5.8 Intercepts of Polynomial Functions

Study Guide and Exercise Book Pages
107 to 109

Tools
- grid paper
- graphing calculator
- computer algebra system
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad*® 4
- BLM 5–3 Chapter 5 Review
- BLM 5–4 Chapter 5 Practice Test
- BLM 5–5 Chapter 5 Case Study

Key Terms
- *x*-intercepts
- zeros
- roots
- real roots

Definitions of Key Terms can be found on the Online Learning Centre at **ww.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- It may be necessary to review the different number systems, including natural numbers, whole numbers, integers, rational numbers, and real numbers.
- Review with students how to determine the *x*-intercepts of a function.
- Review with students the concept that a function is positive if it is above the *x*-axis, and negative if it is below the *x*-axis.

Example

- Review with students how to use the **Zero** operation on a graphing calculator. When there are multiple zeros, students sometimes have difficulty understanding the concept of the calculator requiring a left bound, a right bound, and a guess to have enough information to calculate a particular zero.
- Ensure that students set their windows appropriately and choose an *x*-scale of 1 so that they immediately see the zeros for this example: 0, 2, and −2.
- To review the concepts students have learned in this chapter, you may wish to revisit the difference between zeros of odd or even order, polynomials of odd or even degree, and polynomial functions with odd or even symmetry.
- For part **c)**, you may wish to remind students that the function must be in factored form before each of the factors is set to zero.
- Assign students another polynomial function that they must first factor, and then graph by plotting the zeros. After zeros are plotted and their order noted, students can easily see the relationship between this concept and end behaviours. You may also wish to have students add a *y*-intercept to their graphs. The *y*-intercept will help to determine if the end behaviours are correct (**question 10** follows this process).

Questions

- Encourage students to use technology when they are determining the *x*-intercepts (or zeros) or roots of polynomial functions.
- It may help some students to do a more in-depth review of factoring.
- Have students work in peer groups with a student "factoring expert" who can help those who may be having difficulty understanding factoring.
- Remind students to always common factor first.
- Ask students to name the shapes of the functions in **question 1**, and to picture them mentally.
- For **question 3**, have students first graph using pencil and paper. Review the process as a set of steps:
  - Fully factor.
  - Plot the zeros.
  - Take note of zeros that are of order 2.
  - Determine the end behaviours of the function.
  - Draw a curve through the points.
  - Check by verifying that the *y*-intercept falls on the curve.
  - Check work with the graphing calculator. Remind students that it would be better to enter the original equation into the calculator. This would reveal any mistakes they may have made in the factoring process.

Common Errors

- Students often forget that a zero of order 2 touches, but does not cross, the x-axis.

R_x Have students use technology to graph a number of these types of equations, each with a zero of order 1, 2, and 3. Have them zoom into each zero and describe what the function looks like around the zero.

Differentiated Instruction

- Have students work in groups of two to four. Assign a different problem to each group, which they write on a large sheet of paper. Each group adds the first line of the solution to their problem, and then passes the paper to the next group. Each group then adds the second line to the problem on the sheet they now have, and then passes the sheet again. Continue until the solution is complete. Display the completed posters around the room.

- **Question 6** shows how using technology helps to factor polynomials that are too difficult to factor using the methods learned thus far. Ask students how to find the y-intercepts easily by looking at the original equation. Can there be more than one y-intercept? Why?
- **Question 7** can initiate a good class discussion. Have students explain their viewpoint using diagrams on the board. You may wish to discuss non-real roots and non-real (complex) numbers.
- In **question 8**, remind students that a function is positive when its y-value is positive.
- **Question 9** shows students that a function does not change sign before and after a zero of order 2.
- When creating the interval tables in **questions 8** and **9**, some students may want to use interval notation instead of inequalities.
- As an extension to **question 11**, have students discuss how the graph of $y = x^3 - 4x^2$ differs from the graph of $y = x^3 + 4x^2$.
- As an extension to **question 13b**), you may wish to ask students how the value of the leading coefficient will affect the y-intercept. This extension will also help students complete **question 15**.
- Assign **questions 14** and **15** to students who require a challenge. Perhaps have students work on these questions in pairs.
- Ask students to determine the type of symmetry in **question 14**.
- Students may have difficulty finding the equation for **question 15** because they may not realize that they have to calculate the value of the leading coefficient. Students may have more success with this question if it is scaffolded.
- Have students look through all the questions and make a list of all the new terminology that they have learned throughout this chapter. Students may be surprised by the number of new terms they have learned in this chapter.
- You may wish to use **BLM 5–3 Chapter 5 Review** to help students identify areas in which they need to further their understanding.
- Provide students with **BLM 5–4 Chapter 5 Practice Test** to prepare them for the chapter test.

Case Study

- You may wish to have students complete **BLM 5–5 Chapter 5 Case Study**, which incorporates the learning from this chapter.
- The price function is linear; students should recognize this by the pattern in the table. Students may miscalculate the rate of change if they forget to include the differences in the x-values or neglect the fact that the price is decreasing while the number of pairs is increasing. They may need to be reminded that the slope-point form for the equation of a line can be used to determine the equation.
- Students may be confused by the function notation in the revenue statement and not realize that they need to multiply only the price function by x.
- The cost function here is quadratic, so the second differences are equal.
- To obtain the correct equation for profit, students must be careful to subtract all the terms of the cost function from the revenue function.
- Students should use a graphing calculator to determine the optimum number of pairs of shoes needed.

Technology Suggestions

- To illustrate an alternative method for solving the **Example**, you could use the **Zeros** function under the **Algebra** menu in TI-Nspire™ CAS.
- Consider having students use *The Geometer's Sketchpad*® as a visual method for solving **question 3**. They can plot a point on the graph and measure the coordinates. Then, they can watch the coordinates change dynamically as they move the point toward the intercepts.
- Consider having students solve **question 6** in two ways using a CAS. In the first, they can use the **Factor** function to quickly derive the factored form of the polynomial. In the second, they can use the **Zeros** function. Ask them to compare the results. As an extension, ask students to repeat this exercise with a polynomial that does not have integral factors.
- As an extension to **question 14**, have students solve the question using *The Geometer's Sketchpad*®. Then, have them plot the function using a slider to control the value of the coefficient of x^2. Ask them to show conditions for four real roots, two real roots, and no real roots, and how the graph indicates which case is appropriate.
- For **question 15**, students can use a CAS to expand the multiplication of the binomial factors, and then solve for the value of a using the coordinates of the given point.

Ongoing Assessment

- You may wish to assess achievement by having students complete **BLM 5–5 Chapter 5 Case Study**.

Summative Assessment

- You may wish to use the chapter test that you can find in the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12.**

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15
Reasoning and Proving	6, 15
Reflecting	4, 14
Selecting Tools and Computational Strategies	7
Connecting	3, 6–8, 14
Representing	n/a
Communicating	3, 4, 7, 11, 12, 14