **b**).

Questions

- Review the fact that $2^{2x} = (2^2)^x = (2^x)^2$.
- Students may need to review how to rewrite powers, such as $8^4 = (2^3)^4 = 2^{12}$.
- For **question 1**, remind the students that an expression such as $7^0 = 1$, except that 0^0 is not defined.
- For **questions 2e**) and **f**), students may need the hint that they have to rewrite both sides of the equation, since this is the first occurrence of this type of question.
- **Question 3** is meant to be done with a scientific calculator, not a graphing calculator. Help students to find an approximate solution that they know is accurate to one decimal place. As an extension, have them find an answer that is accurate to two decimal places. Discuss why students cannot complete these questions algebraically at this time.
- Students may become confused evaluating **question 3c**) if they do not cancel the negative signs as their first step.
- After having completed **question 3**, it may be valuable to discuss why equations such as $3^x = 4$ do not have a solution. Graphing both sides can visually show why there is no solution.
- Before completing **question 4**, remind students that it is possible to evaluate numbers that are fractions with negative integral exponents by flipping the fractions. For example, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2$. It may be very helpful for students to do a few examples showing the long method with all of the steps for them to understand that $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2$.

Common Errors

- Students may have difficulty recognizing that a pair of numbers are related. For example, students may not know that 8 and 32 are both powers of 2.

R_x You may wish to create lists of special numbers on chart paper. List powers of 2, 3, 4, 5, 6, and 7 so that students can see them daily and recognize them more quickly. You may want to create flash cards with these special numbers. For example, when you hold up a card with the number 64 on it, students should recognize that it is equal to 2^6, 4^3, and 8^2.

Differentiated Instruction

- Write equations such as **question 2** on cue cards for students. Using the **inside/outside circle,** one student shares his or her equation and the other describes how to complete the first step in the solution. Rotate the outside circle until each pair has met and exchanged questions. Have students return to their seats and complete the solution.

For example,

$$\left(\frac{3}{5}\right)^{-2} = \frac{1}{\left(\frac{3}{5}\right)^2}$$
$$= \frac{1}{\frac{9}{25}}$$
$$= 1 \div \frac{9}{25}$$
$$= 1 \times \frac{25}{9}$$
$$= \frac{25}{9}$$

- For **question 5**, refer students to the **Example**.
- For **question 8**, it may be necessary to do a practice question to help students understand how to create a function that models half-life. Review the relationship between exponential growth and exponential decay; the relationship between an exponential equation with base 2 and an exponential equation with base $\left(\frac{1}{2}\right)$; and $2^{-1} = \left(\frac{1}{2}\right)$. You may wish to have students research the term *half-life* on the Internet.
- Before attempting **question 8c**), students must be introduced to the extra step that is required to solve questions such as $2(5)^x = 250$. Students must isolate the power first by dividing both sides by 2. Encourage students to use systematic trial with a scientific calculator, and check their answer with a graphing calculator.
- For **question 10**, remind students that both sides of the exponential equation have to be written as powers of the base number 5.
- Students may need help when they are working on questions such as **question 11**, in which they have to rewrite 2^{2x} as $(2^x)^2$ in order to solve the exponential equation.
- For **question 11**, it may be necessary to review with students how to factor trinomials. Remind students that when the quadratic equation is solved, it is necessary to still solve the resulting exponential equations.
- For **question 12**, students may need to review the steps needed to enter the function $y = 0.1\left(2^{\frac{x}{2}} + 2^{-\frac{x}{2}}\right)$ using the **Y=** editor of a graphing calculator.

Technology Suggestions

- Consider using *The Geometer's Sketchpad*® for parts **c**) and **d**) of the **Example**. You can dynamically generate a table of values as a point is moved along one of the functions using the **Tabulate** feature from the **Graph** menu. Consider an alternative solution using the **Solve** function on a CAS.
- **Questions 1** to **10** can be solved or checked using a CAS.
- For **questions 6** and **7**, consider an alternative solution using *The Geometer's Sketchpad*® (GSP) to simultaneously work with a graph and a table of values. Graph both sides of the equation. Plot a point on the exponential function, and tabulate the coordinates of the point. Then, move the point in small increments, and tabulate the coordinates after each increment. See **T4–4 How to Do Section 4.3 #6 Using *The Geometer's Sketchpad*®** for detailed instructions. Go to **www.mcgrawhill.ca/books/mct12** and follow the links to this GSP sketch.
- Part **d**) of **question 8** is a good candidate for the **Solve** function of a CAS.

- Use *The Geometer's Sketchpad®* for a dynamic solution to **question 12**, especially locating the minimum point. Plot a point on the curve, measure the coordinates, and then move it until the y-value is a minimum.

ONGOING ASSESSMENT

- Ensure that students recognize numbers that have a common base, and that they can apply the proper exponent rule at each step in a solution. For example, use the following to assess students' understanding: Solve $\left(\frac{1}{4}\right)^{x+2} = \left(\sqrt{32}\right)^{2x-6}$ algebraically. Communicate clearly throughout your solution. You may wish to use **A–7 Communicating** to assist you.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	8, 11, 12
Reasoning and Proving	3
Reflecting	8, 11, 12
Selecting Tools and Computational Strategies	2, 11, 12
Connecting	4, 8, 11, 12
Representing	2, 12
Communicating	5, 11, 12