4.6 Solving Problems Using Logarithms

Study Guide and Exercise Book Pages
77 to 79

Tools
- grid paper
- scientific calculator
- graphing calculator
- markers and chart paper

Related Resources
- G–1 Grid Paper
- BLM 4–3 Chapter 4 Review
- BLM 4–4 Chapter 4 Practice Test
- BLM 4–5 Chapter 4 Case Study

Teaching Suggestions

Key Concepts

- The change of base formula will enable students to complete all logarithms using their calculator.
- You may wish to explain to students that logarithmic scales are useful for making easy comparisons in relationships in which the dependent variable changes over several orders of magnitude. The pH scale introduced in **question 9** is a very common example that students may have encountered in Chemistry class. The key feature of this logarithmic scale is that an increase of 1 on the pH scale corresponds to a tenfold increase in hydronium concentration.

Example

- You may wish to review how to estimate the answer for part **a**) without a calculator: graph $y = 15^x$ and $y = 20$, and estimate where the graphs intersect.
- Highlight the key points on the graph in part **c**) with students: the asymptote at $x = 0$ and the points (1, 0) and (4, 1). Remind students that they could have drawn the graph by hand by recognizing it is the inverse of the exponential function $y = 4^x$, and taking the inverse of a few points from this graph.
- As an extension, have students solve $4 = 2(1.04^t)$. Assist students who incorrectly multiply 2×1.04.

Questions

- Review with students the first step in solving questions such as **question 3d**).
- For **questions 4c**) and **5b**), press **TRACE**, type a value for x, and press **ENTER**. The corresponding y-value will be displayed. For **question 4c**), have students explain why the values of x are 10 and 100. Have students guess the answer to **question 5b**) before they find it on the graphing calculator.
- For **questions 4** and **5**, encourage students to write the equations of other logarithmic functions and to investigate their graphs. Review domains and ranges of logarithmic functions.
- Review substituting into formulas.
- For **questions 6** to **12**, have students work together in groups, assigning one of the questions to each group. Have groups create a solution for their assigned question using markers and large sheets of paper. They could then present the solution to their classmates. You may wish to mark the group presentations using a rubric.
- Encourage students to research the different applications of logarithms in **questions 6** to **12** on the Internet, and to find other applications of logarithms in real-life situations.
- For **questions 7** and **8**, review how to determine the parameters i and n in the formula for compound interest when given information about the annual rate of interest and the number of compounding periods per year.
- Remind students to use the correct units when they find an answer to **question 8c**).
- Before beginning **question 9**, have students name some other substances that are acidic or alkaline.

Common Errors

- Students often have difficulty with the logarithm problems that involve sound levels and the Richter scale.

R_x Discuss thoroughly with students the meaning of the variables found in each equation. It would be useful to create a sound intensity comparison chart with "no sound" having a sound intensity of 10^{-12} W/m^2 and a loudness of 0 dB, all the way up to jet aircraft having a sound intensity of 10^4 W/m^2 and a loudness of 160 dB. A similar chart can be created for the Richter scale, with headings True Intensity of Earthquake, Richter Scale Magnitude, and Event in History.

Differentiated Instruction

- Use **cooperative task groups** to complete the application problems in this section.
- Add new rules and formulas to the **word wall**.

- **Questions 10** and **11** provide another commonly encountered logarithmic scale, the decibel scale, which is used for comparing sound intensities. This may be of interest to students considering a future in the recording industry. Note that the given intensities are approximations and can vary depending on distance from the sound and variations in the source of the sound.
- Students may be confused when attempting to answer **question 10**. Students are asked to find the value of $\frac{I_2}{I_1}$; they do not have to find I_2 or I_1.
- The Richter scale, introduced in **question 12**, provides a means to compare the intensities of earthquakes. The key feature of this logarithmic scale is that an increase of x on the Richter scale corresponds to a 10^x increase in intensity.
- In **question 12a**), students are asked to find the value of the quantity $\frac{I}{I_0}$; they do not have to find I or I_0.
- Assign **questions 13** to **15** to challenge students.
- For **question 15**, ensure that students check their answer; otherwise, they may not recognize the inadmissible value.
- To help review logarithms, a BingoLogarithm game could be set up. On a bingo card, students mark the answers to questions about logarithms that you ask. Another way to review would be to put the answers to questions in each corner of the classroom. Students go to the corner that has the correct answer to the question that you read out.
- You may wish to use **BLM 4–3 Chapter 4 Review** to help students identify areas in which they need to further their understanding.
- Provide students with **BLM 4–4 Chapter 4 Practice Test** to prepare them for the chapter test.

Case Study

- You may wish to have students complete **BLM 4–5 Chapter 4 Case Study**, which incorporates the learning from Chapter 4.
- For **question 1a**), students should be able to connect the fact that the tenfold difference in hydronium ion concentration means that the base is 10.
- You may point out that the pH scale is more convenient for representing small concentrations of ions.
- For **question 1c**), students need to substitute for the pH and solve for the concentration by writing the equation in exponential form.
- Under optimum conditions, many bacteria can divide every 20 min. Even with a more conservative division cycle of 2 h, the theoretical resulting concentration of bacteria in **question 2b**) is very large. In reality, there are several controlling factors, such as limitations of food and space, production of waste, and immune responses, to help keep the population in check.
- **Question 2c**) requires students to determine the initial concentration of *E. coli* bacteria present when the patient arrived at the clinic. To do this, students should realize they can let 5000 represent the final concentration and rearrange the equation to solve for the initial quantity.
- Based on the results of this test, the laboratory would likely be asked to conduct other tests to determine the cause of the patient's discomfort.

Ongoing Assessment

- You may wish to assess students' comprehension of this section by asking questions such as "How often did you check the Example to help you with questions? For which questions?"

Summative Assessment

- You may wish to use the chapter test that you can find in the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12.**

Technology Suggestions

- The **Solve** function on a CAS can be used to check answers to **question 3**.
- For **questions 4** and **5**, consider using *The Geometer's Sketchpad*® as an alternative method. Plot a point on the graph, and measure the coordinates. Watch the coordinates change dynamically as the point is moved along the function.
- Consider solving one or more of **questions 6** to **13** using a CAS.
- **Question 15** is a good candidate for an alternative solution using a CAS.
- For **question 16**, a lab could be booked for the class. This would provide an opportunity for students to prepare a report to their classmates electronically.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	6–13, 15
Reasoning and Proving	14
Reflecting	4, 5, 7, 8
Selecting Tools and Computational Strategies	15
Connecting	11, 12, 14, 15
Representing	14
Communicating	5, 13, 14, 16