5.1 Direct Variation

***Principles of Mathematics 9*, pages 238–245**

A

1. Determine the constant of variation for each direct variation.

a) The distance travelled by a car varies directly with time. The car travels 270 km in 3 h.

b) The distance travelled on a trip varies directly with the amount of gas used. A car travelled 375 km and used 25 litres of gas.

c) The money earned by an employee varies directly with time. The employee earned $320 in 40 h.

2. The cost, C, in dollars, of building a patio varies directly with its width, w, in metres.

a) Find an equation relating C and w if the cost of building a patio with a width of 4 m is $300.

b) What does the constant of variation represent?

c) Use the equation to determine the cost of a patio with a width of 7 m.

3. The total cost of potatoes varies directly with the mass, in kilograms, bought. Potatoes cost $2.18/kg.

a) Choose appropriate letters for variables. Make a table of values showing the cost of 0 kg, 1 kg, 2 kg, 3 kg, 4 kg, and 5 kg of potatoes.

b) Graph the relationship.

c) Write an equation for the relationship in the form $y = kx$.

B

4. A marina charges $9.50 per hour to rent a boat.

a) Describe the relationship between the cost of the boat rental and the time, in hours, the boat is rented for.

b) Illustrate the relationship graphically and represent it with an equation.

c) Use your graph to estimate the cost of renting the boat for 12 h.

d) Use your equation to determine the exact cost of renting the boat for 12 h.

5. A rental agency charges $8 per hour to rent a canoe.

a) Describe the relationship between the cost of the canoe rental and the time, in hours, the canoe is rented for.

b) Illustrate the relationship graphically and represent it with an equation.

c) Use your graph to estimate the cost of renting the canoe for 8 h.

d) Use your equation to determine the exact cost of renting the canoe for 8 h.

6. A parking lot charges $14.50 per day for long-term parking at the airport.

a) Describe the relationship between the cost of the long-term parking and the time, in days, the car is parked for.

b) Illustrate the relationship graphically and represent it with an equation.

c) Use your graph to estimate the cost of parking the car for 6 days.

d) Use your equation to determine the exact cost of parking the car for 6 days.

7. The cost of a certain type of cookies varies directly with the number of packages of cookies that are purchased. The cookies cost $3.50/package.

   a) Choose appropriate letters for variables. Make a table of values showing the cost of 0 packages, 1 package, 2 packages, 3 packages, and 4 packages.

   b) Graph the relationship.

   c) Write an equation for the relationship in the form $y = kx$.

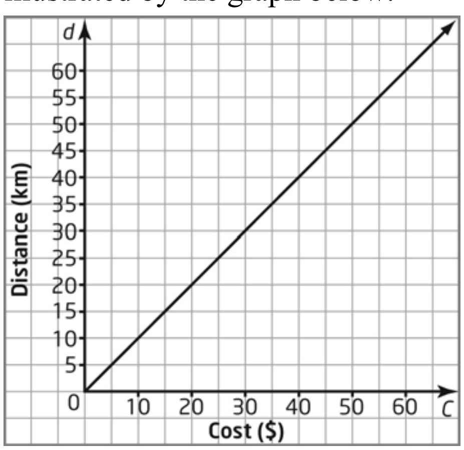
8. Alison has a part-time job as a lifeguard. Alison's pay varies directly with the time, in hours, she works. She earns $9.75/h.

   a) Explain why this relationship is considered a direct variation.

   b) Write an equation representing Alison's regular pay.

   c) Graph this relationship, using pencil and paper or technology.

9. Describe a situation that could be illustrated by the graph below.



C

10. To raise money for a local charity, students organized a walk-a-thon. For the walk-a-thon, the amount of money raised by each student varied directly with the number of kilometres walked. Dieter raised $320 by walking 20 km.

    a) Graph this direct variation for distances from 0 km to 20 km, using pencil and paper or technology.

    b) Write an equation relating the money Dieter raised and the distance, in kilometres, that he walked.

    c) How much would he have raised by walking 25 km?

