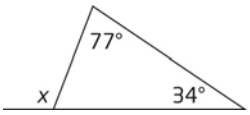
7.1 Angle Relationships in Triangles

***Principles of Mathematics 9*, pages 364–373**

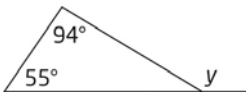
A

1. Find the measure of each exterior angle.

a)



b)



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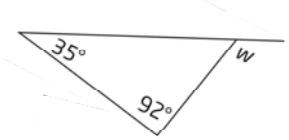
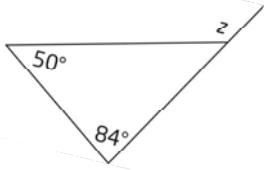
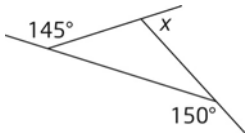


d)



2. Find the measure of each unknown exterior angle.

a)



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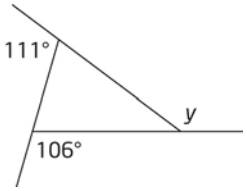

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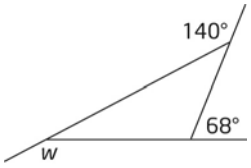
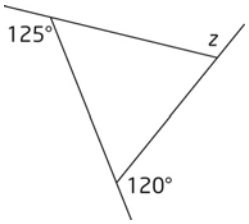


d)



B

3. The measures of two of the exterior angles of a triangle are given. Find the measure of the third exterior angle.

a) 80° and 110°

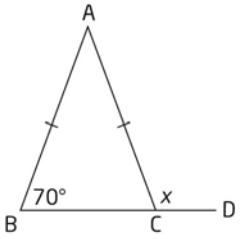
b) 60° and 105°

c) 65° and 140°

d) 80° and 130°

4. Find the measure of the exterior angle labelled x for each isosceles triangle.

a)



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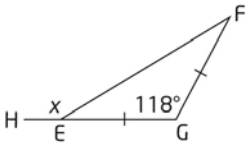

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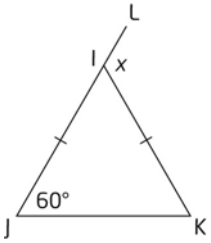

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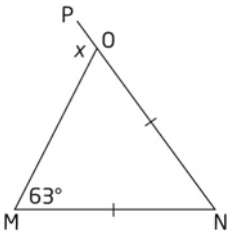
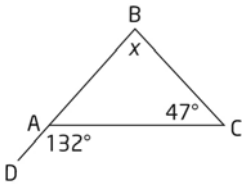


5. Find the measure of each unknown angle.

a)



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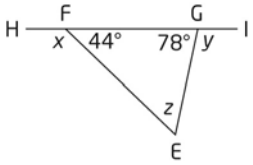

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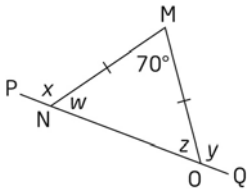
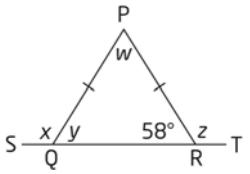
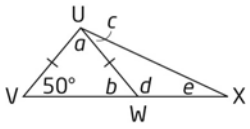


d)



e)

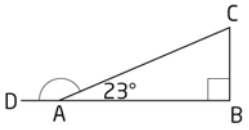


6. **a)** What type of triangle has three exterior angles equal?

b) What type of triangle has two exterior angles equal?

c) What type of triangle has no exterior angles equal?

7. A boat ramp is shown in the diagram.

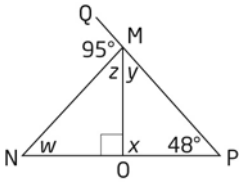


a) Find the measure of the exterior angle at the foot of the ramp.

b) Find the measure of the interior and exterior angles at the top of the ramp.

