Chapter 4 Practice Test

1. Simplify. Express your answers using only positive exponents.

a) $w^{-8} \times w^2$　　　　**b)** $(x^3)^{-4}$

c) $\dfrac{-24h^{-4}}{-3h^{-10}}$　　　　**d)** $(6a^{-4}b^3)^{-2}$

e) $\left(\dfrac{9}{xy}\right)^{-2}$　　　　**f)** $\left(\dfrac{3c^{-2}}{4d}\right)^{-3}$

2. Evaluate. Express your answers as integers or as fractions reduced to lowest terms.

a) $-\left(\dfrac{3}{4}\right)^{-2}$　　　　**b)** $\dfrac{(9.2)^{\frac{1}{3}}(9.2)^{\frac{1}{2}}}{9.2^{\frac{5}{6}}}$

c) $\sqrt[4]{16^3}$　　　　**d)** $\dfrac{4}{5}\left(\dfrac{256}{625}\right)^{-\frac{1}{2}}$

3. Simplify. Express your answers using only positive exponents.

a) $v^{\frac{1}{5}} \times v^{-\frac{3}{10}}$　　　　**b)** $\sqrt[3]{n^8} \div \sqrt{n^3}$

c) $\left(x^{\frac{3}{4}}\right)^{-\frac{2}{9}}$　　　　**d)** $\dfrac{s^{-\frac{1}{3}}t^{\frac{2}{3}}}{s^{-\frac{5}{12}}t^{\frac{7}{9}}}$

4. Violent storms, such as tornadoes and hurricanes, can be described using the formula $D = 9.4t^{\frac{2}{3}}$. In this formula, D is the diameter of the storm, in kilometres, and t is the time, in hours, that the storm lasts. If a hurricane lasts for 18 h, what is its diameter, to the nearest kilometre?

5. a) Graph the functions $y = \left(\dfrac{1}{3}\right)^x$ and $y = 9$.

b) Determine the point of intersection from the graph.

c) Use another method to determine the x-value of the point of intersection.

6. a) Sketch the graph of an exponential function that has the following properties:
- domain $\{x \in \mathbb{R}\}$
- range $\{y \in \mathbb{R},\ y > 0\}$
- y-intercept 3
- horizontal asymptote $y = 0$
- increasing function

b) Is it possible to sketch the graph of another exponential function with the same properties as the exponential function in part a)? Explain your answer.

7. Solve. Express your answers as integers or as fractions reduced to lowest terms.

a) $4^x = \dfrac{1}{64}$　　　　**b)** $9^{3x} = 27^{x-3}$

c) $\left(\dfrac{16}{49}\right)^x = \dfrac{343}{64}$　　　　**d)** $5^{\frac{1}{2}(x+2)} = 25^{2(x-1)}$

8. On March 1, Juveria heard the principal say there would be a surprise celebrity guest at this year's prom. The next day, Juveria told her friend. The following day, each of them told another friend. The next day, each of these people told another friend, and so on.

a) Write an equation that relates the number, n, of people who have heard the plan to the number of days, d, after March 1.

b) On what date will 128 people be told the principal's plan?

c) After what date will more than 1000 people be told the principal's plan?

9. A car is purchased for $24 000, and its value decreases 15% every year.

a) Write an equation representing this situation.

b) After how many years will the car be worth 10% of its original value?

c) By what amount does the car depreciate during the first year? during the fifth year? Explain why these amounts are significantly different.

10. Determine the solution to the equation $3.4^x = 100\ 000$, to one decimal place.

11. Determine the point of intersection of the functions $y = 4^{5x-2}$ and $y = 16^{x+2}$.

12. Solve $27^{x-1} = 3^{x+1}$. Describe how the graphs of these functions interact.

13. Jason is a salesperson at a shoe store. The table shows his monthly earnings.

Month	Earnings ($)
January	3100
February	3131
March	3162
April	3194
May	3226

   a) Construct an equation that models Jason's monthly earnings.
   b) Based on this model, predict his earnings for September.
   c) Do you think this model is realistic? Explain.

14. Write each equation in logarithmic form.

   a) $3^4 = 81$ **b)** $2^{-3} = \dfrac{1}{8}$
   c) $2401 = 7^4$ **d)** $5^0 = 1$

15. Evaluate each of the following logarithms.
   a) $\log_6 216$ **b)** $\log_2 128$
   c) $\log 0.0001$ **d)** $\log \dfrac{1}{10^{-7}}$

16. Explain why $\log_b \sqrt{b} = \dfrac{1}{2}$.

17. Explain why the base on a logarithm cannot be negative, 0, or 1.

18. a) Explain how to use a scientific calculator to evaluate $\log_2 75$ using two methods.
   b) Use one of these methods to find the value of $\log_2 75$, to two decimal places.

19. a) Sketch the graph of $y = 2^x$ and $y = \log_2 x$.
   b) Complete the following table.

	$y = 2^x$	$y = \log_2 x$
Domain		
Range		
x-intercepts		
y-intercepts		
Vertical asymptote		
Horizontal asymptote		
Intervals of increase		
Intervals of decrease		

20. An investment of $3000 earns 3.5%, compounded semi-annually.
   a) Write an equation that models the amount, A, in dollars, of the investment after time, t, in years.
   b) What is the amount of the investment after 5 years?
   c) How long will it take to double in value?

21. The magnitude, M, of an earthquake is measured using the Richter scale, which is defined as $M = \log\left(\dfrac{I}{I_0}\right)$, where I is the intensity of the earthquake being measured and I_0 is the intensity of a standard, low-level earthquake.
   a) How many times as intense as a standard earthquake is one measuring 4.5?
   b) What is the magnitude of an earthquake that is 100 000 times as intense as a standard earthquake?

22. The pH scale is used to determine how acidic or alkaline a substance is. The pH scale is defined as $\text{pH} = -\log [H^+]$, where H^+ is the hydronium ion concentration in a substance, measured in moles per litre.
   a) Orange juice has a hydronium ion concentration of approximately 0.0005 mol/L. Determine its pH.
   b) Distilled water has a pH of 7. Determine its concentration of hydronium ions.