Chapter 4

Solve Exponential Equations

Curriculum Expectations

Exponential Functions

Solving Exponential Equations Graphically

A1.1 determine, through investigation with technology, and describe the impact of changing the base and changing the sign of the exponent on the graph of an exponential function

A1.2 solve simple exponential equations numerically and graphically, with technology (e.g., use systematic trial with a scientific calculator to determine the solution to the equation $1.05^x = 1{,}276$), and recognize that the solutions may not be exact

Sample problem: Use the graph of $y = 3^x$ to solve the equation $3^x = 5$.

A1.3 determine, through investigation using graphing technology, the point of intersection of the graphs of two exponential functions (e.g., $y = 4^{-x}$ and $y = 8^{x+3}$), recognize the x-coordinate of this point to be the solution to the corresponding exponential equation (e.g., $4^{-x} = 8^{x+3}$), and solve exponential equations graphically (e.g., solve $2^{x+2} = 2^x + 12$ by using the intersection of the graphs of $y = 2^{x+2}$ and $y = 2^x + 12$)

Sample problem: Solve $0.5^x = 3^{x+3}$ graphically.

A1.4 pose problems based on real-world applications (e.g., compound interest, population growth) that can be modelled with exponential equations, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation

Sample problem: A tire with a slow puncture loses pressure at the rate of 4%/min. If the tire's pressure is 300 kPa to begin with, what is its pressure after 1 min? After 2 min? After 10 min? Use graphing technology to determine when the tire's pressure will be 200 kPa.

Solving Exponential Equations Algebraically

A2.1 simplify algebraic expressions containing integer and rational exponents using the laws of exponents $\left(\text{e.g., } x^3 \div x^{\frac{1}{2}}, \sqrt{x^6y^{12}}\right)$

Sample problem: Simplify $\dfrac{a^3b^2c^3}{\sqrt{a^2b^4}}$ and then evaluate for $a = 4$, $b = 9$, and $c = -3$. Verify your answer by evaluating the expression without simplifying first. Which method for evaluating the expression do you prefer? Explain.

A2.2 solve exponential equations in one variable by determining a common base (e.g., $2^x = 32$, $4^{5x-1} = 2^{2(x+11)}$, $3^{5x+8} = 27^x$)

Sample problem: Solve $3^{5x+8} = 27^x$ by determining a common base, verify by substitution, and investigate connections to the intersection of $y = 3^{5x+8}$ and $y = 27^x$ using graphing technology.

A2.3 recognize the logarithm of a number to a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation (i.e., the undoing or reversing) of exponentiation, and evaluate simple logarithmic expressions

Sample problem: Why is it possible to determine $\log_{10}(100)$ but not $\log_{10}(0)$ or $\log_{10}(-100)$? Explain your reasoning.

A2.4 determine, with technology, the approximate logarithm of a number to any base, including base 10 [e.g., by recognizing that $\log_{10}(0.372)$ can be determined using the LOG key on a calculator; by reasoning that $\log_3 29$ is between 3 and 4 and using systematic trial to determine that $\log_3 29$ is approximately 3.07]

A2.5 make connections between related logarithmic and exponential equations (e.g., $\log_5 125 = 3$ can also be expressed as $5^3 = 125$), and solve simple exponential equations by rewriting them in logarithmic form (e.g., solving $3^x = 10$ by rewriting the equation as $\log_3 10 = x$)

A2.6 pose problems based on real-world applications that can be modelled with given exponential equations, and solve these and other such problems algebraically by rewriting them in logarithmic form

Sample problem: When a potato whose temperature is 200°C is placed in an oven maintained at 200°C, the relationship between the core temperature of the potato T, in degrees Celsius, and the cooking time t, in minutes, is modelled by the equation $200 - T = 180(0.96)^t$. Use logarithms to determine the time when the potato's core temperature reaches 160°C.

Chapter 4 Planning Chart

Section	Study Guide and Exercise Book Pages	Teacher's Resource Blackline Masters	Assessment	Tools
4.1 The Exponent Laws	60–63	• G–2 Placemat • BLM 4–1 Chapter 4 Prerequisite Skills • T4–1 How to Do Section 4.1 Example Using TI-Nspire™ CAS	• BLM 4–2 Chapter 4 Self-Assessment Checklist • A–4 Selecting Tools and Computational Strategies	• scientific calculator • computer algebra system
4.2 Solving Exponential Equations Graphically	64–67	• G–1 Grid Paper • T–3 *Fathom*™ • T4–2 How to Do Section 4.2 #4 Using *The Geometer's Sketchpad*® • T4–3 How to Do Section 4.2 #7 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS	• A–7 Communicating	• grid paper • graphing calculator • computer • *Fathom*™ • *The Geometer's Sketchpad*®
4.3 Solving Exponential Equations Numerically	68–70	• G–1 Grid Paper • T4–4 How to Do Section 4.3 #6 Using *The Geometer's Sketchpad*®	• A–7 Communicating	• grid paper • scientific calculator • graphing calculator • computer with *The Geometer's Sketchpad*®
4.4 Points of Intersection	71–73	• G–1 Grid Paper • T4–5 How to Do Section 4.4 Example Using TI-Nspire™ CAS		• grid paper • graphing calculator • computer algebra system
4.5 Logarithms	74–76	• G–1 Grid Paper • T4–6 How to Do Section 4.5 #4 Using a Scientific Calculator and a Graphing Calculator	• A–2 Reasoning and Proving • A–7 Communicating	• grid paper • scientific calculator • graphing calculator
4.6 Solving Problems Using Logarithms	77–79	• G–1 Grid Paper • BLM 4–3 Chapter 4 Review • BLM 4–4 Chapter 4 Practice Test • BLM 4–5 Chapter 4 Case Study		• grid paper • scientific calculator • graphing calculator • markers and chart paper

Chapter 4 Blackline Masters Checklist

	BLM	Title	Purpose
4.1 The Exponent Laws			
	G–2	Placemat	Student Support
	A–4	Selecting Tools and Computational Strategies	Assessment
	BLM 4–1	Chapter 4 Prerequisite Skills	Practice
	BLM 4–2	Chapter 4 Self-Assessment Checklist	Assessment
	T4–1	How to Do Section 4.1 Example Using TI-Nspire™ CAS	Technology
4.2 Solving Exponential Equations Graphically			
	G–1	Grid Paper	Student Support
	T–3	*Fathom*™	Technology
	A–7	Communicating	Assessment
	T4–2	How to Do Section 4.2 #4 Using *The Geometer's Sketchpad*®	Student Support
	T4–3	How to Do Section 4.2 #7 Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS	Technology
4.3 Solving Exponential Equations Numerically			
	G–1	Grid Paper	Student Support
	A–7	Communicating	Assessment
	T4–4	How to Do Section 4.3 #6 Using *The Geometer's Sketchpad*®	Technology
4.4 Points of Intersection			
	G–1	Grid Paper	Student Support
	T4–5	How to Do Section 4.4 Example Using TI-Nspire™ CAS	Technology
4.5 Logarithms			
	G–1	Grid Paper	Student Support
	A–2	Reasoning and Proving	Assessment
	A–7	Communicating	Assessment
	T4–6	How to Do Section 4.5 #4 Using a Scientific Calculator and a Graphing Calculator	Technology
4.6 Solving Problems Using Logarithms			
	G–1	Grid Paper	Student Support
	BLM 4–3	Chapter 4 Review	Practice
	BLM 4–4	Chapter 4 Practice Test	Practice
	BLM 4–5	Chapter 4 Case Study	Practice
	BLM 4–6	Chapter 4 BLM Answers	Answers