Chapter 2 Review

2.1 Graphs of Sinusoidal Functions

1. **a)** Determine $y = \sin x$ for each angle and all the related angles, from 0° to 360°.

x	$y = \sin x$	
	Exact	**Approximate**
0°		
30°		
45°		
60°		
90°		

b) Graph $y = \sin x$.
c) Explain why $y = \sin x$ is a function.

2. For the functions $y = \sin x$ and $y = \cos x$ on the interval $0° \le x \le 360°$, determine
a) the amplitude
b) the period
c) the domain and range
d) the x- and y-intercept(s)
e) the intervals of increase and decrease

3. A bicycle wheel has diameter 24 in. Let the origin be at the centre of the wheel. Consider a point on the tire along the positive x-axis.
a) Sketch a graph of the horizontal displacement of this point versus the angle of rotation for one cycle. Which function, sine or cosine, models the horizontal displacement? Explain.
b) Sketch a graph of the vertical displacement of this point versus the angle of rotation for one cycle. Which function models the vertical displacement? Explain.

2.2 Translations of Sinusoidal Functions

4. Determine the phase shift and the vertical translation.
a) $y = \sin (x + 14°) – 12$
b) $y = \sin (x – 5°) + 7$
c) $y = \cos (x – 36°) – 4$
d) $y = \cos (x + 85°) + 10$

5. Determine the phase shift and the vertical shift. Sketch two cycles of the function.
a) $y = \sin (x – 30°) + 3$
b) $y = \cos (x + 40°) – 1$

6. **Use Technology**
a) Graph the function $y = \sin^3 x$ on the interval $0° \le x \le 720°$.
b) Predict how the graph of $y = \sin^3 (x + 90°) – 2$ will differ from the graph of $y = \sin^3 x$. Check by graphing.
c) Predict how the graph of $y = \cos^3 x – 2$ will compare to the graph of $y = \sin^3 (x + 90°) – 2$. Check by graphing.

2.3 Stretches, Compressions, and Reflections of Sinusoidal Functions

7. Determine the vertical stretch or compression and the horizontal stretch or compression.
a) $y = 5 \sin 3x$
b) $y = –3 \cos 2x$
c) $y = \frac{1}{2} \sin \frac{1}{3}x$
d) $y = 6 \cos \frac{1}{4}x$

8. Determine the amplitude and period, and then sketch a graph of two cycles.
a) $y = 2 \sin 3x$
b) $y = 4 \cos \frac{1}{2}x$
c) $y = -\frac{1}{2} \sin 4x$
d) $y = 3 \cos 2x$

9. The graph of $f(x) = \sin x$ is vertically stretched by a factor of 5, horizontally stretched by a factor of 3, and reflected in the x-axis. Write an equation for the new function, $g(x)$.

978-07-090900-7

2.4 Combining Transformations of Sinusoidal Functions

10. Determine the amplitude, period, phase shift, and vertical translation of each function.

a) $y = 2 \sin [3(x + 18°)] + 9$

b) $y = \frac{1}{5} \cos [5(x - 37°)] + 4$

c) $y = -7 \sin [12(x + 46°)] - 3$

d) $y = 4 \cos \left[\frac{2}{5}(x - 10°)\right] - 8$

11. a) Describe the transformations that must be applied to $f(x) = \sin x$ to obtain $g(x) = 3 \sin 4x + 2$. Apply each transformation, one step at a time, to sketch one cycle of $g(x)$.

b) State the domain and range of $g(x)$.

c) Suppose $g(x)$ is shifted 30° to the right to obtain $h(x)$. Write the equation of $h(x)$, and then graph $h(x)$.

12. Consider the function $g(x) = 2 \cos [3(x + 60°)] - 1$.

a) State the amplitude, period, phase shift, and vertical translation.

b) Transform the graph of $f(x)$ to $g(x)$. Show each step in the transformation.

c) State the domain and range of $g(x)$.

2.5 Representing Sinusoidal Functions

13. a) Write an equation in the form $y = a \sin kx$ of a sine function that has amplitude 4, period 120°, and a maximum at (30°, 4).

b) Write an equation in the form $y = a \cos kx$ of a cosine function that is reflected in the x-axis, has amplitude 5, period 720°, and a maximum at (360°, 5).

14. Write an equation of a function in the form $y = a \sin [k(x - d)] + c$ for the graph of $f(x) = \sin x$ after it has been vertically compressed by factor $\frac{1}{3}$, horizontally stretched by factor 2, shifted right 16°, and translated down 9 units.

15. Write an equation of a function in the form $y = a \cos [k(x - d)] + c$ for the graph of $f(x) = \cos x$ after it has been reflected in the y-axis, vertically stretched by factor 8, horizontally compressed by factor $\frac{2}{3}$, shifted left 40°, and translated up 4 units.

2.6 Solving Problems Involving Sinusoidal Functions

16. André measured tides at the Bay of Fundy. He recorded his data in a table.

Time	Water Height (m)
7 a.m.	13
10 a.m.	9
1 p.m.	5
4 p.m.	9
7 p.m.	13

a) Construct a cosine model for the water height, h, as a function of n, the number of hours past high tide.

b) Construct a sine model for the water height, h, as a function of n, the number of hours past high tide.

17. The height above ground of a rider on a Ferris wheel can be modelled by the equation $h(t) = 14 \sin [18(t - 5)] + 16$, where t represents time, in seconds, and h represents the height, in metres, of the rider above the ground.

a) What is the initial height of the rider?

b) How long does one rotation take?

c) What is the maximum height of the rider?

978-07-090900-7