5.2 Graphs of Polynomial Functions

Study Guide and Exercise Book Pages
85 to 89

Tools
- grid paper
- graphing calculator
- pins or tacks
- tracing paper
- computer with dynamic geometry software
- computer with Microsoft® *Excel* or Corel® *Quattro Pro*

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad*® 4
- T5–2 How to Do Section 5.2 #7 Using *The Geometer's Sketchpad*®
- T5–3 How to Do Section 5.2 #11 Using a Graphing Calculator or a Spreadsheet

Key Terms
- end behaviour
- leading coefficient
- odd-degree polynomial
- even-degree polynomial

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Review the concept of even and odd numbers.
- Have students make a more detailed version of the chart presented in the **Key Concepts** box. They can create this table in their chapter reference sheets. Doing this exercise will give them a fuller understanding of the **Example**.
- Have students first graph $y = x$, $y = x^2$, $y = x^3$, $y = x^4$, $y = x^5$, and $y = x^6$, using a graphing calculator. Discuss the shapes of these basic functions, the degree, leading coefficient, domain, range, and end behaviour.
- Then, have students use a graphing calculator to graph functions with more than one term, such as $y = 2x + 1$, $y = x^2 + 3x - 5$, $y = 2x^3 - 2x^2 - 4x + 5$, $y = x^4 + 2x^3 - x^2 - 3x - 1$, $y = 3x^5 + 2x^3 - 3x - 1$, and $y = x^6 + x^5 - x^3 + 5x$. Discuss the shapes of these polynomials, the degree, leading coefficient, domain, range, and end behaviour.
- Ask students, "What will happen if the leading coefficient in any of the previous examples becomes negative?" Students should be able to connect with prior learning and explain that a leading negative sign represents a reflection in the x-axis, and so the end behaviours of functions should be opposite.
- Discuss with students the names of the various polynomial functions: linear, quadratic, cubic, quartic, and quintic.
- Remind students that both the maximum and minimum values are y-values.

Example

- Some students may incorrectly say that there is no leading coefficient, since they do not see a number in front of the cubed term. Help them understand that the coefficient is 1.
- Consider asking students to suggest different ways that they could express the end behaviour. For example, they may say, "It starts low and ends high," or "The function extends from quadrant III to quadrant I." This closer consideration might help them remember the concept.
- Help students recognize that although the domain of all polynomials is $\{x \in \mathbb{R}\}$, the range of odd-degree polynomials is $\{y \in \mathbb{R}\}$, and the range of even-degree polynomials is not $\{y \in \mathbb{R}\}$. Ask students why this is.

Questions

- **Question 2** helps students memorize the end behaviours of even- and odd-degree functions. As an extension to this question, have students guess the most likely degree and name of each polynomial function.
- For **question 3e**) and **f**), help students recognize that the leading coefficient is not necessarily the first number in the expression. Have students order the terms of all polynomials from highest to lowest degree.
- Discuss with students why **question 4** asks them to determine the least possible degree. Start by comparing the graphs of $y = x^2$, $y = x^4$, and $y = x^6$. This question will help prepare students for section 5.3.

Common Errors

- Some students may become confused about the four possible pairs of end behaviours for a given polynomial function.

R_x You may wish to play End Behaviour Aerobics to help students memorize the different end behaviours. Students use their arms to represent the direction of the end behaviours.

Differentiated Instruction

- Add the **Key Concepts** to the **word wall**.
- Use **four corners** to determine the end behaviours of a given polynomial function. Hold up a sample equation. Students move to the corner of the room that represents the end behaviours.

