6.2 Strategies for Solving Polynomial Equations

Study Guide and Exercise Book Pages
114 to 116

Tools
- graphing calculator
- computer algebra system
- poster paper
- markers

Related Resources
- G–1 Grid Paper
- T–4 The TI-Nspire™ CAS Calculator
- T6–2 Solve Polynomial Equations Using TI-83 Plus/ TI-84 Plus and TI-Nspire™ CAS

Key Terms
- factors
- polynomial equation
- roots of an equation
- *x*-intercepts
- zeros

Definitions of Key Terms can be found on the Online Learning Centre at **ww.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Some students have difficulty understanding the concept in the first bullet of the **Key Concepts**. That is, if the product of two factors is zero, then one or both of the factors must equal zero. Substitute some values into the equation to emphasize this concept.
- Revisit with students how to determine the x-intercepts of a polynomial function algebraically, by setting the y-value equal to zero in the polynomial function, and solving for x.
- Discuss the different number systems with students, including natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. As a class, you might create a chart on poster paper that summarizes the different systems. Post this chart in the classroom.

Example

- Help students recall different types of factoring.
- Remind students to always common factor first.
- Have students discuss what method they would use to factor the quadratic in the second line of the solution to part **b**).
- Have students explain, in their own words and using proper terminology, the relationship between the solution to a polynomial equation and the roots of the polynomial function.
- To help reactivate students' knowledge of the concepts in the previous section, you may wish to give students three x-intercepts and have them create a polynomial function, in expanded form, that has those three roots. Ask students to share their answers with others to see if they all have the same answer, or if there is more than one answer to this question.

Questions

- Some students may have difficulty understanding that when a function has a double real root, there is only one x-intercept. Suggest that they talk this concept through with a partner to explain why this is.
- In **question 1**, have students note that all of these equations represent finding zeros for a quadratic function. Have students note that each of these quadratic functions has two distinct real zeros. As an extension, ask students, "Do quadratic functions always have two zeros?" Ask them to create a quadratic function with one or no x-intercepts.
- In **question 1e**), help students solve the second bracket, $(2x + 5) = 0$, to get the correct fraction.
- In **question 3**, have students note the patterns in the signs in the expanded and factored forms. For example, if the question has a negative coefficient in the second term and a positive constant, then both brackets of the factored form must have minus signs in them.
- In **question 4**, you may wish to have students write the factored form of the equation after they have found the roots using a graphing calculator.
- Remind students that some equations, such as $x^2 = 4$ in **question 6**, have two solutions, $x \pm 2$, but a question such as $x = \sqrt{4}$ has only one answer, $x = 2$. The $\sqrt{\ }$ sign implies that we are looking for the positive root of 4 only.

COMMON ERRORS

- Some students will still have difficulty factoring questions with multiple steps.

R_x Have students make a **decision tree** to outline the steps for factoring various types of polynomials.

DIFFERENTIATED INSTRUCTION

- Add the Key Terms to the **word wall**.
- Use **think aloud**, and have one group lead the class through one part of **question 9**. Have another group lead the class through the next part. Students should communicate using correct terminology. Provide help as needed.

- Before beginning **question 7**, discuss with students how to factor more difficult trinomials, with coefficients greater than 1. Also, discuss what it means to have an exact answer, and an approximate answer. Why would a fraction be considered an exact answer? Why might a decimal be an approximate answer? Students should give an exact answer for this question.
- In **question 8**, remind students that in order to solve a quadratic equation, it must have a zero on the right-hand side. Before beginning the question, ask students, "What is the first thing you should try to do before attempting these questions?" (They should remove a common factor, if possible.)
- For **question 10**, you may wish to revisit the names of polynomial functions with higher degrees, and their end behaviours.
- In **question 12**, have students analyse each question first and discuss whether they can factor it using methods they have learned so far. They should be able to factor parts **b**), **d**), and **e**) by hand. Knowledge of the factor theorem is needed for the others. Have students discuss what other methods they might use to solve equations that are not easily factored.
- Before trying to solve equations in **question 12**, have students discuss how many roots each equation "should" have. It is possible that students will recognize that quadratics have two roots, cubics have three roots, etc. Try to direct them to this realization. Consider having students graph the equations with technology as an extension to the question. Help them recognize the relationship between the type of equation and the number of roots.
- Students will probably not recognize that some of the roots may be real and others may be complex. Encourage them to learn more about and sketch the graphs of functions that do not have real roots.
- Challenge students to learn more about non-real roots on the Internet.
- For help with **question 13**, refer students to **question 11** in section 6.1.
- Remind students in **question 14** that they can solve a quadratic by using the quadratic formula.