C

8. Find the measure of each unknown angle.



9. Find the angles in a triangle if the measures of the angles are

a) x, $2x$, and $3x$

b) $x - 10$, x, and $x + 10$

10. Do an Internet search for *pentaflexagon*. Then, construct a pentaflexagon using the information you find.

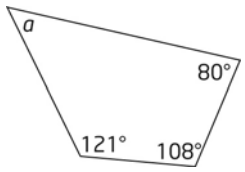
7.2 Angle Relationships in Quadrilaterals

***Principles of Mathematics 9*, pages 374–383**

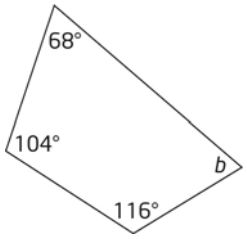
A

1. Find the angle measures a, b, c, d, e, and f.

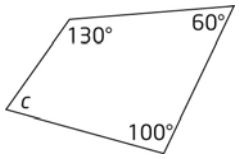
a)



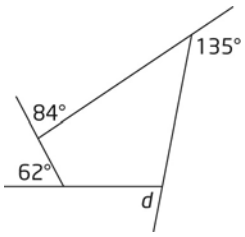
b)



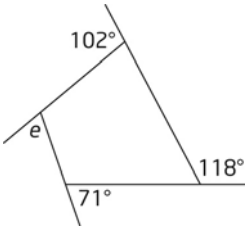
c)



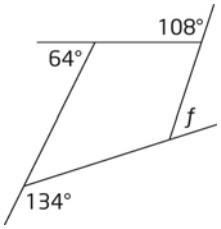
d)



e)



f)



B

2. The measures of three of the interior angles of a quadrilateral are given. Find the measure of the fourth interior angle.

a) 42°, 91°, and 115°

b) 50°, 87°, and 108°

c) 103°, 92°, and 78°

d) 100°, 72°, and 67°

3. The measures of exterior angles at three vertices of a quadrilateral are given. Find the measure of the exterior angle at the fourth vertex.

a) 104°, 95°, and 73°

b) 93°, 78°, and 85°

c) 92°, 79°, and 113°

d) 114°, 85°, and 68°

4. Each row of this table lists measures of three interior angles in a quadrilateral. Find the measure of the fourth interior angle in each quadrilateral.

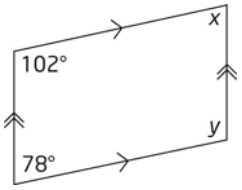
	∠A	∠B	∠C	∠D
a)	102°	77°	48°	unknown
b)	54°	68°	unknown	107°
c)	72°	unknown	82°	101°
d)	unknown	90°	85°	104°

5. Each row of this table lists measures of three exterior angles at three vertices of a quadrilateral. Find the measure of the fourth exterior angle at the fourth vertex in each quadrilateral.

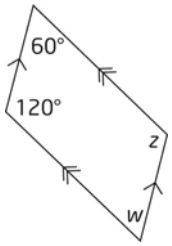
	∠A	∠B	∠C	∠D
a)	100°	80°	63°	unknown
b)	74°	76°	unknown	110°
c)	91°	unknown	78°	103°
d)	unknown	75°	108°	86°

6. Find the measure of each unknown angle.

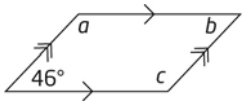
a)



b)

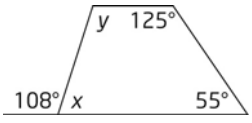


c)

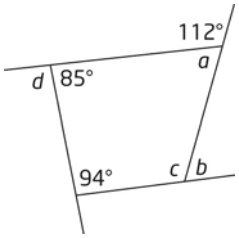


7. Find the measure of each unknown angle.

a)



b)



8. Draw an example of a quadrilateral with each set of interior angles, or explain why the quadrilateral is not possible.

a) $\angle A = 104°$, $\angle B = 85°$, and $\angle C = 93°$

b) $\angle D = 99°$, $\angle E = 84°$, and $\angle F = 102°$

c) $\angle G = 150°$, $\angle H = 130°$, and $\angle I = 84°$

9. Draw an example of a quadrilateral with each set of interior angles, or explain why the quadrilateral is not possible.

a) four acute angles

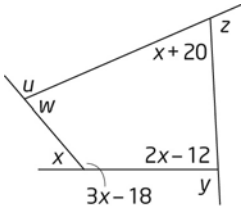
b) exactly two acute angles

c) one right angle and two obtuse angles

10. Calculate the mean measure for the exterior angles of a quadrilateral.

C

11. Find the measure of each unknown angle.



12. Draw a square EFGH and a rectangle JKLM. Construct diagonals EG and JL. Investigate the properties of the two quadrilaterals to answer parts a) to c). Describe how you determined the answer to each question.

a) Does the diagonal divide the square into two congruent triangles?

b) Does the diagonal divide the rectangle into two congruent triangles?

c) Is the diagonal a line of symmetry in the square?

d) Is the diagonal a line of symmetry in the rectangle?

e) Does the diagonal bisect any angles in the square?

f) Does the diagonal bisect any angles in the rectangle?