- **Question 6** is a useful question to complete as part of the lesson for this section. As you work through the question, you may want to write the answers on chart paper and post them in the classroom so students can refer to them throughout the unit. As an extension of this question, have students work in groups and complete the chart for $f(x) = x$, $f(x) = x^3$, and $f(x) = x^4$.
- For **question 7**, review that the graph of $x = a$ is a vertical line. Although students are familiar with reflecting graphs in the x-axis, they may be unfamiliar with line symmetry and point symmetry. You may wish to formally define these terms and visually show how they work. Some students may benefit from folding their paper along the line $x = a$ to check if a graph has line symmetry.
- For **question 7**, students can "prove" a function has point symmetry by using two pieces of paper with the same graph on them. They can then put a pin through point (a, b) and physically rotate one paper 180° (they might use tissue or tracing paper for the second graph). They can see if the graphs match by holding the two pieces of paper up to the light.
- For **question 7**, you may wish to define *point symmetry* as symmetry with respect to the origin: a reflection of the graph in the y-axis followed by a reflection in the x-axis.
- Students may have difficulty remembering the first and second differences of linear and quadratic functions. **Questions 10a**) and **b**) review these concepts. Remind students to find the differences by subtracting the first from the second. It would help some students if they begin filling in the differences at the bottom of the chart.
- **Questions 10c**) and **d**) extend students' knowledge of finite differences to higher powers. Encourage students to extend their knowledge of this concept by investigating the fourth differences in quartic functions and the fifth differences in quintic functions.
- Students may have difficulty determining the answer to **question 10g**). You may want students to work on this question in pairs, or to guide them through the question.
- For **question 11**, note that the value of the leading coefficient can be found with the help of this equation:
  constant value of finite difference = leading coefficient × degree **!** (i.e., degree factorial)
  Most of the remaining questions require the knowledge of this formula, so it would be helpful for students to add it to their reference sheets. Consider adding it to the **word wall** as well.
- After completing **question 13**, students may want to make a chart detailing how the end behaviour of any function can be determined from the degree and the leading coefficient.
- If students are unsure about the finite differences for **question 13d**), they may wish to create a table. Alternatively, you might show students how to create difference tables on the graphing calculator.
- For **question 15**, some students may need a quick review of the terms *profit* and *loss*. Encourage students to use technology to get a visual representation of the graph of the profit function. You might also have them investigate profit and loss on the Internet.
- For **question 16**, review with students how to calculate the volume and surface area of a topless cylinder. Help them to rewrite h in terms of r. Some students may have trouble viewing this equation as a polynomial function because of the presence of π, S, and r.
- You might assign **question 17** to students who need a challenge. However, **questions 18** and **19** should be within all students' grasp.

Technology Suggestions

- You can illustrate line symmetry and point symmetry in **question 7** using *The Geometer's Sketchpad*®. If a graph has line symmetry, students can plot the line of symmetry. They first plot a point on the function that lies on one side of the line of symmetry. They can then use the **Transform** menu to plot a reflection of that point in the line of symmetry. Ask them to drag the point along the function and observe what happens to the image point. If a graph has point symmetry, students can plot the point of symmetry. They can plot a point on the function that lies on one side of the point of symmetry and use the **Transform** menu to plot a 180° rotation of that point about the point of symmetry. Ask students to drag the point along the function and observe what happens to the image point. Refer to **T5–2 How to Do Section 5.2 #7 Using *The Geometer's Sketchpad*®**. You could have students work similarly on the graphs of x^4 and x^5 after working with the BLM.
- You can use the **Lists** on a graphing calculator or a spreadsheet such as Microsoft® *Excel* or Corel® *Quattro Pro* to calculate first differences for **questions 10** to **12**. On a graphing calculator, use the **ΔList** function. On a spreadsheet, you can create a standard template, and then use it repeatedly for different functions. Refer to **T5–3 How to Do Section 5.2 #11 Using a Graphing Calculator or a Spreadsheet** for more details.

Ongoing Assessment

- Students must have a complete understanding of end behaviours of polynomial functions before continuing. You may wish to assess their understanding so far by asking questions such as the following:
  - How often did you check the **Example** in the Study Guide and Exercise Book or the summary chart on your reference sheet?
  - For which questions did you have to refer to the **Example**?
  - Which questions did you find easy? difficult? Why?

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15, 16
Reasoning and Proving	10
Reflecting	4, 10
Selecting Tools and Computational Strategies	11, 12
Connecting	4, 7, 10, 16
Representing	7
Communicating	2, 4–8, 10, 13, 14, 17–19