5.1 Identifying Polynomial Functions

Study Guide and Exercise Book Pages
81 to 84

Tools
- grid paper
- graphing calculator
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 5–1 Chapter 5 Prerequisite Skills
- BLM 5–2 Chapter 5 Self-Assessment Checklist
- T5–1 How to Do Section 5.1 #4d) and h) Using *The Geometer's Sketchpad®*
- A–1 Problem Solving
- A–2 Reasoning and Proving
- A–3 Reflecting
- A–4 Selecting Tools and Computational Strategies
- A–5 Connecting
- A–6 Representing
- A–7 Communicating

Key Terms
- polynomial expression
- degree
- leading coefficient
- constant term
- relation
- function
- vertical line test
- domain
- range
- polynomial function
- constant function

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

- Before students begin the chapter, you may wish to have them complete **BLM 5–1 Chapter 5 Prerequisite Skills** to activate their prior skills.
- Give students **BLM 5–2 Chapter 5 Self-Assessment Checklist** to keep track of their skills and knowledge. Have students return to it throughout the chapter.
- For further teacher support for this chapter, go to the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Key Concepts

- Have students create a reference sheet to which they can add the terminology learned in this lesson.
- Describe a polynomial expression by writing a sample expression on the board, such as $4x^3 + 7x^2 - 3x + 5$. Discuss key terms, such as *degree*, *leading coefficient*, and *constant term*, and have students identify each element in the sample expression.
- Make students aware that expressions such as $2x^3 + 6x$ are also polynomial expressions, but with terms "missing." Ask students to identify the constant term in this expression.
- Have students use proper terminology when discussing polynomials. For example, "whereas $4x^3 + 7x^2 - 3x + 5$ is a polynomial expression, $y = 4x^3 + 7x^2 - 3x + 5$ is a polynomial function."
- You may wish to have students graph sample polynomial functions on a graphing calculator or in *The Geometer's Sketchpad®*, so that they can identify key features of a polynomial graph. Graphs of polynomial functions always have a domain of $\{x \in \mathbb{R}\}$, and they have a number of "bumps," depending on the degree of the equation.
- Review the difference between a relation and a function. Students typically understand what a function is when the vertical line test is introduced. Students tend to have difficulty when given sets of points or mapping diagrams, and then asked to determine whether they are relations or functions.
- Define the concept of a function, using concept attainment. This is done by writing "Function" and "Non-function" on the board, and then writing examples and non-examples of functions under the appropriate heading (or use cards that you have prepared in advance). Discuss with the class what makes the examples in the function column similar to each other. What makes them different from the non-examples? Then, create three more examples of each with the class's help. You could extend this by having students create their own examples. Students could also exchange cards with others, and then place them in the appropriate column on the board.

Example

- Graph each function on a graphing calculator or *The Geometer's Sketchpad®* to help students identify the type of function by its shape.
- You may wish to have students note that each relation in the **Example** is, in fact, a function. This is true not only because they pass the vertical line test, but also because they can be expressed in the form of y on one side of the equal sign and a single expression on the other.

Common Errors

- Many students have difficulty understanding the wording of the definition of a function.

R_x Allow students to quote the vertical line test when determining whether a given relation is a function. The vertical line test seems to be easily understood by all students, perhaps due to its visual nature.

Differentiated Instruction

- Use **concept attainment** to define a polynomial function. Provide examples and non-examples of polynomial functions for students, and have them create some of their own. Have students sketch graphs of the examples and non-examples using the graphing calculator.
- Construct a **word wall** of terminology encountered in this chapter

Common Errors

- Some students may think that $C(r)$ means $C \times r$.

R_x Review function notation with students.

Questions

- This section provides an opportunity to review the different types of functions, such as the sine function, the cosine function, and the exponential function, that were studied in previous chapters.
- For **question 1e**), have students explain why the answer is zero.
- When determining if relations are functions in **question 4**, encourage students to use technology to create visual representations of the functions.
- Students may have difficulty with **questions 4d**) and **h**). Discuss with them what methods are available to prove whether a given relation is a function. They may say that an easy method is to look at the graph on a graphing calculator and see if it passes the vertical line test. However, to accomplish this, the equation must be expressed in the form $y =$. If students are able to put the equation into this form, they will notice $y = a \pm \sqrt{\quad}$, which should alert them to the fact that there are two functions. This may be the best method because students will probably have difficulty drawing a mapping diagram or an accurate table of values for these two question parts.
- For **question 6**, remind students that if there is an x-value that corresponds to two different y-values, then the relation is *not* a function.
- For help with **question 8**, refer students to the **Example**.
- To help students with **question 9**, refer them to the phrase "whole number" in the first bullet of the **Key Concepts**.
- **Question 10** is useful to review work learned in grades 10 and 11.
- **Questions 11** and **12** help to prepare students for a discussion on end behaviours, which is taught in the next section. If students are having difficulty understanding the concept of the end behaviour of a function, it might be a good idea to have them determine the values of y by substituting large positive and negative values for x in the function.
- In **questions 11** to **13**, students may express themselves in different ways. For example, they may say
  - when x increases, y increases
  - when x becomes larger, y becomes larger
  - when x approaches infinity, y approaches infinity

  All of these are good ways to communicate their answer to the first part of **11c**).
- As an extension to **question 14**, have students graph a simple rational function and a square root function on their graphing calculator. Ask them how these functions are different from those in **questions 11** to **13**.
- After completing **question 14** (and the extension to this question suggested above), you may wish to ask students how polynomial functions are generally different from others they have graphed.

Technology Suggestions

- You can use a computer algebra system (CAS), such as the TI-Nspire™ CAS, to find the degree of a polynomial in **question 3**. To do this, direct students to use the **polyDegree** function under the **Algebra/Polynomial Tools** menu.
- Students can use either a graphing calculator or *The Geometer's Sketchpad*® to easily graph **question 4a**) to **c**), and **e**) to **g**). They can then use the vertical line test to determine whether the relation is a function.
- In **question 4d**), x is expressed as a function of y. Students can use *The Geometer's Sketchpad*® to plot this relation directly. When the **New Function** dialog box opens, direct students to press the **Equation** button and then select $x = f(y)$. They can then enter the right side in the usual way. For **question 4h**), students will have to first solve the equation for y. They will need to graph both $y = \pm\sqrt{9-x^2}$ independently. For more details, see **T5–1 How to Do Section 5.1 #4d) and h) Using *The Geometer's Sketchpad*®**.

- If you have a relation that is not in the form $x = f(y)$ or easily solvable for y, you can use other software, such as Graphmatica, that graphs implicit relations.
- You can illustrate part **c**) of **questions 11** and **12** by plotting the functions using *The Geometer's Sketchpad*®. Before changing the scale, right-click on each function and then select **Properties**. Select the **Plot** tab and set the domain to large numbers in each direction. Return to the graphing screen, and drag a number on either axis to change the scale. Have students observe what happens to the function when they do this.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15
Reasoning and Proving	14
Reflecting	4, 7, 14
Selecting Tools and Computational Strategies	4
Connecting	8–10
Representing	4, 15
Communicating	4– 6, 9–15

Ongoing Assessment

- Use **Assessment Masters A–1** to **A–7** to remind students about the Mathematical Processes Expectations and how you may be assessing them throughout this chapter.