Challenge Questions 1

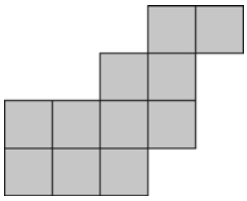
Principles of Mathematics 9 Exercise and Homework Book

1. **Perfect squares** If you take two of each of the numbers from 1 to 13, you now have 26 numbers. Write these numbers in pairs so that the sum of every pair of numbers is a perfect square.

2. **English** When you write the whole numbers in words, "one," "two," "three," and so on, what is the first word that has the letters in alphabetical order?

3. **River crossing** Two grade 6 students and two grade 9 students want to cross a river in a canoe. The canoe is big enough to hold the two grade 6 students or one grade 6 student and one grade 9 student. How many times must the canoe cross the river to get all the students to the other side?

4. **Measurement** The figure is made up of 11 identical squares. The area of the figure is 539 cm^2.

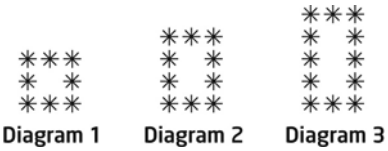


What is the perimeter of the figure?

5. **Whole numbers** Find 5 consecutive whole numbers whose sum is 405.

6. **Whole numbers** Jason chose a whole number less than 10. He multiplied the number by 6 and added 1. The result was a perfect square. What numbers could he have chosen?

7. **Asterisks** The rectangular shapes are made from asterisks.



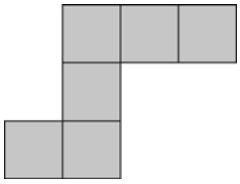
   a) How many asterisks will there be in the 4th rectangle? the 5th rectangle?

   b) How many asterisks will there be in the *n*th rectangle?

   c) How many asterisks will there be in the 26th rectangle?

   d) Which rectangle is made from 92 asterisks?

8. **Measurement** The squares are exactly the same size. The total area of the figure is 384 cm^2. What is the perimeter of the figure?



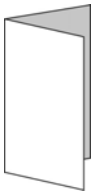
9. **Lifeguards** There are 12 h of lifeguard work available for the weekend. Three lifeguards have each agreed to work a whole number of hours. In how many different ways can the 12 h be divided so that each person works at least 2 h?

10. **Floor tiles** A rectangular floor is tiled with 36 square tiles. The tiles around the outside edge of the rectangle are red, and the tiles on the inside are white. How many red tiles are there? Is there more than one possible answer?

Challenge Questions 2

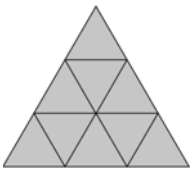
Principles of Mathematics 9 Exercise and Homework Book

1. **Measurement** A square piece of paper is folded in half as shown. The perimeter of each new rectangle formed is 24 cm. What is the perimeter of the original square?



2. **Toothpicks** There are 20 toothpicks in a pile. In how many ways can they be organized into 3 groups, with an even number of toothpicks in each group?

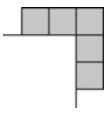
3. **Triangle design** How many triangles are in the 1st row? The 2nd row? The 3rd row? The 4th row? If the pattern continues, how many triangles are in the 10th row? The 20th row? The 100th row? The *n*th row?



4. **Number pattern** Determine the pattern and write the next 3 rows.

1 1 1
1 2 3 2 1
1 3 6 7 6 3 1
1 4 10 16 19 16 10 4 1

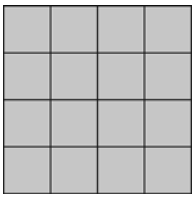
5. **Picture border** A picture 20 cm by 20 cm is to be bordered by 1-cm squares. How many squares are needed for the border?



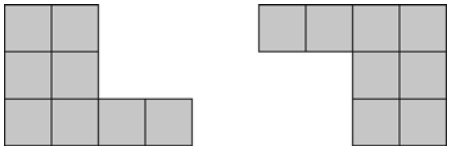
6. **Dividing grids** The diagram shows two ways to divide the grid into two congruent pieces using line segments that connect grid points.