11. The volume of water in a water tank varies with time. The tank contains 200 L of water after 2 min.

    a) Write an equation relating the volume of water and time. What does the constant of variation represent?

    b) Graph this relationship using pencil and paper or technology.

    c) What volume of water is in the tank after 30 min?

    d) How long will it take to fill a water tank that can hold 100 000 L of water?

12. To convert from Canadian (= British Imperial) gallons to litres, multiply by 4.546. Write an equation to convert litres to Canadian gallons. Round the constant of variation to the nearest thousandth.

5.2 Partial Variation

***Principles of Mathematics 9*, pages 246–253**

A

1. Identify each relation as a direct variation, a partial variation, or neither. Justify your answer.

a) $y = 10x$ **b)** $C = 4t + 3$

c) $y = 3x + 2$ **d)** $d = 3t$

2. a) Copy and complete the table of values given that y varies partially with x.

x	y
0	4
1	7
2	
3	13
4	
	25

b) Identify the initial value of y and the constant of variation from the table.

c) Write an equation relating y and x in the form $y = mx + b$.

d) Graph the relation.

e) Describe the graph.

3. a) Copy and complete the table of values given that y varies partially with x.

x	y
0	−3
1	1
2	
3	9
4	
	29

b) Identify the initial value of y and the constant of variation from the table.

c) Write an equation relating y and x in the form $y = mx + b$.

d) Graph the relation.

e) Describe the graph.

B

4. A charitable organization is planning to rent a hall for a fundraiser. The cost of renting the hall is \$200. There is an additional cost of \$3 for each person attending the fundraiser for the entrance fee.

a) Identify the fixed cost and the variable cost of this partial variation.

b) Write an equation relating the cost, C, in dollars, and the number of people, n.

c) Use your equation to determine the total cost if 100 people attend the fundraiser.

5. A cellular phone company offers two types of monthly plans:

• Plan A: 10¢ per minute

• Plan B: a monthly fee of \$10 plus 7.5¢ per minute

a) Graph both relations for 0 to 100 min in 1 month.

b) Classify each relation as a direct variation or a partial variation.

c) Write an equation relating the cost and the number of minutes for each plan.

d) Compare the monthly cell phone plan costs. When is Plan A cheaper than Plan B? When is Plan B cheaper than Plan A?

6. This table shows the amount a printing company charges to print a newsletter.

Number of Newsletters, *n*	Cost, *C* ($)
0	50
200	450
400	850
600	1250
800	1650
1000	2050

a) Identify the fixed cost this company charges to print the newsletter. What do you think this amount might represent?

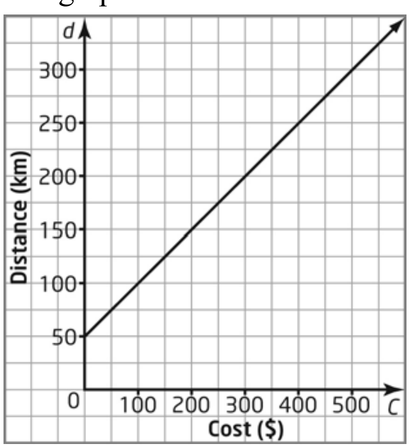
b) Determine the variable cost of printing one newsletter. Explain how you found this.

c) Write an equation representing the price to print the newsletters.

d) What is the cost to print 1200 newsletters?

e) How many newsletters can be printed for $300?

7. Describe a situation that might lead to this graph.



C

8. The prom committee of a school is planning the school prom. The cost of renting the hall for the prom and hiring the serving staff is $825. There is an additional cost of $15 per person for the meal.

a) Identify the fixed cost and the variable cost of this partial variation.

b) Write an equation to represent this relationship.

c) Use your equation to determine the total cost if 150 people attend the prom.

d) How many people can attend the prom for $3450?

9. A health club offers two types of monthly memberships:

• Membership A: $3 per visit
• Membership B: a flat fee of $8 and $2 per visit

a) Graph both relations for 0 to 10 visits.

b) Classify each relation as a direct variation or a partial variation.

c) Write an equation relating the cost and the number of visits for each membership.

