5.7 Difference of Squares of Polynomial Expressions

Study Guide and Exercise Book Pages
105 to 106

Tools
- algebra tiles
- computer with dynamic geometry software

Related Resources
- T–2 *The Geometer's Sketchpad*® 4

Key Terms
- difference of squares

Teaching Suggestions

Key Concepts

- This section provides a review of factoring difference of squares. Students have encountered the method in a previous mathematics course.
- Review the **Key Concepts** with students.
- Have students prove that when you multiply factors of the form $(a + b)(a - b)$, the middle terms are opposites and therefore cancel each other.
- Have students add the rule for factoring differences of squares to their chapter reference sheets.

Example

- You may wish to use algebra tiles to teach the concept of factoring a difference of squares. Alternatively, you could use the virtual algebra tiles in *The Geometer's Sketchpad*® to work through the example with students. (See **T5–6 How to Do Section 5.6 #4f) Using *The Geometer's Sketchpad*® and TI-Nspire™ CAS** for directions for generating these virtual tiles, and an example of how they can be used.)
- Some students may think that a *sum* of squared terms can be factored. They do not make the connection between the name of the method and the fact that a *difference* of squares question always has "something squared at the front of the expression, something squared at the back of the expression, and a minus sign in the middle." Make this point for each of the parts in the **Example**.
- When reviewing part **c**), help students see that the question cannot be completed unless a common factor is extracted first.
- For part **e**), ensure that students have brackets in the correct positions, so that their final answer is equivalent to the one in the Study Guide and Exercise Book.
- You may wish to play the following game to reinforce the concept of difference of squares:
  - **Step 1:** Have each student pick any two consecutive numbers.
  - **Step 2:** Tell them to square each number and subtract the smaller value from the larger value. Let this value be a.
  - **Step 3:** Have students add the two original numbers and let this value be b.
  - **Step 4:** Ask them to explain why a is equal to b and write their solution algebraically.

  You could have students work on this game in pairs. If students are having difficulty, you might work as a class to write the solution algebraically:

  Let n and $(n + 1)$ represent the two consecutive numbers.

$$
\begin{aligned}
(n+1)^2 - n^2 &= (n + 1 - n)\,(n + 1 + n) && \text{Factor using the difference of squares method.}\\
&= (1)(n + 1 + n) && \text{Simplify.}\\
&= n + 1 + n\\
&= (n + 1) + n
\end{aligned}
$$

Common Errors
- Some students believe that they can factor a sum of perfect squares using the method introduced in this section.

R_x Have students check that they have factored questions correctly by expanding and simplifying the factored expressions.

Differentiated Instruction
- Construct a **decision tree** to outline the steps necessary to factor a given expression.

Questions

- Encourage students to work through the problems themselves. You could have them work on the first few questions in pairs. Tell them that they should review the **Example** questions before asking for assistance.
- When students are factoring differences of squares, encourage them to keep the signs in the same order when they factor. For example, $x^2 - 25 = (x + 5)(x - 5)$ and $x^2 - 16 = (x + 4)(x - 4)$
- It may be helpful for some students to add an extra step when factoring the difference of squares. For example,
  $$\begin{aligned} x^2 - 36 &= x^2 - 6^2 \\ &= (x + 6)(x - 6) \end{aligned}$$
- **Question 2** shows students that the variable does not always have to be the first term. As long as "something squared is at the front of the expression, something squared is at the back of the expression, and a minus sign is in the middle," they can use the method of factoring a difference of squares.
- For **question 7**, encourage students to use the Internet to research non-real roots.
- For **question 7a)i)**, have students explain to a partner why $m^2 + 1$ cannot be factored any further.
- In **question 8**, remind students to common factor first.
- You may wish to show students that they could solve **question 9** in different ways:
  - They could solve for a and b, letting $a + b = 82$ and $a - b = 78$, and using elimination to solve.
  - They could solve for a logically by saying that a must be halfway between 82 and 78.

  You could then ask students if using either of these methods is really necessary. Help them realize that they are asked to find $a^2 - b^2$, which is equivalent to multiplying $(a + b)(a - b)$. This is equivalent to multiplying (82)(78).
- In **question 10**, some students may have difficulty finding the square roots of fractions.
- Some students may have difficulty understanding the wording of **question 11**. You may want to read the question as a class or have students work in pairs.
- Assign the **C** questions to students who are not being challenged by the questions in **A** and **B**.
- In **question 12**, students learn that they must factor certain trinomials before applying the difference of squares factoring technique. It may be necessary to review perfect square trinomial factoring with students before doing this question.
- Before factoring the trinomials in **question 12b)**, help students make the question look like **question 12a)**. Students must recognize that they have to common factor a negative sign out of the last three terms to do this question successfully.
- In **question 13**, students learn that they can insert the root of a number that is not a perfect square into the brackets. Remind students that writing the roots as exact values is more acceptable than writing them as approximate values.
- As an extension to **question 13**, ask students, "If these expressions were made into a function, such as $y = x^2 - 5$, what would we know about the function?"
- Challenge students to make up their own multi-step factoring question by inserting a question from section 5.6 into a question from section 5.7 or vice versa. Students can first check if the question they created is factorable, and then have a partner try to factor it.

Technology Suggestions

- **Question 7** is a good application for a CAS because the pattern becomes readily apparent. You can easily extend the concept by asking students to predict and check other similar polynomials.
- It would be instructive in **question 13** to demonstrate the factoring of at least one of the parts using a CAS. You could do the question once using an **EXACT** output, and a second time using an **APPROXIMATE** output. Doing so leads to a discussion of the circumstances under which each of these would be appropriate.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	n/a
Reasoning and Proving	9, 12, 13
Reflecting	9, 13
Selecting Tools and Computational Strategies	9
Connecting	7–9, 12, 13
Representing	9, 10
Communicating	n/a

Ongoing Assessment

- To assess students understanding, have them write a journal entry that explains how to do today's work to a classmate who was absent. Have them include at least five different examples in their journal entry.