How many other ways are there, if rotations and reflections are not permitted?

7. **Geometry** This is a 4 by 4 square.



One way to separate it into two congruent shapes, made up of smaller squares, is shown.



Find at least five other ways.

8. **Latin Squares** The diagram shows a Latin Square. The numbers 1, 2, and 3 have been placed so that each number appears only once in each row and column.

1	2	3
2	3	1
3	1	2

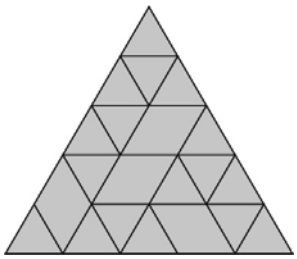
There are 11 other different Latin Squares that use the numbers 1, 2, and 3. Draw them.

Challenge Questions 3

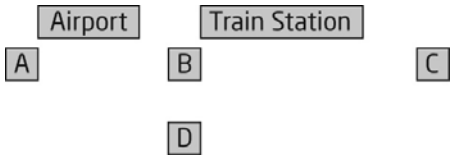
Principles of Mathematics 9 Exercise and Homework Book

1. **Measurement** This question was part of the Canada/U.S. qualifying test for the First World Puzzle Team Championship, held in 1992.

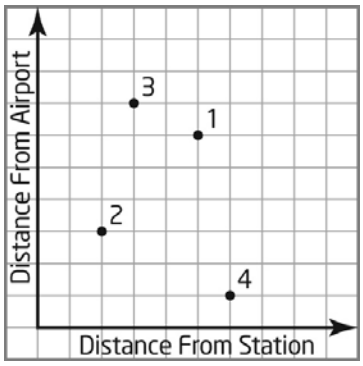
   How many triangles with side length 1 unit, with side length 2 units, with side length 3 units, with side length 4 units, and with side length 5 units are in this figure?



2. **Comparing models** The map shows the locations of 4 houses, the train station, and the airport.



   Match each house with the correct number of the distance graph. Give reasons for your answer.



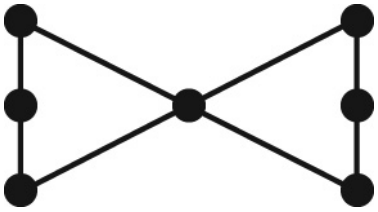
3. **Triangular numbers** the first 4 triangular numbers are shown.

   What are the next 3 triangular numbers?

4. **Summer job** Jennifer works at a hardware store during her summer vacation. She earns $12.50/h for up to 40 h/week. She earns time-and-a-half for hours over 40 h/week. One week she worked 45 h. How much did she earn?

5. **Measurement** The perimeter of an isosceles triangle is 8 cm. The length of each side is a whole number. How long is the shortest side?

6. **Number puzzle** Place the digits 1, 2, 3, 4, 5, 6, and 7 in the circles so that the sum of each line of connected circles is the same.



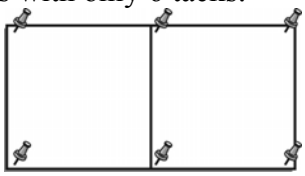
7. **Grouping numbers** In how many different ways can you place the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 into groups so that the sum of the numbers in each group is 15?

8. **TV watching** Sketch a graph to show the length of time you spend watching TV versus the day of the week.

Challenge Questions 4

Principles of Mathematics 9 Exercise and Homework Book

1. Bulletin board To hang a picture on a bulletin board, Masao uses 4 thumbtacks, 1 in each corner. For two pictures of the same size, Masao can overlap the corners and hang both pictures with only 6 tacks.

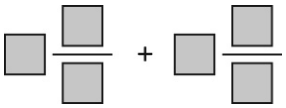


a) What is the minimum number of tacks Masao needs to hang 6 pictures of the same size in a row?

b) Write an expression for finding the number of tacks needed to hang any number of pictures in a row.

2. Driving distances A delivery van and a bus left a highway garage at the same time and travelled in opposite directions. The van travelled at 80 km/h, and the bus travelled at 75 km/h. How far apart were the two vehicles after 3.5 h?