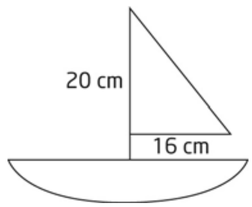
d) Compare the monthly membership costs. When is Membership A cheaper than Membership B? When is Membership B cheaper than Membership A?

5.3 Slope

***Principles of Mathematics 9*, pages 254–263**

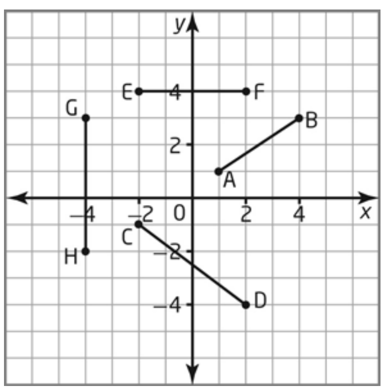
A

1. Determine the slope of the sail on the toy sailboat.



2. A set of stairs is to be built so that each step has a vertical rise of 20 cm over a horizontal run of 27.5 cm. Find the slope, to the nearest hundredth.

3. Calculate the slope of each line segment, where possible.



a) AB

b) CD

c) EF

d) GH

B

4. a) A line segment has one endpoint of A(2, 5) and a slope of $\frac{2}{3}$. Find the coordinates of another possible endpoint B.

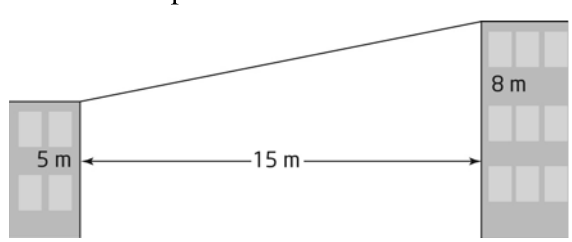
b) A line segment has one endpoint of A(−3, 2) and a slope of $-\frac{3}{5}$. Find the coordinates of another possible endpoint B.

c) A line segment has one endpoint of A(4, −1) and a slope of 0. Find the coordinates of another possible endpoint B.

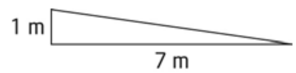
d) A line segment has one endpoint of A(−5, −3) and an undefined slope. Find the coordinates of another possible endpoint B.

5. Two ramps are being built with the same slope. The first ramp is three times the height of the second ramp. Does the first ramp have to be three times as long as the second ramp? Explain.

6. A steel wire goes between the tops of two walls that are 15 m apart. One wall is 8 m high. The other is 5 m high. What is the slope of the steel wire?



7. A wheelchair ramp is built so that the height of the ramp is 1 m and the length of the base of the ramp is 7 m. What is the slope of the wheelchair ramp, to the nearest thousandth?



C

8. A square-based pyramid has a height of 155 m and a base width of 238 m. Find the slope of the pyramid, to the nearest tenth.

9. A ladder is leaning up against a wall of a building so that it reaches 10 m up the wall. The bottom of the ladder is 1.25 m from the base of the wall.

   a) What is the slope of the ladder?

   b) Has the ladder been placed according the safety standards, which state that the ladder should have a slope of between 6.3 and 9.5 when it is placed up against a building?

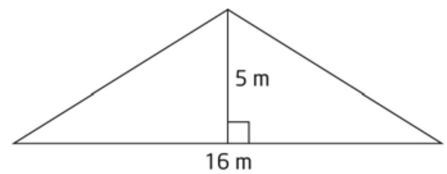
10. Use the classifications below to determine the pitch of each of the following roofs.