13. Find the measures of the interior angles of a quadrilateral so that the measures have each ratio.

a) 2:2:3:3

b) 3:3:5:5

7.3 Angle Relationships in Polygons

***Principles of Mathematics 9*, pages 384–393**

A

1. Find the sum of the interior angles of a polygon with

a) 12 sides

b) 17 sides

c) 22 sides

d) 24 sides

2. Find the measure of each interior angle of a regular polygon with

a) 8 sides

b) 10 sides

c) 14 sides

d) 24 sides

3. How many sides does a polygon have if the sum of its interior angles is

a) 1440°?

b) 720°?

c) 1980°?

d) 2340°?

4. How many sides does a polygon have if each of its interior angles measures

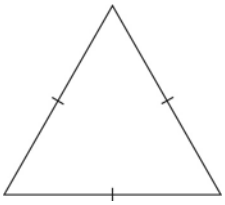
a) 120°?

b) 144°?

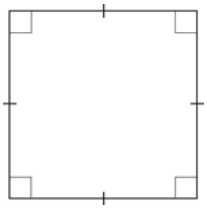
c) 135°?

B

5. **a)** The polygon shown has three equal sides and three equal angles. Write the name of a regular polygon with three sides.



b) The polygon shown has four equal sides and four equal angles. Write the name of a regular polygon with four sides.



6. Copy this table and fill in the missing entries. To find the number of triangles in the polygon, draw diagonals from one vertex.

Polygon	Number of Sides	Number of Diagonals From One Vertex	Number of Triangles in the Polygon	Sum of Interior Angles
pentagon	5	2	3	540°
hexagon				
octagon	8			
dodecagon	12			

7. **a)** Use the formula for the sum of the interior angles of a polygon to show that each interior angle of an equilateral triangle measures 60°.

b) Use the formula for the sum of the interior angles of a polygon to show that each interior angle of a rectangle measures 90°.

8. Ron who is a furniture-maker is designing an octagonal wooden container.

a) At what angle will the adjacent sides of the wooden container meet if its shape is a regular octagon?

b) Do you think the angles between the adjacent sides of the wooden container will all be equal if one pair of opposite sides are twice as long as the other sides?

c) Check your answer to part b) by making a drawing and measuring the angles.

9. **a)** Draw a ten-sided polygon.

b) Calculate how many diagonals you can draw from any one vertex of this polygon. Check your answer by drawing all possible diagonals from one of the vertices.

c) Calculate the sum of the interior angles of the polygon. Check you answer by measuring the angles on your drawing.

10. **a)** Find the measure of each interior angle of a regular 9-sided polygon. Use a second method to check your answer.

b) Find the measure of each interior angle of a regular 15-sided polygon. Use a second method to check your answer.

c) Find the measure of each interior angle of a regular 24-sided polygon. Use a second method to check your answer.

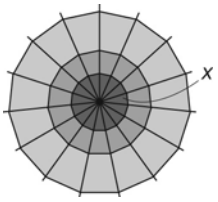
d) Write an expression for the measure of each interior angle of a regular polygon with n sides.

C

11. The sides of a gazebo form a regular 15-sided polygon.

a) Calculate the angle between adjacent sides of the gazebo.

b) Calculate the angle between the 15 adjacent roof supports in the gazebo.



c) Draw a plan of the gazebo.

d) Calculate the angle between adjacent sides in a gazebo with 10 sides.

e) Calculate the angle between the 10 adjacent roof supports in a gazebo with 10 sides.

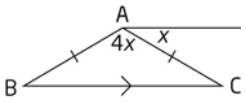
12. A table is to be constructed from wood and used on the patio of a house.

a) At what angle will the adjacent sides of the table meet if its shape is a regular 12-sided polygon?

b) Do you think the angles between the adjacent sides of the table will all be equal if one pair of opposite sides are twice as long as the other sides?

c) Check you answer to part b) by making a drawing and measuring the angles.