3. Smallest sum Use each of the digits 1, 3, 4, 6, 7, and 8 only once. Make the smallest possible sum by arranging the numbers as indicated in the diagram. What is the sum?



4. Counterfeit coin What is the minimum number of mass comparisons an inspector needs to make to find 1 counterfeit coin in a collection of 40 coins? Assume that the counterfeit coin is lighter than the others.

5. Toothpicks The trapezoids are made from toothpicks.

a) How many toothpicks are in the fourth diagram? the fifth diagram?

b) Describe the pattern in words.

c) Write an expression that represents the number of toothpicks needed for the nth diagram in terms of n.

d) Using the expression from part c) how many toothpicks are there in the 50th diagram? The 80th diagram?

6. Leap year If New Year's Day in a leap year is on a Tuesday, on what day of the week does July 1 fall?

7. Cruise departure The ship will leave the harbour at 08:30. You have to be on board 20 min before departure. It takes 25 min to drive to the ship from your hotel. You must allow 15 min to check out of the hotel. It will take you 20 min to pack. You need half an hour to eat breakfast and at least 45 min to shower and dress. For what time should you place your wake-up call?

8. Coins How many different combinations of coins have a value of \$0.28? Copy and complete the table to find out.

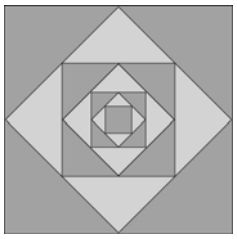
Combinations	\$0.25	\$0.10	\$0.05	\$0.01
1	1	0	0	3
2	0	1	2	8
3				

9. Estimating distances About how many kilometres will you travel on school property this school year?

Challenge Questions 5

Principles of Mathematics 9 Exercise and Homework Book

1. **Square design** The area of the large square is 64 cm^2.



Each smaller square is formed by joining the midpoints of the sides of the next larger square. What is the area of the smallest square?

2. **Difference of squares** The whole number 5 can be written as the difference of the squares of two whole numbers.

$$5 = 3^2 - 2^2$$

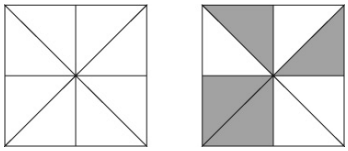
What other whole numbers between 1 and 10 can be written as the difference of the squares of two whole numbers?

3. **Student council** In a student council election, there are 2 candidates for president: Ali and Beth. There are 3 candidates for vice-president: Connie, Devo, and Eleanor. There are 3 candidates for treasurer: Franco, Gino, and Helen. One possible set of winners is Beth, Dev, and Helen. How many others are there?

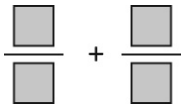
4. **Chores** Kim has two chores at home. Every 4 days, she must clean the gerbil cage. Every 6 days, she must clean the canary cage. Last Monday, she did both jobs. On what day of the week will she next do both jobs?

5. **Making change** List the different ways you can make change for a dollar using only quarters and nickels.

6. **Designs** The square has been divided into 8 triangles. One way to shade 4 of the 8 triangles is shown. How many other different ways to shade 4 triangles are there?



7. Copy the diagram. Fill in the boxes with the digits 2, 3, 4, and 5 to make the greatest possible sum.



8. **Shopping**. You spend $2.75 in a store and receive $7.25 change from $10.00. Notice that the arrangement of the digits in the amount you spent is a rearrangement of the digits in your change. Find 4 other pairs of amounts spent and change from $10.00 that share this property.

9. **Bus schedule** The Beach bus leaves every 20 min, and the Sand bus leaves every 45 min. If they leave together at noon, when is the next time that they will eave together?

10. **Guessing game** Is it possible to guess any number from 1 to 1024 in 10 guesses or fewer if you are told on each guess that it is correct, too large, or too small? Explain.

11. **Phone calls** About how many hours do all the high school students in Ontario spend on the telephone in a year?