    Shallow: $m \leq \frac{3}{12}$

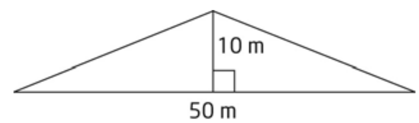
    Medium: $\frac{3}{12} < m \leq \frac{6}{12}$

    Steep: $m > \frac{6}{12}$

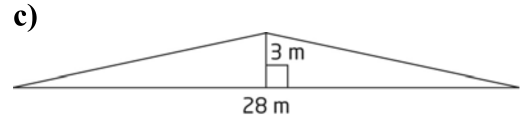
    a)



    b)



    c)



5.4 Slope as a Rate of Change

***Principles of Mathematics 9*, pages 264–271**

A

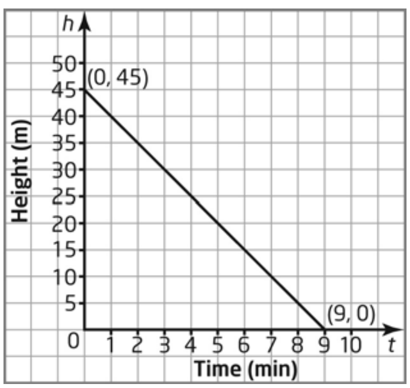
1. A heron can travel an average of 400 km in 10 h. What is the rate of change of distance?

2. A small bird can flap its wings 120 times in 30 s. What is the rate of change of wing flaps?

3. The average resting adult heart beats 720 times in 10 min. What is the rate of change of heart beats?

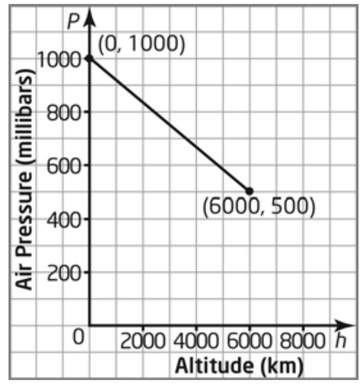
4. This graph shows the height above ground of a skier over time.

a) Calculate the slope of the graph.



b) Interpret the slope as a rate of change.

5. This graph shows the relationship between atmospheric pressure and altitude.



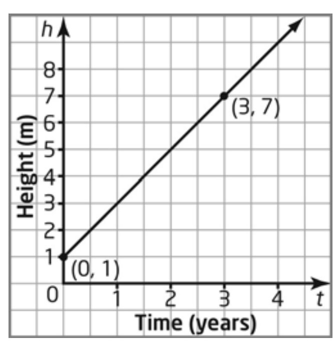
a) Calculate the slope of the graph.

b) Interpret the slope as a rate of change.

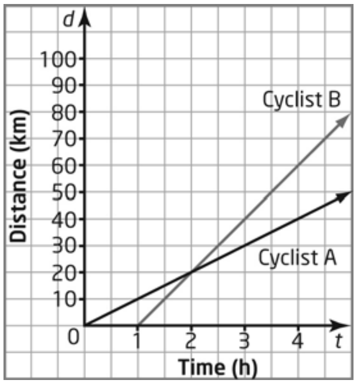
B

6. The price of a litre of milk increased from \$1.25 in 2004 to \$1.35 in 2006. What is the average price increase per year?

7. This graph shows the height of a tree over a 5-year growing period. Calculate the rate of change of height per year.



8. This distance-time graph shows two cyclists that are travelling at the same time.



a) Which cyclist has the greater speed, and by how much?

b) What does the point of intersection of the two lines represent?

9. This table shows the average undergraduate tuition fees for full-time students in Ontario in engineering by year. Is the rate of change constant over the 5-year period?

Average Undergraduate Tuition Fees for Full-Time Students in Ontario in Engineering

Year	Tuition ($)
1999–2000	4456
2000–2001	4742
2001–2002	5011
2002–2003	5302
2003–2004	5968

Source: Statistics Canada, Centre for Education Statistics. Last modified: 2004-09-01.

10. A water tank is being filled with water at a constant rate. After 20 s, the tank contains 200 L of water.

a) Graph this relation.

b) The water tank will overflow if it contains more than 300 L of water. How long will it take to fill the water tank? Mark this point on your graph.

11. Selam is on the track team at school. He runs every day after school. One day he ran 6 km in 30 min.

a) Calculate the rate of change of Selam's distance from his starting point.

b) Graph Selam's distance as it relates to time.

c) Explain the meaning of the rate of change and how it relates to the graph.

C

12. A music store is holding a special clearance sale on a $1500 piano. Initially there is a discount of 10%, Every 4 h, an additional 10% is taken off the latest price.

a) Make a table showing the price over the 12 h the sale is in effect.

b) Graph the price over the 12 h of the sale.

c) Explain the shape of the graph.