Technology Suggestions

- If students are using TI-Nspire™ CAS, you might distribute **T–4 The TI-Nspire™ CAS Calculator**.
- Encourage students to use a graphing calculator to determine the x-intercepts of polynomial functions. This will give them the opportunity to see the relationship between the x-intercepts and the roots of the equation. You may want to distribute **T6–2 Solve Polynomial Equations Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS**.
- Using a CAS to help factor expressions can help students reinforce their factoring skills.
  - Using TI-Nspire™ CAS, use the **Factor** command. After constructing a calculator page, press (menu), then select **Algebra**, and then **Factor**.
  - Using TI-89, press **F2**, and then select **factor**(.

  Students can also use an online factoring calculator to assist them. Go to **www.mcgrawhill.ca/books/mct12** and follow the links.
- You can use a calculator to determine the x-intercepts, or zeros.
  - You can find the zeros on TI-83 Plus/TI-84 Plus by first graphing both functions and then pressing **2nd**, **TRACE**, and selecting **zero**. Move to the left of an x-intercept and press **ENTER**. Move to the right of the *same* x-intercept and press **ENTER** twice. The zero will be computed. Do the same for the second x-intercept.

- Using TI-Nspire™ CAS, press (menu), select **Algebra**, and then **Zeros**. Enter the expression followed by the comma key, and then the variable. Caution students that this must be an expression, rather than an equation. They must also enter the comma and the variable inside the closing bracket.
- Using TI-89, press **F2** to access the **Algebra** menu. Then, select **zeros**(. Enter the expression followed by the comma key, and then the variable. Ensure that you close the bracket of the **zero** command.

- You can solve an equation using a graphing calculator.
  - For TI-Nspire™ CAS, press (menu), select **Algebra**, and then **Solve**. Enter the equation, followed by a comma key and the variable.
  - For TI-89, press **F2** and select **solve**(. Enter the equation followed by a comma and the variable, and then close the bracket of the solve command.
  - Using TI-83 Plus/TI-84 Plus, press **2nd**, **0** to go to the **CATALOG**. Press **ALPHA**, **LN** to go to the commands beginning with S, and then scroll down to the **solve**(command. Press **ENTER**. Enter the expression, a comma, the variable, a guess, and, in curly brackets, the low bound and the high bound. Press **ENTER**. For example, **solve**($x^2 + x, x, 0.5, \{-1,2\}$) results in a solution of 0, which is close to the guess of 0.5 and within the lower and upper bound of -1 and 2. If there is no answer within the lower and upper bound, an error message is displayed. This feature is not as effective as graphing to see the zeros, but it is a way of testing various parts of the domain for a solution.
- The **intersect** function on a graphing calculator can be used to help solve an equation. For example, use the following methods to help with **question 5**:
  - Using TI-83 Plus/TI-84 Plus, graph both quadratic functions: $f(x) = 2x^2 - 2x - 12$ and $g(x) = x^2 - x - 6$. Press **2nd**, **TRACE**, and select **5: intersect**. Use the arrow keys to move to the left of one of the x-intercepts, and press **ENTER**. Move to the right of the *same* x-intercept, and press **ENTER**. Press **ENTER** again and the calculator will compute the x-intercept. Follow the same procedure with the other x-intercept.
  - Using TI-Nspire™ CAS, you can find the intersection points by entering the equations into a **Graphs & Geometry** page. Press (menu), choose **Points & Lines**, and then select **Intersection Point(s)**. Move the pointer on the screen and click on each graph. The labelled intersection points will appear.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	13, 14
Reasoning and Proving	10
Reflecting	5
Selecting Tools and Computational Strategies	12, 14
Connecting	2, 5
Representing	13
Communicating	5, 6, 9–11, 14

Ongoing Assessment

- To assess students' understanding, ask, "What questions did you find easy? difficult? Why?" Help students revisit methods with which they are having difficulty. Assign additional problems to help them practise these concepts.