13. Find the measure of $\angle BCA$ in the diagram shown.



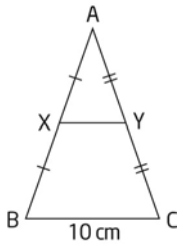
7.4 Midpoints and Medians in Triangles

***Principles of Mathematics 9*, pages 394–400**

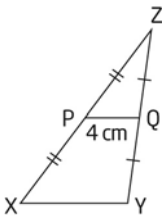
A

1. Calculate the length of line segment XY in each triangle.

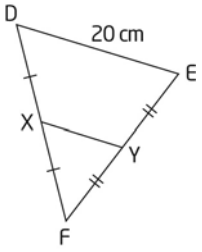
a)



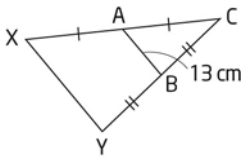
b)



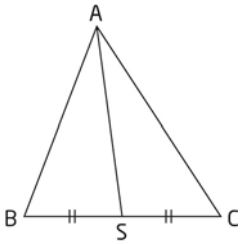
c)



d)



2. The area of ΔABC is 20 cm^2.

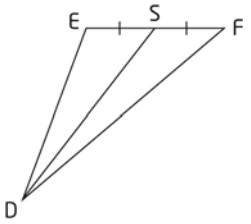


a) Calculate the area of ΔABS.

b) Calculate the area of ΔACS.

B

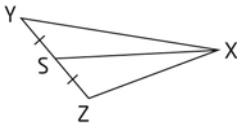
3. The area of ΔDEF is 26 cm^2.



a) Calculate the area of ΔDES.

b) Calculate the area of ΔDFS.

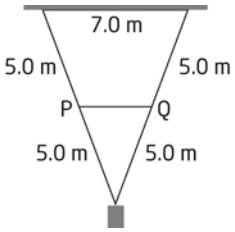
4. The area of ΔXYZ is 15 cm^2.



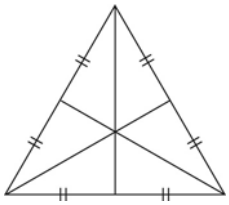
a) Calculate the area of ΔXYS

b) Calculate the area of ΔXZS

5. Calculate the length of the cross-brace PQ in this bridge support.



6. a) Make a conjecture about whether the medians in an equilateral triangle bisect each angle at the vertex.

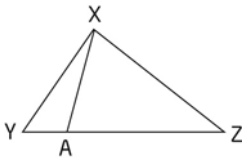


b) Describe how you can see if your conjecture is correct by folding a diagram of an equilateral triangle.

c) Describe how you could use geometry software to see if your conjecture is correct.

d) Use one of the two methods you described to test your conjecture. Describe your results.

7. Virginia conjectures that $\angle AXZ$ in this diagram will be obtuse when point A is located anywhere on the side YZ. Use a counter-example to show that this conjecture is false.

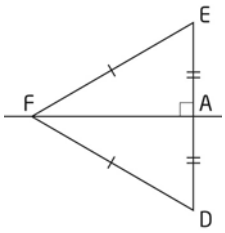


8. Here are two conjectures about isosceles triangles with an 80° interior angle. For each conjecture, either draw a counter-example or explain why you think the conjecture is true.

a) The 80° angle is always opposite one of the two equal sides.

b) The 80° angle is always opposite the unequal side.

9. Carey constructed ΔDEF with DE = EF = DF. He then constructed the midpoint of DE at A and drew a perpendicular line through DE at A. Will this right bisector pass through the vertex F? Justify your answer.



C

10. Determine whether the right bisectors of the sides of a triangle intersect at a single point.

- If you are using pencil and paper, draw the right bisectors in at least one example of each type of triangle.
- If you are using geometry software, construct a triangle and the right bisector of each side. Drag each vertex to various new locations. Does changing the shape of the triangle affect how the right bisectors intersect?

Do you think that the right bisectors intersect at a single point in all triangles? Explain your reasoning.