13. A health club membership costs \$600 for 1 year. The health club is holding a membership drive and is reducing the price of club memberships over the next 8 h. Initially there is a discount of 5%. Every 2 h the discount is increased by 5%.

a) Make a table showing the cost of a health club membership over the next 8 h.

b) Graph the price over the 8 h of the membership drive.

c) Explain the shape of the graph.

14. A liquid is being poured slowly onto a level surface, making a circular pattern.

a) Find the circumference of the circular pattern when the radius is 20 cm.

b) Write an equation relating the circumference, C, of the circular pattern to the radius, r.

c) Find the rate of change of circumference with respect to radius of the relation.

5.5 First Differences

***Principles of Mathematics 9*, pages 272–278**

A

1. Look at each equation. Predict whether it represents a linear relation or a non-linear relation. Use a graphing calculator to confirm your answers.

a) $y = 3x - 8$

b) $y = -4x + 2$

c) $y = 3x^2 - 2$

d) $y = 3^x$

e) $y = -\frac{2}{3}x + 5$

f) $y = \frac{5}{x}$

2. Copy each table and include a third column to record first differences. Classify each relation as linear or non-linear.

a)

x	*y*
0	2
1	6
2	10
3	14

b)

x	*y*
–3	–4
–1	–1
1	1
3	4

3. These tables show the distance travelled by a canoeist. Without graphing, determine if each relation is linear or non-linear.

a) In still water:

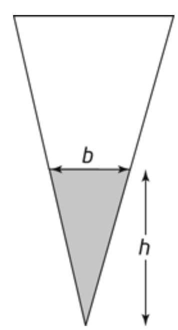
Time (s)	Distance (m)
0	0
1	1
2	2
3	3
4	4
5	5

b) With the current:

Time (s)	Distance (m)
0	0
1	1
2	3
3	6
4	10
5	15

B

4. The triangle's base is one-half its height. The triangle is painted from the bottom up.

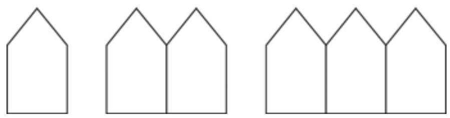


a) Create a table comparing the height of the painted portion to its area as the height increases.

b) Use first differences to determine whether the relation is linear.

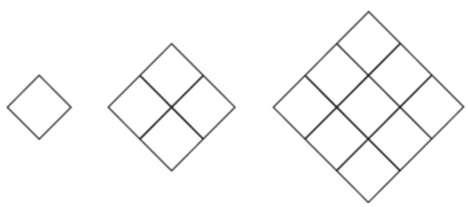
5. Use first differences to determine which relations are linear and which are non-linear. Write an equation representing each linear relation. Extrapolate the relation to predict the outcome for the seventh step.

a)



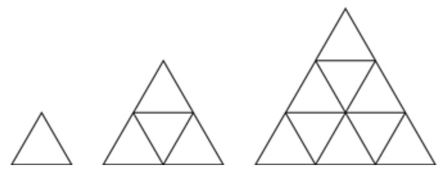
Number of Pentagons	Number of Segments
1	
2	
3	
4	

b)



Base Side Length	Total Number of Tiles
1	
2	
3	
4	

6. Use first differences to predict which relationships are linear and which are non-linear for the following pattern. Write an equation representing each linear relation. Extrapolate the relation to predict the outcome for the seventh step.



a)

Base Side Length	Number of Triangles
1	
2	
3	
4	

b)

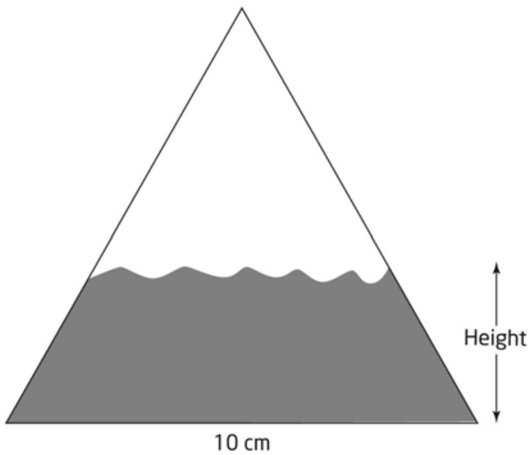
Base Side Length	Number of Segments
1	
2	
3	
4	

c)

Base Side Length	Number of Horizontal Lines in Shape
1	
2	
3	
4	

C

7. A triangular piece of cardboard is 10 cm wide. Its base is half its height. It is dipped in water and is wet from the bottom up.

