4.4 Points of Intersection

Study Guide and Exercise Book Pages
71 to 73

Tools
- grid paper
- graphing calculator
- computer algebra system

Related Resources
- G–1 Grid Paper
- T4–5 How to Do Section 4.4 Example Using TI-Nspire™ CAS

Teaching Suggestions

Key Concepts

- For the key concept "An exponential equation can be solved graphically by determining the x-coordinate of the point of intersection of the two corresponding exponential functions," have students explain the meaning in their own words, using sketches. Discuss what has been learned in previous sections.

Example

- Discuss that *any* equation may be solved by graphing the left-hand and right-hand side, and finding the x-coordinate of the point of intersection. Demonstrate by solving an equation other than an exponential equation.
- Assign different groups a different line in the solution to part **b**). Have the groups solve their equation by graphing. What do they notice? Note that the groups' graphs look different, but the intersection point is always at $x = -2$.
- Although the **Example** allows students to use technology when graphing, review how to graph each side of the original question by hand. Review the concept of a horizontal asymptote and remind students that although it appears that the graph of $y = 2^{-x}$ touches the x-axis, the graph does not cross the x-axis and does not become negative. Since $y = 2^{-x} = \left(\frac{1}{2}\right)^x$, students can draw the graph without technology. Point out that $y = 2^{-x}$ is a reflection of $y = 2^x$ in the y-axis, thereby helping students to connect their learning to that of Chapter 2. Help them draw $y = 4^{x+3}$ by reviewing the concept that $x + 3$ will shift the original graph 3 units to the left.

Questions

- Review with students how to enter the equations of exponential functions using the **Y=** editor of a graphing calculator. If there is more than x in the exponent, they must put brackets around the exponent.
- Remind students that sometimes exponential equations have to be solved by dividing both sides by the same number:

$$3(2^{x+1}) = 48$$
$$\frac{3(2^{x+1})}{3} = \frac{48}{3}$$
$$2^{x+1} = 16$$

- Encourage students to answer **question 1** both algebraically and graphically.
- **Question 3** gives the result $0 = -13$. For most students, the meaning of this will only become clear when the graphs are compared. In fact, drawing graphs of one of the intermediate steps ($2^{4x+6} = 2^{4x-7}$) will be useful, since students can clearly observe that these graphs are identical except for a horizontal shift (therefore, there is no intersection point).
- Have students make up an equation that has no solution. Have students start by drawing two functions that have no intersection point.

Common Errors
- Some students may still have trouble applying the various exponent laws when manipulating a question.

R_x Having the laws posted in the classroom will help. You may wish to start each lesson with review questions that involve each of the exponent laws.

Differentiated Instruction
- Have students use **cooperative task groups** to design a mini-lesson on how to teach **question 4**, or a similar question, to absent students. Have groups think of how they would best present their lesson, and what questions they would ask the class to prompt for answers. Have groups present.

- For **question 6**, review common factoring, and then review common factoring when the common factor is in exponential form, such as

$$\begin{aligned} 3^{x+2} - 3^x &= 3^x(3^2 - 1) \\ &= 3^x(9 - 1) \\ &= 3^x(8) \end{aligned}$$

- Have students graph **question 8** by hand to review graphing straight lines.
- Have students solve **questions 8** and **10** algebraically and graphically.
- Review percent increase and percent decrease. Have students state the yearly percent increase for **question 11** and the yearly percent decrease for **question 12**. This will help students to complete **questions 13** and **14**.
- For **question 13**, review with students the meaning of the units kilopascals (kPa).
- To summarize this lesson, assign a question similar to the following: Do the graphs of each of the following pairs of functions intersect? If they do, find the intersection point algebraically. If they do not, explain why.
  a) $y = 7^{x-2}$ and $y = 49^{x+2}$
  b) $y = 3(6^{4x+2})$ and $y = 108$ (Students must divide by 3 first.)
  c) $y = 8^{x-1}$ and $y = 4^{\frac{3x}{2} - \frac{3}{2}}$ (Graphs are identical and yield an answer of $0 = 0$.)

Technology Suggestions

- Use *The Geometer's Sketchpad*® as an alternative for part **a**) of the **Example**. You can generate a table of values as a point is moved along one of the functions using the **Tabulate** feature from the **Graph** menu.
- Consider solving the **Example** using TI-Nspire™ CAS. See **T4–5 How to Do Section 4.4 Example Using TI-Nspire™ CAS** for detailed instructions.
- For **questions 1** and **2**, consider checking one of these questions using *The Geometer's Sketchpad*® and the other using TI-Nspire™ CAS.
- For **questions 4** to **7**, consider solving at least one question using *The Geometer's Sketchpad*® and one using TI-Nspire™ CAS.
- **Question 8** is an opportunity to use the **Solve** function on a CAS and graph in *The Geometer's Sketchpad*®.
- For **questions 9** and **10**, consider checking one of these using *The Geometer's Sketchpad*® and the other using TI-Nspire™ CAS.
- **Questions 11** and **12** can also be done with a graphing calculator. Review how to create a scatter plot on a graphing calculator (be sure to clear the RAM first) and use the exponential regression function.
- Remind students that they can answer **question 11c**) by adding the equation $y = 850$ to the graphing calculator, and then finding an intersection point.
- Consider an alternative solution for **question 13b**) using TI-Nspire™ CAS.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	11–14
Reasoning and Proving	13, 14
Reflecting	8
Selecting Tools and Computational Strategies	9, 14
Connecting	2, 6, 10, 13, 14
Representing	9
Communicating	2, 4–7

Ongoing Assessment

- You may wish to ask students questions such as "What questions did you find easy? difficult? Why?" to find out which concepts students need more practice with.