Chapter 5

Polynomial Functions

Curriculum Expectations

Polynomial Functions

Investigating Graphs of Polynomial Functions

B1.1 recognize a polynomial expression (i.e., a series of terms where each term is the product of a constant and a power of x with a non-negative integral exponent, such as $x^3 - 5x^2 + 2x - 1$); recognize the equation of a polynomial function and give reasons why it is a function, and identify linear and quadratic functions as examples of polynomial functions

B1.2 compare, through investigation using graphing technology, the graphical and algebraic representations of polynomial (i.e., linear, quadratic, cubic, quartic) functions (e.g., investigate the effect of the degree of a polynomial function on the shape of its graph and the maximum number of x-intercepts; investigate the effect of varying the sign of the leading coefficient on the end behaviour of the function for very large positive or negative x-values)

Sample problem: Investigate the maximum number of x-intercepts for linear, quadratic, cubic, and quartic functions using graphing technology.

B1.3 describe key features of the graphs of polynomial functions (e.g., the domain and range, the shape of the graphs, the end behaviour of the functions for very large positive or negative x-values)

Sample problem: Describe and compare the key features of the graphs of the functions $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, and $f(x) = x^4$.

B1.4 distinguish polynomial functions from sinusoidal and exponential functions [e.g., $f(x) = \sin x$, $f(x) = 2^x$)], and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions

B1.5 substitute into and evaluate polynomial functions expressed in function notation, including functions arising from real-world applications

Sample problem: A box with no top is being made out of a 20-cm by 30-cm piece of cardboard by cutting equal squares of side length x from the corners and folding up the sides. The volume of the box is $V = x(20 - 2x)(30 - 2x)$. Determine the volume if the side length of each square is 6 cm. Use the graph of the polynomial function $V(x)$ to determine the size of square that should be cut from the corners if the required volume of the box is 1000 cm^3.

B1.6 pose problems based on real-world applications that can be modelled with polynomial functions, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation

B1.7 recognize, using graphs, the limitations of modelling a real-world relationship using a polynomial function, and identify and explain any restrictions on the domain and range (e.g., restrictions on the height and time for a polynomial function that models the relationship between height above the ground and time for a falling object)

Sample problem: The forces acting on a horizontal support beam in a house cause it to sag by d centimetres, x metres from one end of the beam. The relationship between d and x can be represented by the polynomial function $d(x) = \frac{1}{1850}x(1000 - 20x^2 + x^3)$. Graph the function, using technology, and determine the domain over which the function models the relationship between d and x. Determine the length of the beam using the graph, and explain your reasoning.

Connecting Graphs and Equations of Polynomial Functions

B2.1 factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring)

Sample problem: Factor: $x^4 - 16$; $x^3 - 2x^2 - 8x$.

B2.2 make connections, through investigation using graphing technology (e.g., dynamic geometry software), between a polynomial function given in factored form [e.g., $f(x) = x(x - 1)(x + 1)$] and the x-intercepts of its graph, and sketch the graph of a polynomial function given in factored form using its key features (e.g., by determining intercepts and end behaviour; by locating positive and negative regions using test values between and on either side of the x-intercepts)

Sample problem: Sketch the graphs of $f(x) = -(x - 1)(x + 2)(x - 4)$ and $g(x) = -(x - 1)(x + 2)(x + 2)$ and compare their shapes and the number of x-intercepts.

B2.3 determine, through investigation using technology (e.g., graphing calculator, computer algebra systems), and describe the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function [e.g., the real roots of the equation $x^4 - 13x^2 + 36 = 0$ are the x-intercepts of the graph of $f(x) = x^4 - 13x^2 + 36$]

Sample problem: Describe the relationship between the x-intercepts of the graphs of linear and quadratic functions and the real roots of the corresponding equations. Investigate, using technology, whether this relationship exists for polynomial functions of higher degree.