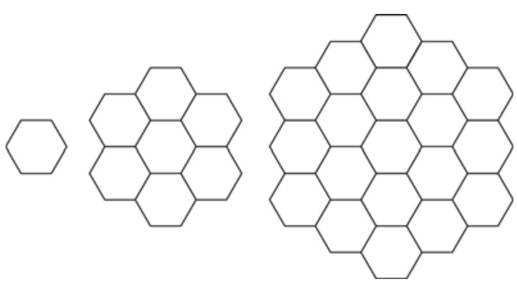


a) Create a table comparing the height of the wet cardboard to its area as the height increases from 0 cm to 5 cm.

b) Use first differences to determine whether the relation is linear.

c) What is the area of the wet cardboard if the height is 16 cm?

8. The first few figures in a pattern are shown.



a) Copy and complete the table.

Figure Number	Number of Hexagons in Pattern
1	
2	
3	
4	

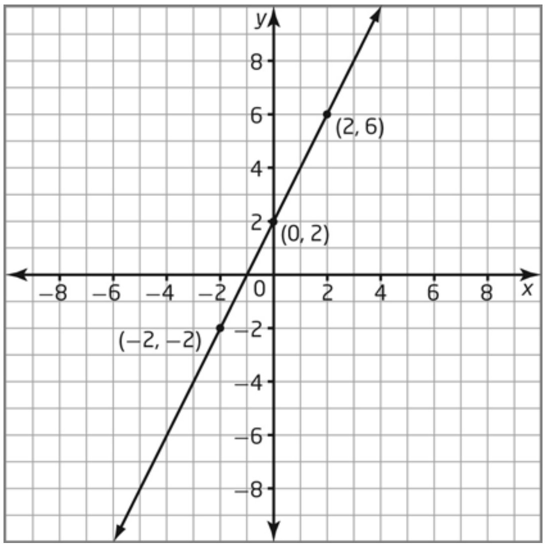
b) Use first differences to predict if the relationship is linear or non-linear.

5.6 Connecting Variation, Slope, and First Differences

***Principles of Mathematics 9*, pages 279–287**

A

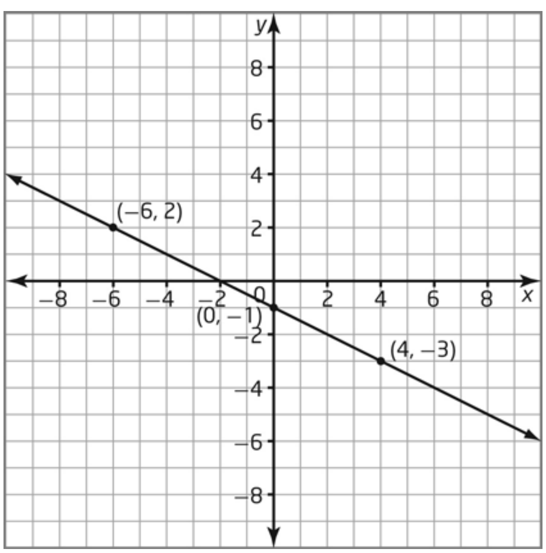
1. a) Determine the slope.



b) Determine the vertical intercept.

c) Write an equation for the relation.

2. a) Determine the slope.



b) Determine the vertical intercept.

c) Write an equation for the relation.

