7.6 Solving Problems Involving Properties of Circles

Study Guide and Exercise Book Pages
152 to 154

Tools
- geometry set
- grid paper
- computer with dynamic geometry software

Related Resources
- G–1 Grid Paper
- BLM 7–3 Chapter 7 Review
- BLM 7–4 Chapter 7 Practice Test
- BLM 7–5 Chapter 7 Case Study
- T7–6 How to Do Section 7.6 #4 Using *The Geometer's Sketchpad®*

Common Errors
- Students often have difficulty drawing a diagram from given information.

R$_x$ Quiz students on their circle terminology before having them attempt to draw diagrams for questions in this section. Encourage students to highlight key words in their description. Encourage them to double-check their diagram or trade their diagrams with a partner to see if their partner has understood the problem the same way.

Differentiated Instruction
- Use **think aloud**. Have one group lead the class through one of the problems by drawing the diagram and then using proper circle terminology to explain their solution.

Teaching Suggestions

Key Concepts

- You may wish to discuss where the geometry of circles may apply to real life. Where are circles used in design, architecture, and nature? In what situations might you need to measure part of a circle?

Example

- Note that when taking the square root of both sides, the negative root is not relevant, and therefore not mentioned.
- Ensure that students calculate exact answers, and do not round until the very last step.

Questions

- Before beginning the questions, review the terminology of circles from sections 7.4 and 7.5. Have students explain the meaning of each term with a diagram and using their own words.
- Encourage students to use the Internet to find other three-dimensional structures and buildings that are circular and cylindrical.
- Remind students to use the square root symbol when calculating length or distance using the Pythagorean theorem.
- Remind students to clearly show each step of their solution and to communicate clearly throughout the presentation of their solution.
- Remind students that an angle, such as $\angle ACB$, is an angle centred at point C.
- Encourage students to draw neat diagrams using their geometry set.
- Remind students that a tangent line is perpendicular to the radius at the point of contact.
- **Question 2** assesses students' understanding of the seventh circle property, learned in section 7.5.
- **Question 3** assesses students' understanding of the first circle property, learned in section 7.5.
- For **question 4**, remind students of the formula for the area of a triangle.
- **Question 5** assesses students' understanding of the first circle property, learned in section 7.5. Remind them of the meaning of the term *congruent*. Many students will have difficulty with part **b**). They will not know what is required to "prove" that two triangles are congruent. Help them to make the connection that if the lengths of the three sides of one triangle are equal to the corresponding lengths of the three sides of the other triangle, then the triangles must be congruent (SSS congruency theorem). You may wish to have students investigate the following: "Given three side lengths (or three sticks) of lengths 6 cm, 4 cm, and 4 cm, how many different triangles can be created?"
- Students may be confused when drawing a diagram for **question 6**, because the point of tangency is drawn first, and point B, which is further along the tangent line, is drawn last. Previous word problems were always worded the other way around. Have students double-check their diagram with the wording of the question to see if their diagram makes sense.
- **Question 8** assesses students' understanding of the second circle property, presented in section 7.5.

- **Question 9** assesses students' understanding of the third circle property, presented in section 7.5.
- In **question 10**, you may wish to tell students that they will need to use the formula for the area of a sector for the left section of garden, and the formula for the area of a triangle for the two right-angled triangles.
- For **question 11**, encourage students to make a circular design of their own.
- As an extension to **question 11**, you may wish to review section 7.4 by asking students to find the area of the circle's sector, and the area of the circle's segment.
- For **question 12**, explain to students that they again have to prove that the triangles are congruent in order to justify their answers. This time, they can state that if two sides and the contained angle (the right angle) of one triangle are equal to two sides and the contained angle of the other triangle, then the triangles are congruent.
- Assign **question 13** to students who need a challenge. Students may not understand that the runner makes almost one revolution around the track before leaving the circle along the tangent line. Help students use the Pythagorean theorem to find the distance travelled along the tangent line, and then use primary trigonometric ratios to find the central angle in the diagram. Finally, they can use the formula for the length of an arc to find the distance travelled along the circular track.
- Challenge students to create a real-life circle problem that could arise in the construction business (perhaps similar to **question 10**). Encourage them to design a problem that requires the use of a variety of circle properties and formulas in its solution.
- Challenge students to research a career as an air traffic controller and the use of radar screens in this career.
- You may wish to use **BLM 7–3 Chapter 7 Review** to help students identify areas in which they need to further their understanding.
- Provide students with **BLM 7–4 Chapter 7 Practice Test** to prepare them for the chapter test.

Case Study

- In **question 1**, ensure that students understand that this is a volume problem, and not a surface area problem.
- For question **2a**), the three cushions are rectangular and uniform. Therefore, students need to calculate the area for only one cushion and then multiply by 3.
- For part **c**), students may need to be reminded of the formula for a trapezoid. They may also require some guidance on how to use the thickness of this section to calculate the surface area. They must use the Pythagorean theorem to calculate the surface area for the ends. You may refer them to similar questions, such as question 8 in **BLM 7–4 Chapter 7 Practice Test**.
- For **questions 2d**) and **e**), students will have to remember to double the areas to account for both arms and both ends.
- Students may question the need to calculate the area of all parts. This is necessary because unless the couch is disassembled, some parts will be inaccessible.

Technology Suggestions

- After working out the problem in the **Example** using pencil and paper, use *The Geometer's Sketchpad*® to present an alternative solution.
- **Question 1** is a good candidate for an alternative solution using *The Geometer's Sketchpad*®. To draw a tangent, construct a radius to a point on the circle. Then, construct a line through the point perpendicular to the radius.
- Use *The Geometer's Sketchpad*® to create a dynamic solution for **question 4**, especially part **b**). Create triangle interiors for △CED and △CFD, and measure the areas. Then, drag points E and F. Refer to **T7–6 How to Do Section 7.6 #4 Using *The Geometer's Sketchpad*®** for more details.
- **Question 5** is a good candidate for illustration using *The Geometer's Sketchpad*®. After developing the conjecture, discuss whether *The Geometer's Sketchpad*® can be used to provide a valid proof.
- Consider assigning one or more of **questions 6** to **11** to be done using *The Geometer's Sketchpad*®.
- **Question 12** is a good candidate for illustration using *The Geometer's Sketchpad*®. Wait until students have stated and supported their conjecture before measuring the areas of the triangles.
- **Question 13** is an excellent candidate for a dynamic solution, since it is not immediately obvious where the runner leaves the circular track. Consider asking for at least two different solutions that use technology.

Ongoing Assessment

- You may wish to assess achievement by having students complete **BLM 7–5 Chapter 7 Case Study.**

Summative Assessment

- You may wish to use the chapter test that is in the Instructor Centre on the Online Learning Centre at **www.mcgrawhill.ca/books/mct12.**

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	10, 11, 13
Reasoning and Proving	5, 12
Reflecting	4
Selecting Tools and Computational Strategies	10, 13
Connecting	2, 4
Representing	6, 13
Communicating	4, 5, 8, 9, 11, 12