5.6 Factoring Polynomial Expressions

Study Guide and Exercise Book Pages
101 to 104

Tools
- grid paper
- computer algebra system
- algebra tiles
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad*® 4
- T5–6 How to Do Section 5.6 #4f) Using *The Geometer's Sketchpad*® and TI-Nspire™ CAS

Key Terms
- greatest common factor
- common factoring
- factor by grouping
- binomial common factor
- decomposition

Teaching Suggestions

Key Concepts

- There are two aspects of the decomposition method for factoring trinomials that give students difficulty:
  - Some students write the second-last step as $2x(3x - 1)\ 1(3x - 1)$, rather than $2x(3x - 1) - 1(3x - 1)$.
  - Some students have trouble understanding how the binomial common factor can be common factored to produce the final answer.
- Discuss with sketches of graphs why the fifth bullet in the **Key Concepts** says that quadratic polynomial expressions of the form $ax^2 + bx + c$ can *sometimes* be factored using decomposition.
- Make sure students remember to always common factor first. You may wish to use examples to explore why this is a good rule to follow.
- Have students add new rules and examples to their chapter reference sheets.

Example

- Discuss with students why there is only one step to part **a**). Why does the resulting trinomial not factor further?
- Discuss with students that factoring by grouping can sometimes be done more than one way. For example, part **b**) could also be completed by rearranging the given expression and putting the two x-terms first. Then, take a common factor of x out of the first two terms and y out of the last two terms. The final answer will be the same as the one derived in the **Example**.
- After teaching part **c**), it would be useful to connect the factoring of a trinomial to students' knowledge of FOIL, or multiplying a binomial by a binomial. Tell students that by using FOIL, we know that the product of two binomials is a trinomial, such as $(a^2 + 3a + 2)$. To factor a simple trinomial, tell students to think of FOIL backward to find the two binomials, as follows:
  - The first term of the trinomial is a^2. According to first times first, the unknown variable must be a. So, you have $(a\quad)(a\quad)$.
  - Since "last times last" must equal 2, the numbers at the back of each bracket have to be 2 and 1. So, you now have $(a\ \ 2)(a\ \ 1)$.
  - The outside and inside terms must add to $3a$. Therefore, the 2 and the 1 both have to be positive. This results in $(a + 1)(a + 2)$.

  Some students will prefer this way of factoring. You may wish to have students extend this method and show how it works for trinomials with a leading coefficient not equal to 1.
- As an extension to part **f**), help students understand that it is important to always common factor first. Have them factor the trinomial first, and then pull out the common factors. A good example is part **g**), where it is not possible to factor the trinomial unless the common factor is pulled out first.
- For part **g**), students typically have a difficult time writing all the steps with proper form and the correct number of brackets.
- You may wish to use TI-Nspire™ CAS in this lesson.

COMMON ERRORS

- Many students have difficulty factoring trinomials when the leading coefficient is a number other than 1.

R_x Introduce other methods of factoring trinomials, and discuss the pros and cons of each method.

DIFFERENTIATED INSTRUCTION

- Have students work in **cooperative task groups** to research different methods of factoring trinomials. Have each group choose a different method, or assign one to them. The group will then present one or two examples to the class, using their method.
- You may wish to use algebra tiles to teach this lesson.

ONGOING ASSESSMENT

- To help assess student understanding, ask, "How often did you check the **Example** in the Study Guide and Exercise Book to help you with the questions? For which questions did you check the **Example**?"

Questions

- Encourage students to practise finding pairs of numbers that multiply to a specified number. This will help them when they are factoring more difficult trinomials. Use this idea to make a warm-up game for this section.
- Since students may have difficulty finding the greatest common factor (GCF) of a pair of larger numbers, you may wish to review how this can be done. Some students may have difficulty working horizontally to find the factors of numbers; it may be better for them to work vertically. For example, instead of factoring $24 = 2 \times 2 \times 2 \times 3$, it may be easier for students to factor 24 as follows:

  $$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times 2 \times 6 \\ &= 2 \times 2 \times 2 \times 3 \end{aligned}$$

  Some students may find it helpful to draw a factor tree for larger numbers.
- It may be helpful for some students to show how each term is divided by the common factor to the right of the question. For example, $x^3 + x^2 = x^2(x + 1)$, where the common factor is x^2 and $\frac{x^3}{x^2} = x$ and $\frac{x^2}{x^2} = 1$.
- Encourage students to check that they have factored questions correctly by expanding and simplifying the factored expressions.

Technology Suggestions

- When discussing the **Example**, you can use the **Factor** function on a CAS as an alternative method for working through the question, or to check answers.
- As an alternative method for **question 4**, consider having students use the virtual algebra tiles provided with *The Geometer's Sketchpad*® (the most likely source for this file is the folder Samples/Sketches/Algebra). Algebra tiles provide an opportunity for differentiated instruction, especially favouring the visual learner. Refer to **T5–6 How to Do Section 5.6 #4f) Using *The Geometer's Sketchpad*® and TI-Nspire™ CAS** for details.
- **Question 5** is a good candidate for using a CAS. By removing the necessity for repeated factoring, students can concentrate on the development of the pattern, progressing upward in the development of cognitive skills.
- Consider having students use a CAS for **questions 8** and **9**. A CAS is a good choice for factoring higher degree polynomials, especially those that do not have an obvious common factor. If time permits, demonstrate one or more of this type of polynomial.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	10, 11
Reasoning and Proving	9
Reflecting	7
Selecting Tools and Computational Strategies	4, 8, 9
Connecting	5, 10, 11
Representing	10, 11
Communicating	7