3. Make a table of values and graph each relation. Draw a right triangle on your graph to find the slope.

a) $y = 3x - 2$

b) $y = -2x + 1$

c) $y = \frac{1}{2}x$

d) $y = -0.5x - 1$

B

4. Use the rule of four to represent this relation in three other ways.

x	y
0	1
1	3
2	5
3	7
4	9

a) Use a graph.

b) Use words.

c) Use an equation.

5. Use the rule of four to represent this relation in three other ways.

x	y
0	3
1	1
2	–1
3	–3
4	–5

a) Use a graph.

b) Use words.

c) Use an equation.

6. A cleaning service charges $50 plus $10 per room to clean an apartment. Represent the relation using numbers, a graph, and an equation.

7. The cost of renting a bicycle is $20.00 plus $2.00/h.

a) Graph this relation.

b) Identify the slope and the vertical intercept of the line. What do they represent?

c) Is this a direct or a partial variation? Explain.

d) Write an equation relating the cost and the rental hours.

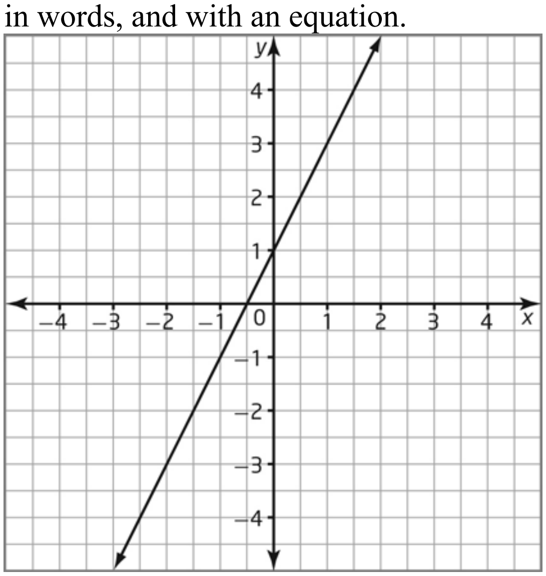
8. d varies directly with t. When $t = 5$, $d = 11$.

a) Find the slope and the vertical intercept of the line.

b) Write an equation for this relation.

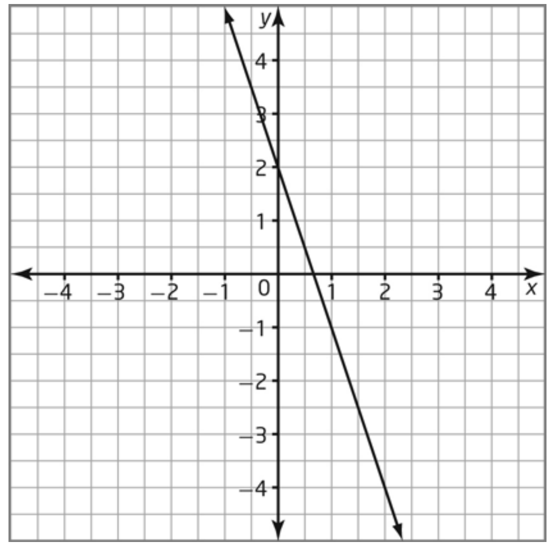
c) Graph this relation.

9. Complete the rule of four for this relation by representing it numerically, in words, and with an equation.



C

10. Complete the rule of four for this relation by representing it numerically, in words, and with an equation.



11. Complete the rule of four for the relation $y = 3x + 2$ by representing it numerically, graphically, and in words.

12. A water tank is being filled. The table shows the volume, in kilolitres, of water for the elapsed time, in minutes.

Time (min)	0	20	40	60	80
Volume of Water (kL)	20	50	80	110	140

a) Confirm that this relation is linear.

b) Graph this relation.

c) Find the slope of the graph as both a fraction and a decimal. Is the slope constant? What does the slope represent?

d) Write an equation for the volume of water in terms of the time.

e) Use your graph or equation to find the volume of water after 30 min.

Chapter 5 Review

***Principles of Mathematics 9*, pages 288–289**

1. Semir works during the weekends at a restaurant. He earns $10.50/h. His pay varies directly with the time, in hours.

a) Choose appropriate letters for variables. Make a table of values showing Semir's pay for 0 h, 1 h, 2 h, 3 h, and 4 h.

b) Graph the relationship.

c) Write an equation in the form $y = kx$.

