4.5 Logarithms

Study Guide and Exercise Book Pages
74 to 76

Tools
- grid paper
- scientific calculator
- graphing calculator

Related Resources
- G–1 Grid Paper
- T4–6 How to Do Section 4.5 #4 Using a Scientific Calculator and a Graphing Calculator
- A–2 Reasoning and Proving
- A–7 Communicating

Key Terms
- inverse
- logarithmic function
- common logarithm

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12.**

Teaching Suggestions

Key Concepts

- In this section, students are introduced to the term *logarithm* and the logarithmic function as the inverse of an exponential function.
- Describe an inverse: the reflection of a function in the line $y = x$. Choose a few points on an exponential function and show how students can reverse the coordinates of the points to draw the inverse function.
- Have students state the domain and range of the exponential function and notice that the domain of the logarithmic function equals the range of the exponential function, and vice versa.
- You may wish to discuss why exponential functions are defined only for positive numbers not equal to 1.
- Alert students to the fact that the **LOG** calculator button represents $\log_{10}$.
- Have students add the rules and diagrams to their chapter reference sheets.

Example

- It is easier for students to convert from logarithmic to exponential form. Students convert logarithmic equations to exponential equations, and convert exponential equations to logarithmic equations.
- Part **e**) of the **Example** models how to evaluate a logarithm graphically. Have students write their own version and provide a guide for how to do this.

Questions

- You may wish to review how to find the inverse of a function algebraically.
- Review domains and ranges for a variety of logarithmic graphs.
- Encourage students to be creative when solving logarithmic expressions such as in **questions 8, 9**, and **13**, and to solve the logarithmic examples using different methods (e.g., calculator, exponent rules, graphing).
- Use **questions 2** and **3** to review some of the exponent laws.
- For **question 4a**), have students say "2 to what exponent equals 128?" Verbalizing questions will help students to memorize how a logarithm works.
- For **question 4**, you may wish to review powers of 2, 3, 4, 5, 6, and 7, so students will recognize that, for example, 216 is a power of 6.
- Use **question 6** to assess students' understanding of a logarithm. A complete solution with good communication may include not only the exponential form, but also graphs of the exponential form.
- For **question 8**, have students review the **Key Concepts** to find that log is equivalent to $\log_{10}$. Although students may recognize that **question 8** can be done with their scientific calculator, have students do it without a calculator.
- Students will find **question 10** easier if they plot points on the exponential function and find the inverse of the points, rather than finding the inverse of the equation algebraically.
- For **question 11**, remind students to visually relate the graph of a logarithmic function to the concept that it is not possible to find the logarithm of the number zero or the logarithm of a negative number. Have students compare the values in each of the columns and comment.

Common Errors

- Some students do not understand the concept of a logarithm and must rely on memorization to answer questions.

R_x Reinforce the relationship between the logarithmic function and its exponential inverse. Have students sketch graphs for logarithmic functions and describe the relationship between the logarithmic graph and its inverse, the exponential graph.

Differentiated Instruction

- Use **concept attainment** to teach this section. Use a chart with the headings "Logarithm" and "Non-Logarithm" that lists examples and non-examples of logarithms under the appropriate heading. Have students identify and define the concept of logarithms. Then, apply their defined concept to generate their own examples and non-examples.
- Add new rules to the **word wall.**

- For **question 12,** some students will need help getting started. Remind them that $x = 2$ and $y = 8$. This question provides an opportunity to assess students' reasoning and communicating skills.
- **Question 13** connects material learned in previous sections to the concept of a logarithm. Refer students to part **e**) of the **Example**.
- For **question 14,** review the key features of a logarithmic graph. Help students to first draw an asymptote at $x = 0$, then the key point (1, 0) and a second point at (7, 1). With their graph drawn, students may also want to estimate an answer to part **c**) before using a graphing calculator.
- **Question 15** shows students that the **LN** button performs similarly to the **LOG** button. It also introduces the change of base formula. Have students use this new rule to evaluate the answers to **question 4** again.
- For **question 16,** review the parameters in the formula for compound interest and how to determine the parameters i and n when given information about the annual rate of interest and the number of compounding periods per year. Students will require the change of base formula from **question 15e**) or will need to use systematic trial to answer **question 16b**).
- Have students research real-life data from Statistics Canada that can be modelled by the graphs of exponential functions.

Technology Suggestions

- For part **e**) of the **Example,** show students how to use the **LOG** key on a scientific or graphing calculator to evaluate a logarithm for any given base. Although this must be done by rote at this stage, it sets the stage for **question 15,** as well as the change of base formula in section 4.6. Ensure students learn the proper sequence of keystrokes for the evaluation of logarithms on their own model of calculator. You may wish to point out the other kind of logarithm, the natural logarithm, that is available on the calculator.
- Students can verify answers for **question 4** using the **LOG** key on a scientific or graphing calculator. See **T4–6 How to Do Section 4.5 #4 Using a Scientific Calculator and a Graphing Calculator** for detailed instructions.
- For **question 13b**), students can verify using the **LOG** key.

Ongoing Assessment

- You may wish to use **question 4** to assess students' understanding of logarithms. This question requires an understanding of exponents and requires students to use their heads, not their calculator. You may wish to use **A–2 Reasoning and Proving** and **A–7 Communicating** to assist you.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	16
Reasoning and Proving	2
Reflecting	6, 12, 14–16
Selecting Tools and Computational Strategies	8
Connecting	9, 12, 14, 15
Representing	16
Communicating	2, 5–7, 9, 12–15