Challenge Questions 6

Principles of Mathematics 9 Exercise and Homework Book

1. **Order of operations** Use six 9s and the order of operations to write an expression that equals 100.

2. **Numbers** Find three consecutive whole numbers who sum is 144.

3. **Time** How many hours are in 1 000 000 s?

4. **Seating arrangements** Justine, Chris, and Meelang plan to travel together in a van. They will all sit in the front seat. In how many different arrangements can they sit in each of the following situations?
   a) all three can drive
   b) only Chris and Meelang can drive
   c) only Justine can drive

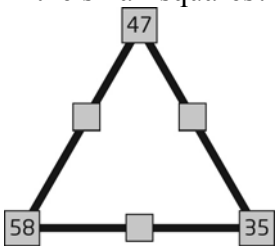
5. **Coin collection** In a collection of coins, the number of dimes and nickels add to 5. The numbers of dimes and quarters add to 7. The numbers of nickels and quarters add to 8. How many of each type of coin are there?

6. **Averages** The average of two numbers is 21. When a third number is included, the average of the three numbers is 23. What is the third number?

7. **Cutting cheese** A block of cheese, covered with wax, measures 12 cm by 12 cm by 10 cm. The block is cut into 2-cm cubes.
   a) How many cubes are there?
   b) How many cubes have wax on 3 faces?
   c) How many cubes have wax on 2 faces?
   d) How many cubes have no wax on them?

8. **Consecutive integers** Find 3 consecutive integers whose product is 1716.

9. **Squares** The numbers in the large squares are found by adding the numbers in the small squares. What are the numbers in the small squares?



10. **Melting Ice** You fill a glass half full of water from the tap. Then, you add enough ice cubes to the water to fill the glass. Sketch a graph of temperature versus time from the moment you add the ice to the water until the moment when the ice has all melted.

11. **Train trip** Your train leaves at 08:15. The bus trip to the train station takes 25 min. The bus stop is a 5-min walk from your place. You should get to the train station to buy your ticket 15 min before the train leaves. It will take you 55 min to get dressed, eat breakfast, and pack. For what time should you set your alarm clock?

12. **Trees** Estimate the number of trees within a 2 km radius of your school.

13. **Making a cube** What is the edge length of a cube that can be made with 294 cm^2 of cardboard? What assumptions have you made?

14. **Islands** About how many islands are there in Ontario?

Challenge Questions 7

Principles of Mathematics 9 Exercise and Homework Book

1. Products Determine the pattern and predict the next two lines.

$101 \times 101 = \square$

$202 \times 202 = \square$

$303 \times 303 = \square$

2. Team points A team gets 2 points for a win, 1 point for a tie, and no points for a loss. The Bears have played 28 games. They have 27 points and 7 losses. How many wins do they have?

3. Letters Assume that the following pattern continues.

A, BBB, CCCCC, DDDDDDD, …

a) How many letter Ms will there be?

b) How many letter Zs will there be?

4. Buying chicken Chicken pieces come in boxes of 6, 9, and 20. You can buy 21 pieces by buying two boxes of 6 pieces and one box of 9 pieces.

6 + 6 + 9 = 21

Can you buy

a) 41 pieces?

b) 42 pieces?

c) 43 pieces?

d) 44 pieces?

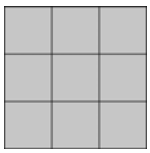
5. Pattern If the pattern continues, find the following product.

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{24}\right)$$

6. Purchase price The amount of a purchase is $12.43. How can the exact amount be paid without using a $10.00 bill, but using the smallest number of bills and coins?

7. Pop machine Students are allowed into the school at 08:00. Classes start at 09:00. There are 15-min breaks that begin at 10:30 and 14:30. Lunch is from 12:00 to 13:00. School is dismissed at 16:00. Students must leave the building by 17:00. Students are allowed to use the pop machine in the school cafeteria before and after school and during their breaks and lunch hour. The machine is filled twice a day, at 07:00 and 14:00. Sketch a graph of the number of cans in the machine versus time on a hot Monday in September.

