5.3 Comparing Polynomial Functions

Study Guide and Exercise Book Pages
90 to 93

Tools
- graphing calculator
- computer with dynamic geometry software

Related Resources
- T–2 *The Geometer's Sketchpad*® 4
- T5–4 How to Do Section 5.3 #9 Using *The Geometer's Sketchpad*®

Key Terms
- odd-degree function
- even-degree function
- odd order
- even order
- local minimum points
- local maximum points

Definitions of Key Terms can be found on the Online Learning Centre at **ww.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- The first bullet, stating that a polynomial function can be represented in four different ways, is a good opportunity for classroom discussion. Discuss what *algebraically*, *numerically*, *graphically*, and *verbally* mean. How can students represent a polynomial function in all four of these ways?
- In order to prove how many x-intercepts an odd-degree function and an even-degree function can have, it is important to complete a number of examples with students, using technology. Alternatively, take an investigative approach. Students will discover how graphs behave when the factored form of the function has a zero of order 1, 2, or 3. They will also discover the minimum and maximum number of turning points even- or odd-degree functions can have.
- Discuss with students the difference between local maximum and minimum points and maximum and minimum points of the function.
- Some students may have difficulty proving algebraically whether functions are even or odd. Review simplifying algebraic expressions such as $(-x)^2 = x^2$ and $(-x)^3 = -x^3$.
- Have students add a table of rules and accompanying diagrams to their chapter reference sheets.

Example

- Students typically have trouble understanding what has happened when they prove algebraically that a function has even symmetry. Have students summarize their result. For example, "We have shown that if $-x$ is substituted into the function, the function does not change. Therefore, the function looks identical for negative x-values as it does for positive x-values."
- In part **c**), have students again compare the exponent of each term: the first term has an odd exponent, and the second and third have even exponents. Consequently, the function cannot have even or odd symmetry.
- Help students recognize that if they are asked to state what kind of symmetry a function has, they can look at the exponents of each term to determine whether the symmetry is odd, even, or neither. However, if they are asked to prove what type of symmetry a function has, they must use the algebraic method.

Questions

- In **question 1**, students using technology will be able to quite easily see the difference between a zero of order 1 and a zero of order 2. Suggest that students look closely at the zero of order 3 in **question 1d**). Have them note that, as they zoom in, the function curves like a cubic graph as it crosses the x-axis.
- For **question 2**, challenge students to find the degree and sign of the leading coefficient for questions such as $f(x) = (2x - 5)^2(3 - 7x)$.
- To help with the terminology in question 3, have students describe the difference between a function with an even degree or odd degree, zeros with even or odd order, and functions with even or odd symmetry.
- After completing **question 8** with technology, and having determined the roots, have students write the function in factored form, without factoring.

Common Errors

- Students can become confused when even and odd functions are introduced because they equate the new terms with even-degree and odd-degree functions.

R_x After teaching the difference between the two concepts, have students use their communication skills to describe the difference to a partner. As a review, have students discuss the difference each day for a few days.

Differentiated Instruction

- Use **timed retell** to summarize the **Key Concepts** of this lesson.
- Kinesthetic learners may find it helpful to graph even functions and then fold the graphs along the *y*-axis. They could then graph odd functions and, keeping the origin constant, observe what happens to the graph when it is rotated through 180°.

Ongoing Assessment

- **Questions 3**, **6**, and **8** will be helpful in assessing whether students understand the difference between zeros of odd or even order functions with odd, even, or no symmetry and odd-degree or even-degree polynomial functions.

- **Question 9** would make an excellent investigation lesson. How many x-intercepts can a cubic have? Can you draw a cubic with 0, 1, 2, 3, 4, and more x-intercepts? How many turning points can a cubic have? Can you draw one with 0, 1, 2, 3, and more turning points? Repeat these questions for linear, quadratic, quartic, and quintic functions. Have students summarize their results in a chart. (**Questions 10** to **13** scaffold part of this process.)
- Before answering **question 16**, have students define an odd function.
- Encourage students to extend their knowledge of polynomial functions to quintic functions and beyond by graphing and comparing the functions to linear, quadratic, cubic, and quartic functions.

Technology Suggestions

- If using TI-83 Plus/TI-84 Plus for **questions 1**, **7**, and **8**, use the **ZBox** function from the **ZOOM** menu to zoom in on areas of interest. Then, turn on the **TRACE** function to investigate the location of x-intercepts, and extreme points. In *The Geometer's Sketchpad*®, drag the origin to move the entire graph and change the scale by dragging any number along either axis.
- Review how to find maximum and minimum points on a graphing calculator using the **CALC** option and trapping the point between a left and right bound.
- Students can easily create and use sliders to investigate function plots using *The Geometer's Sketchpad*® in **question 9**. Refer to **T5–4 How to Do Section 5.3 #9 Using *The Geometer's Sketchpad*®** for more details.
- You can graph the functions in **questions 13** and **14** using *The Geometer's Sketchpad*®. Plot a point on the function and measure its coordinates. Drag the point along the function, and use the measurements to determine x-intercepts and extreme points.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	n/a
Reasoning and Proving	6, 17
Reflecting	8, 10–13
Selecting Tools and Computational Strategies	15
Connecting	3, 16
Representing	9
Communicating	5, 9, 14, 16, 17