7.3 Volume of Three-Dimensional Objects

Study Guide and Exercise Book Pages
139 to 143

Tools
- different-shaped prisms
- grid paper
- cardboard
- scissors
- beakers and water
- computer with dynamic geometry software
- scientific calculator
- graphing calculator
- computer algebra system

Related Resources
- G–1 Grid Paper
- T-4 The TI-Nspire™ CAS Calculator
- T7–1 How to Do Section 7.1 #2 to 5 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS
- T7–3 How to Do Section 7.3 #18 Using *The Geometer's Sketchpad®*

Key Terms
- volume

Definitions of Key Terms can be found on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12**.

Teaching Suggestions

Key Concepts

- Some students may have trouble visualizing three-dimensional objects. Have different-shaped prisms available for students, such as rectangular prisms and pyramids. This will enable students to look at them from different perspectives.
- Encourage students who are having trouble visualizing three-dimensional objects to visit interactive web sites that allow them to see three-dimensional shapes in a rotation or from different perspectives.
- Have students practise drawing diagrams of common three-dimensional objects.
- Have students brainstorm real-life situations in which the volume of an object needs to be found. For example, a company that sells diesel fuel has to know what volume of fuel a tanker truck can carry so it can calculate how many customers can be served by one truck.
- Have students look through all the practice questions and create a list of formulas that will be required throughout this section.
- Have students search for all the required volume formulas on the Internet.
- Direct students to the formulas for the volume of various three-dimensional figures, starting on page VI of the Study Guide and Exercise Book.
- Add formulas for the volume of objects to a chapter reference sheet.

Example

- Have students draw diagrams for the problem and for each part of the solution.
- Have students verify the number of gallons that will fit into the propane tank by using their choice of online conversion tool.
- Suggest that students find and use online tools, such as a conversion tool or table, or a volume calculator. For a sample of these tools, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Questions

- Encourage students to observe and make scale models of three-dimensional objects. Tell them to relate these to other objects, such as a propane tank. Students will then have a better understanding of shapes, such as cylinders and hemispheres, that can be combined to make new composite shapes.
- Have a discussion with students about the nature of objects around them. Have them notice how objects are simply composites of basic shapes. Artists often make use of this concept when sketching. Ask,
  - If objects are composites of basic shapes, and we can model basic shapes mathematically, do you think we can model the world around us mathematically?
  - Could we find the area and volume of all of the objects in this room?
  - How might this knowledge be used in professions such as architecture and design?
- Encourage students to use online conversion tables and an online conversion calculator to convert one type of unit to another. For a sample of these tools, go to **www.mcgrawhill.ca/books/mct12** and follow the links.

Common Errors

- Students may be confused by the variety of units used in this section.

R_x Consider doing a daily review of the basics from section 7.1, including the names and sizes of various units.

Differentiated Instruction

- Use **cooperative task groups** to complete application problems.
- Add new formulas to a **word wall**.

- To practise estimation, have students picture how large the volumes in **questions 1** and **2** are, and perhaps think of a shape of that size. For example, 200 ft^3 is about the volume of a small garden shed, and 208 cm^3 is about the volume of a Rubik's Cube.
- If students have trouble conceptualizing the very small decimal numbers they calculate in **question 3**, have them convert them to larger numbers, such as 1 000 000 cm^3.
- For **question 4**, define *engine displacement* as "the volume swept by all of the pistons inside the cylinders of an internal combustion engine in a single movement." This question may confuse some students. Help them simplify the problem by suggesting that they convert 400 $in.^3$ to cubic centimetres.
- For **question 7**, revisit with students that a cubed root is equivalent to a term with exponent $\frac{1}{3}$.
- As an extension to **question 8**, calculate the cost of the soil. Ask students questions relating to the number of bags holding 2 ft^3 that will fit into Enid's car trailer.
- Before working on **question 13**, define the term *congruent*. Then, have students work in groups to construct a box out of paper or cardboard. They could label the sides of the box with the dimensions given. Help students to determine a domain for x and to set the window of the graphing calculator appropriately. Remind them how to find the maximum point on a graphing calculator.
- For **question 15**, bring in several beakers of water and have students experiment by filling the beakers with water and estimating which of the shapes will hold more water.
- In **question 16**, discuss with students the meaning of the word *better*, and the difference between "better for the company" and "better for the customer."
- For **question 21**, you may wish to discuss other real-life applications of the surface-area-to-volume ratio. For example, why do we not leave small children unattended in a car in the summer time? Children, with their smaller body mass, overheat much more rapidly than adults, with their larger body mass. Why might polar bears be more suited to the Arctic than snakes? Compare their surface-area-to-volume ratios and discuss how quickly each animal will freeze.
- Students could work together to create a three-dimensional model of the mailbox in **question 25**, or some other shape from the section.
- Challenge students to create their own volume word problem and have a partner solve it. Students may wish to design a three-dimensional item that can be useful in their house or at their future jobs. You may wish to have students present their problem to the whole class and explain why their problem is a useful one to solve.

Technology Suggestions

- Encourage students to use technology, such as *The Geometer's Sketchpad*®, to create drawings of three-dimensional objects.
- If students are using TI-Nspire™ CAS, distribute **T–4 The TI-Nspire™ CAS Calculator**.
- Consider assigning **question 1** or **2** to be done using the unit conversion software in a graphing calculator. (See **T7–1 How to Do Section 7.1 #2 to 5 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS** for more information on how to use this function on a graphing calculator.)
- For **question 7**, revisit with students the keystrokes that are used to determine cube roots using a scientific calculator or a graphing calculator.

- **Questions 7** to **10** involve solids with a uniform cross section, which can be easily modelled using *The Geometer's Sketchpad*®. Consider using this alternative approach for at least one of these questions. Then, the model can be altered dynamically to observe the effect on the volume measurement.
- You can solve **question 13** dynamically using *The Geometer's Sketchpad*®. Plot the equation that determines the volume. Then, draw a point on the function, and drag it to determine the maximum value of the volume as the value of x is changed.
- For **question 17**, revisit with students how to use a spreadsheet on a computer. Note that you can also program a graphing calculator to accept these inputs and to output the desired calculations. Alternatively, you can define functions on a computer algebra system (CAS).
- **Question 18** is a good candidate for simulation using *The Geometer's Sketchpad*®. You can control the volume with a slider, and then automatically draw a sphere and a cube with this volume, with a continual readout of the two surface areas. As an extension, calculate the ratio of the surface areas. See **T7–3 How to Do Section 7.3 #18 Using *The Geometer's Sketchpad*®** for instruction on how to do this.
- **Questions 19** and **22** can also be completed by entering the appropriate formulas into a spreadsheet.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	12–14
Reasoning and Proving	4, 15, 16
Reflecting	3, 23
Selecting Tools and Computational Strategies	13, 17, 19, 20
Connecting	4, 7, 14, 18
Representing	9, 13, 17
Communicating	3, 4, 13–16, 18, 19, 21–23, 25

Ongoing Assessment

- Questions similar to **question 10** may be useful as an assessment for this section.