8. Measurement The perimeter of the figure is 12 units.



Sketch your answers to the following.

a) Remove 1 square and keep the perimeter the same.

b) Remove 2 squares and keep the perimeter the same.

c) Remove 1 square and increase the perimeter by 2.

d) Remove 2 squares and increase the perimeter by 2.

e) Remove 2 squares and increase the perimeter by 4.

9. Book pages If you open a book and the product of the page numbers on the two facing pages is 45 156, what are the page numbers?

10. GCF The greatest common factor of two numbers, m and n, is 14.

If $m = 2 \times 5 \times 7^2$, name three numbers that could be n.

Challenge Questions 8

Principles of Mathematics 9 Exercise and Homework Book

1. Consecutive numbers The number 63 can be written as the sum of consecutive whole numbers as follows.

$63 = 20 + 21 + 22$

a) Find another way to write 63 as the sum of consecutive whole numbers.

b) Find four consecutive whole numbers that add to 138.

2. Number puzzle Copy the diagram and fill in the squares with the numbers 1 to 8 so that consecutive numbers are not adjacent in any direction – horizontally, vertically, or diagonally.

3. Calendar In one year, December had exactly four Tuesdays and four Saturdays. On what day did December 1 fall that year?

4. Railway crossing A train travelling at 90 km/h passes a car sitting at a railway crossing. It took 45 s for the train to pass. How long was the train, in metres.

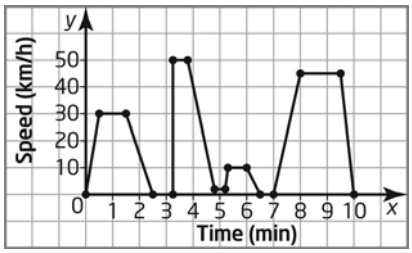
5. Coffee shop A coffee shop on the corner of a busy intersection is open 24 h a day. Sketch a graph of the number of customers in the shop versus the time of day.

6. Line-up You are standing in line at the cafeteria. You are the seventh from the front and eighth from the end. How many people are in the line?

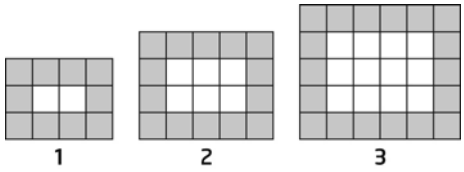
7. Perfect squares The number 2601 is a 4-digit number that is a perfect square, because $51^2 = 2601$. What is the smallest 4-digit number that is a perfect square and that has all even digits?

8. Library books For all the books in your school library, estimate the total number of words.

9. Car trip The graph shows the speed of a car for 10 min. Write a story to explain the graph.



10. Design Each rectangular design is made with grey border squares and white interior squares. Each square has a side length of 1 unit.



a) How many grey squares are in the fourth diagram? the fifth diagram?

b) Write an expression for the number of grey squares in terms of the length of the design, l.

c) How many grey squares are there in the design with a length of 14? 40?

11. Difference of squares The number 16 can be written as the difference of two squares.

$$16 = 25 - 9$$
$$= 5^2 - 3^2$$

What other whole numbers between 10 and 20 can be written as the difference of two squares?

Challenge Questions 9

Principles of Mathematics 9 Exercise and Homework Book

1. **Birthday** Two days ago, Robert was 16 years old. Next year, Robert will be 19 years old. What is today's date and when in Robert's birthday?

2. **Animal speeds** This table shows the maximum speeds of some animals.

Animal	Maximum Speed (km/h)
Ostrich	80
Coyote	72
Elephant	40
Porcupine	18

How many minutes less than a porcupine does a coyote take to run 1 km? What assumptions have you made?

3. **Apartments** An apartment building has six floors. The top floor has one apartment. Each of the other floors has twice the number of apartments as the floor above it. How many apartments are there?

4. **Walking** Sketch a graph that shows the distance you walk in a school day. Label the vertical axis "Distance Walked" and the horizontal axis "Time of Day." Represent a complete 24-h period, starting at midnight. Compare your graph with a classmate's and explain any differences.