2. Matthew cycles 50 km to a friend's home. The distance, d, in kilometres, varies directly with the time, t, in hours.

a) Find an equation relating d and t if $d = 24$ when $t = 1.5$. What does the constant of variation represent?

b) Use the equation to determine how long it will take Matthew to reach his destination.

3. The volume of juice varies directly with the volume of water used to prepare it. Tommy used 2 L of water to make 2.5 L of juice.

a) Explain why this relation is a direct variation.

b) Graph this relation.

4. Identify each relation as a direct variation, a partial variation, or neither. Justify your answer.

a) $y = 5x + 2$

b) $C = \pi d$

c) $y = x^2 - 1$

5. Identify each relation as a direct variation, a partial variation, or neither. Justify your answer.

a) $y = -2x - 3$

b) $F = 2.5a$

c) $y = -x^2 + 2$

6. a) Copy and complete the table of values given that y varies partially with x.

x	*y*
0	5
1	9
2	
3	17
4	
	37

b) Identify the initial value of y and the constant of variation from the table.

c) Write an equation relating y and x in the form $y = mx + b$.

d) Graph the relation. Describe the graph.

7. A company is having business cards printed. The cost to design the business card is $25. There is an additional charge of $0.02 per business card printed.

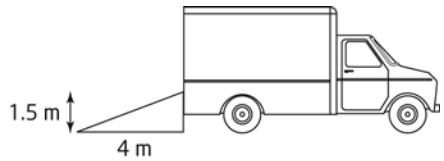
a) Identify the fixed cost and the variable cost for this partial variation.

b) Write an equation representing this relationship.

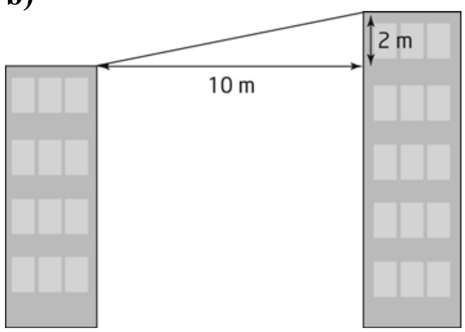
c) Use your equation to determine the total cost of 500 business cards.

8. Determine the slope of each object.

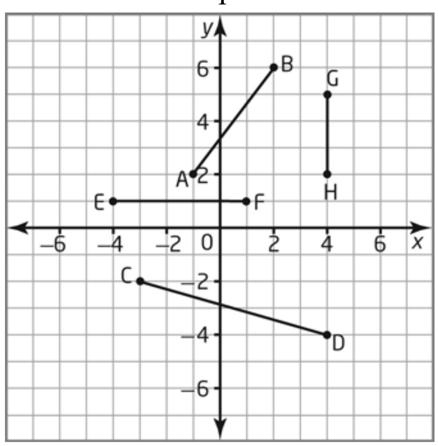
a)



b)



9. Calculate the slope of each line segment.



a) AB

b) CD

c) EF

d) GH

10. A plant is growing at a constant rate. The plant was 4 cm tall after 1 month. The plant was 32 cm tall after 9 months. If you graphed the growth of the plant with respect to time, what would the slope of the graph be? Express it as a rate of change.

11. For safety reasons, an extension ladder should have a slope of between 6.3 and 9.5 when it is placed against a wall. Determine if each of the following ladders has been placed within the safe range.

a) A ladder reaches 4 m up the wall. The foot of the ladder is 0.5 m from the wall.

b) A ladder reaches 3 m up the wall. The foot of the ladder is 0.6 m from the wall.

12. Use first differences to determine whether each relation is linear or non-linear.

a)

x	*y*
0	5
1	11
2	17
3	23
4	29

b)

x	*y*
0	14
1	8
2	3
3	–1
4	–4

13. a) Confirm that this relation is linear.

x	*y*
0	–4
1	1
2	6
3	11
4	16

b) Calculate the slope.

c) Write an equation for the relation.

d) Graph the relation.