11. a) Investigate whether the three medians of a triangle intersect at a single point. Describe your findings.

b) Draw a triangle in which the medians of the sides intersect at a single point. Can you draw a circle that has this point as its centre and intersects the triangle at exactly three points? If so, describe the properties of the circle.

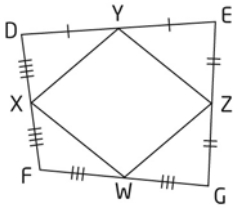
7.5 Midpoints and Diagonals in Quadrilaterals

***Principles of Mathematics 9*, pages 401–407**

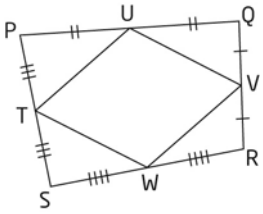
A

1. Which line segments in the following diagrams are parallel?

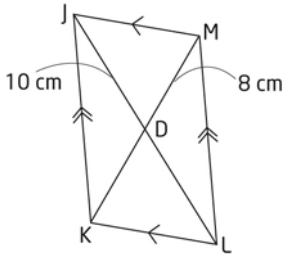
a)



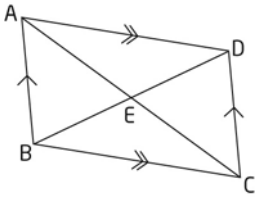
b)



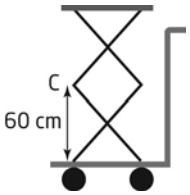
2. Calculate the lengths of KD, LD, KM and LJ.



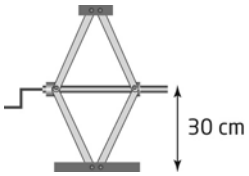
3. Calculate the lengths of AE and BE given that AC measures 22 cm and BD measures 20 cm.



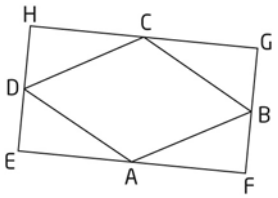
4. Shankar is using a scissor lift trolley to move equipment in a hospital. How high will the platform be when the point C is 60 cm above the base?



5. Sam is using a scissor jack to change a tire on his car. How high will the top of the jack be when the shaft is 30 cm from the base?



6. Construct parallelogram EFGH. Let A be the midpoint of EF, B be the midpoint of FG, C be the midpoint of GH, and D be the midpoint of HE. Connect AB, BC, CD and DA to form a new parallelogram ABCD. Under what conditions is ABCD a rhombus?



7. For each statement, either explain why it is true or draw a counter-example to show that it is false.

a) Any diagonal of a trapezoid bisects its area.

b) Any line segment joining the midpoints of opposite sides of a rectangle bisects its area.

8. On grid paper, draw a rectangle ABCD and mark the midpoints of the four sides. Label these midpoints W, X, Y, and Z.

a) What type of quadrilateral is WXYZ?

b) How is the area of WXYZ related to the area of ABCD? Explain your reasoning.

c) What shape will WXYZ become if ABCD is shrunk to form a square? Support your answer with a drawing.

d) Will the relationship between the areas of WXYZ and ABCD change when ABCD is shrunk into a square. Explain.

9. a) Draw a quadrilateral STUV with ST = SV and UT = UV.

b) At what angle do the diagonals of the quadrilateral intersect?

c) Join the midpoints of the sides of the quadrilateral to form a smaller quadrilateral WXYZ. What type of quadrilateral is WXYZ?

d) Make a conjecture about how the area of WXYZ is related to the area of STUV.

e) Describe how you can use geometry software to test your conjecture.

10. Use Technology

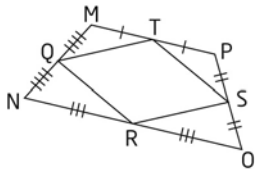
a) Construct $\angle$DEF with ED = EF. Construct a line perpendicular to DE at point D and a line perpendicular to EF at F. Label the intersection of these lines G.

b) Show that DG = FG.

c) Show that EG bisects $\angle$DEF.

C

11. In this diagram, line segments joining the midpoints of the four sides of a quadrilateral form a smaller quadrilateral inside the original quadrilateral.



a) How do you think the area of the smaller quadrilateral compares to the area of the original quadrilateral?