5. **Calendar math** If two months in a row have a Friday the 13th, what months are they? Explain.

6. **Fence posts** Three sides of a triangular field are 40 m, 44 m, and 52 m. A fence is to be built around the field with a post in each corner and the posts 4 m apart. How many fence posts are needed?

7. **Piano tuners** About how many piano tuners are there in Canada?

8. **Compact discs** Tania has a collection of compact discs. When she puts them in piles of 2, she has 1 left over. She also has 1 left over when she puts them in piles of 3 and piles of 4. She has none left over when she puts them in piles of 7. What is the smallest number of compact discs she can have?

9. **Running** Lauryn and Yolanda had a 100-m race. Lauryn beat Yolanda by 10 m. For the second race, Yolanda suggested that Lauryn start 10 m behind the starting line. Yolanda thought this way of starting would give her a fair chance.
   a) Who won the race and by how much?
   b) What assumptions did you make?

10. **Bean toss** In a bean toss game, each person throws two bags.
Scoring A and B gives 18 points.
Scoring A and C gives 15 points.
Scoring B and C gives 13 points.
What will you score if you toss both bags in B?

11. **Sentence** Copy the sentence. Then, fill in the blank with a number, written in words, to make the sentence true.
This sentence has _____ letters.

12. **Even numbers** Six consecutive even numbers are written on a piece of paper. If the sum of the first three numbers is 60, what is the sum of the last three?

13. **Books** How many books are in the library at your school?

Challenge Questions 10

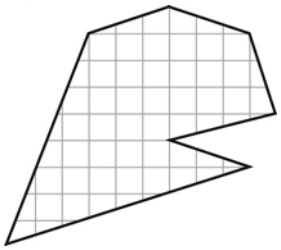
Principles of Mathematics 9 Exercise and Homework Book

1. Perfect squares When you add the digits of the number 45, you create a perfect square.

$4 + 5 = 9$ or 3^2

How many other numbers between 1 and 100 have digits that add to give perfect squares?

2. Measurement Calculate the area of the figure, in square units, if each grid square represents 1 square unit.



3. Numbers Use any of the arithmetic operations and brackets, if necessary, to combine each set of numbers so that they equal the number in brackets.

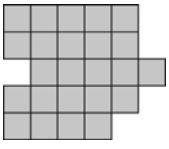
a) 4, 5, 12 [8]
b) 3, 7, 14 [10]
c) 4, 6, 7, 8 [15]
d) 3, 5, 6, 11 [19]

4. Ages The number 72 can be factored as follows.

$72 = 36 \times 2$
$= 18 \times 2 \times 2$
$= 9 \times 2 \times 2 \times 2$
$= 3 \times 3 \times 2 \times 2 \times 2$

Use this technique to find the age of each person in a group of teenagers, if the product of their ages is 661 500.

5. Squares What is the total number of squares in this diagram?



6. Perfect cubes The number 8 is a perfect cube.

$2^3 = 2 \times 2 \times 2 = 8$ or $2^3 = 8$

The number 27 is also a perfect cube.

$3^3 = 3 \times 3 \times 3 = 27$ or $3^3 = 27$

You can write 8 as the sum of two consecutive odd numbers.

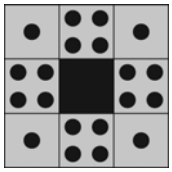
$3 + 5 = 8$

You can write 27 as the sum of three consecutive odd numbers.

$7 + 9 + 11 = 27$

a) The next perfect cube is 64 because $4^3 = 64$. Write 64 as the sum of 4 consecutive odd numbers.
b) The next perfect cube is 125. Write 125 as the sum of five consecutive odd numbers.
c) The next perfect cube is 216. Write 216 as the sum of six consecutive odd numbers.
d) How does the square of the number that is cubed fit into the sum of the consecutive odd numbers?
e) Use this pattern to write 7^3 as the sum of seven consecutive odd numbers.

7. Placing counters The diagram shows how 20 counters have been placed in 8 squares to that there are 6 counters in each row of 3 squares.



Rearrange the counters so that there are 7 counters in each row of squares.