Chapter 5 Planning Chart

Section	Study Guide and Exercise Book Pages	Teacher's Resource Blackline Masters	Assessment	Tools
5.1 Identifying Polynomial Functions	81–84	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 5–1 Chapter 5 Prerequisite Skills • T5–1 How to Do Section 5.1 #4d) and h) Using *The Geometer's Sketchpad*®	• BLM 5–2 Chapter 5 Self-Assessment Checklist • A–1 Problem Solving • A–2 Reasoning and Proving • A–3 Reflecting • A–4 Selecting Tools and Computational Strategies • A–5 Connecting • A–6 Representing • A–7 Communicating	• grid paper • graphing calculator • computer with dynamic geometry software
5.2 Graphs of Polynomial Functions	85–89	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • T5–2 How to Do Section 5.2 #7 Using *The Geometer's Sketchpad*® • T5–3 How to Do Section 5.2 #11 Using a Graphing Calculator or a Spreadsheet		• grid paper • graphing calculator • pins or tacks • tracing paper • computer with dynamic geometry software • computer with Microsoft® *Excel* or Corel® *Quattro Pro*
5.3 Comparing Polynomial Functions	90–93	• T–2 *The Geometer's Sketchpad*® 4 • T5–4 How to Do Section 5.3 #9 Using *The Geometer's Sketchpad*®		• graphing calculator • computer with dynamic geometry software
5.4 Evaluating Polynomial Functions	94–97	• T–2 *The Geometer's Sketchpad*® 4 • T5–5 How to Do Section 5.4 Example Using TI-83Plus/ TI-84 Plus and TI-Nspire™ CAS	• A–5 Connecting	• graphing calculator • computer with dynamic geometry software
5.5 Solving Problems Involving Polynomial Functions	98–100	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4		• grid paper • graphing calculator • computer with dynamic geometry software
5.6 Factoring Polynomial Expressions	101–104	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • T5–6 How to Do Section 5.6 #4f) Using *The Geometer's Sketchpad*® and TI-Nspire™ CAS		• grid paper • computer algebra system • algebra tiles • computer with dynamic geometry software
5.7 Difference of Squares of Polynomial Expressions	105–106	• T–2 *The Geometer's Sketchpad*® 4		• algebra tiles • computer with dynamic geometry software
5.8 Intercepts of Polynomial Functions	107–109	• G-1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 5–3 Chapter 5 Review • BLM 5–4 Chapter 5 Practice Test • BLM 5–5 Chapter 5 Case Study		• grid paper • graphing calculator • computer algebra system • computer with dynamic geometry software

Chapter 5 Blackline Masters Checklist

	BLM	Title	Purpose
5.1 Identifying Polynomial Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad®*	Student Support
	BLM 5–1	Chapter 5 Prerequisite Skills	Practice
	BLM 5–2	Chapter 5 Self-Assessment Checklist	Assessment
	T5–1	How to Do Section 5.1 #4d) and h) Using *The Geometer's Sketchpad®*	Technology
	A–1	Problem Solving	Assessment
	A–2	Reasoning and Proving	Assessment
	A–3	Reflecting	Assessment
	A–4	Selecting Tools and Computational Strategies	Assessment
	A–5	Connecting	Assessment
	A–6	Representing	Assessment
	A–7	Communicating	Assessment
5.2 Graphs of Polynomial Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad®*	Student Support
	T5–2	How to Do Section 5.2 #7 Using *The Geometer's Sketchpad®*	Technology
	T5–3	How to Do Section 5.2 #11 Using a Graphing Calculator or a Spreadsheet	Technology
5.3 Comparing Polynomial Functions			
	T5–4	How to Do Section 5.3 #9 Using *The Geometer's Sketchpad®*	Technology
5.4 Evaluating Polynomial Functions			
	T5–5	How to Do Section 5.4 Example Using TI-83 Plus/TI-84 Plus and TI-Nspire™ CAS	Technology
5.5 Solving Problems Involving Polynomial Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad®*	Student Support
	A–5	Connecting	Assessment
5.6 Factoring Polynomial Expressions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad®*	Student Support
	T5–6	How to Do Section 5.6 #4f) Using *The Geometer's Sketchpad®* and TI-Nspire™ CAS	Technology
5.7 Difference of Squares of Polynomial Expressions			
	T–2	*The Geometer's Sketchpad®*	Student Support
5.8 Intercepts of Polynomial Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad®*	Student Support
	BLM 5–3	Chapter 5 Review	Practice
	BLM 5–4	Chapter 5 Practice Test	Practice
	BLM 5–5	Chapter 5 Case Study	Practice
	BLM 5–6	Chapter 5 BLM Answers	Answers