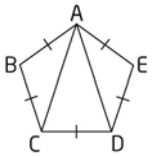
b) Describe how you could confirm your conjecture.

12. Use congruent triangles to show that the diagonals of a rhombus bisect each other.

13. a) Draw a quadrilateral EFGH with EF = GH and EH = GF.

b) Show that this quadrilateral must be a parallelogram.

14. For the regular pentagon in the diagram shown, show that AC = AD.

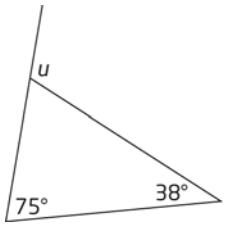


Chapter 7 Review

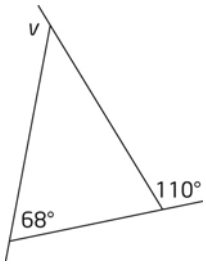
***Principles of Mathematics 9*, pages 408–409**

1. Find the measure of each unknown angle.

a)



b)



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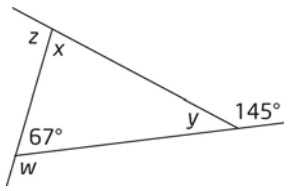
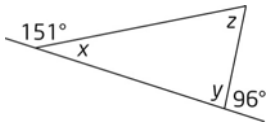
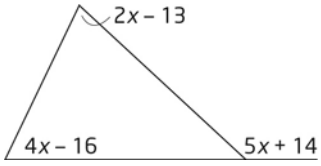


d)



2. Explain why the angle relationships shown are not possible.



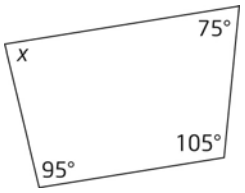
3. For each description , draw an example of the triangle or explain why it cannot exist.

a) a triangle with an obtuse exterior angle

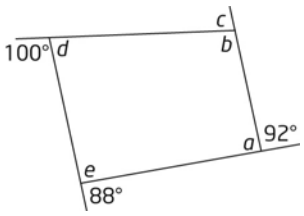
b) a triangle with three acute exterior angles

4. Calculate the measure of each unknown angle.

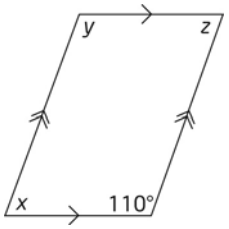
a)



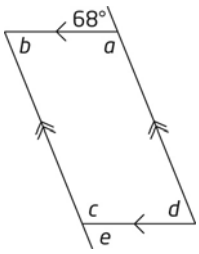
b)



c)



d)



5. For each description, draw an example of the quadrilateral or explain why it cannot exist.

a) a quadrilateral with three acute interior angles

b) a quadrilateral with four acute interior angles

6. Find the sum of the interior angles of each polygon.

a) pentagon

b) heptagon (7-sided figure)

c) pentadecagon (15-sided figure)

d) undecagon (11-sided figure)

e) octagon

7. Find the measure of each interior angle of a regular polygon with

a) 6 sides

b) 7 sides

c) 12 sides

d) 15 sides

8. How many sides does a polygon have if the sum of its interior angles is

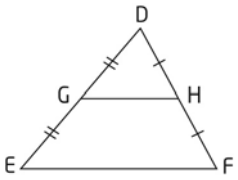
a) 1080°?

b) 1260°?

9. a) Construct a regular hexagon.

b) Describe the method you used.

10. Show that the area of ΔDGH is one quarter of the area of ΔDEF.



11. For each of these statements, either explain why it is true or draw a counter-example to show that it is false.

a) The median to the vertex opposite the unequal side of an isosceles triangle bisects the angle at the vertex.

b) The medians of an isosceles triangle are equal in length.

12. For each of these statements, either use a diagram to help explain why the statement is true, or draw a counter-example and explain why the statement is false.

a) A line segment joining the midpoints of opposite sides of a rhombus bisects its area.

b) A line segment joining the midpoints of two sides of a parallelogram bisects the area of the parallelogram.

c) A line segment joining the midpoints of the parallel sides of a trapezoid bisects its area.

d) The diagonal of a trapezoid bisects the area.

13. Describe how you can use geometry software to determine the types of quadrilaterals in which the diagonals bisect the area of the quadrilateral.