CHAPTER

# 5

# Modelling With Graphs

**Vocabulary**

direct variation
constant of variation
partial variation
slope
rise
run
rate of change
first differences

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

## Curriculum Expectations

### Mathematical Process Expectations

Throughout this course, students will:

**PROBLEM SOLVING**

**MPS.01** develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

**REASONING AND PROVING**

**MPS.02** develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

**REFLECTING**

**MPS.03** demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

**SELECTING TOOLS AND COMPUTATIONAL STRATEGIES**

**MPS.04** select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

**CONNECTING**

**MPS.05** make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

**REPRESENTING**

**MPS.06** create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

**COMMUNICATING**

**MPS.07** communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

**Strand:**
Linear Relations

## Overall Expectations

By the end of this course, students will:

**LRV.01** apply data-management techniques to investigate relationships between two variables;

**LRV.02** demonstrate an understanding of the characteristics of a linear relation;

**LRV.03** connect various representations of a linear relation.

## Specific Expectations

***Understanding Characteristics of Linear Relations***

By the end of this chapter, students will:

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**RE2.02** construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

**RE2.03** identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;

**RE2.04** compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

**RE2.05** determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

***Connecting Various Representations of Linear Relations***

By the end of this chapter, students will:

**RE3.01** determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;

**RE3.02** describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;

**RE3.03** determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

**RE3.04** describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).

**Strand:**
Analytic Geometry

## Overall Expectations

By the end of this course, students will:

**AGV.01** determine the relationship between the form of an equation and the shape of its graph with respect to linearity and non-linearity;

**AGV.02** determine, through investigation, the properties of the slope and $y$-intercept of a linear relation;
**AGV.03** solve problems involving linear relations.

## Specific Expectations

### *Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph*

By the end of this chapter, students will:
**AG1.01** determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation).

### *Investigating the Properties of Slope*

By the end of this chapter, students will:
**AG2.01** determine, through investigation, various formulas for the slope of a line segment or a line;
**AG2.03** determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, $p$, in this equation).

### *Using the Properties of Linear Relations to Solve Problems*

By the end of this chapter, students will:
**AG3.03** describe the meaning of the slope and $y$-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);
**AG3.04** identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, $C$ is the cost of holding a party in a hall and $n$ is the number of guests; $n$ is restricted to whole numbers of 100 or less, because of the size of the hall, and $C$ is consequently restricted to \$50 to \$2550).

## Chapter Problem

The Chapter Problem is introduced in the Chapter Opener. Have students discuss their understanding of the toothpick patterns. There are many patterns that can be drawn out of the diagram in the Chapter Problem. Allow students to explore them. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 291. Alternatively, you may wish to assign the Chapter Problem when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment.

# Chapter 5 Planning Chart

| Section<br>Suggested Timing | Student Text<br>Page (s) | Teacher's Resource<br>Blackline Masters | Assessment | Tools |
|---|---|---|---|---|
| **Chapter 5 Opener**<br>• 15 min | 234–235 | | | |
| **Get Ready**<br>• 80 min | 236–237 | • BLM 5.GR.1 Practice: Get Ready | • BLM 5.GR.2 Get Ready Self-Assessment Checklist | |
| **5.1 Direct Variation**<br>• 80 min | 238–245 | • BLM G10 Grid Paper<br>• BLM T6 *Fathom*™<br>• BLM 5.1.1 Practice: Direct Variation | • BLM A9 Communication General Scoring Rubric | **Tools**<br>• grid paper<br>**Technology Tools**<br>• *Fathom*™<br>• computers<br>• graphing calculators |
| **5.2 Partial Variation**<br>• 80 min | 246–253 | • BLM G10 Grid Paper<br>• BLM 5.2.1 Practice: Partial Variation | • BLM 5.2.2 Achievement Check Rubric<br>• BLM A7 Thinking General Scoring Rubric | **Tools**<br>• grid paper |
| **5.3 Slope**<br>• 80 min | 254–263 | • BLM G10 Grid Paper<br>• BLM 5.3.1 Practice: Slope | • BLM A16 My Progress as a Mathematician<br>• BLM A23 News Report Checklist | **Tools**<br>• grid paper |
| **5.4 Slope as a Rate of Change**<br>• 80 min | 264–271 | • BLM T6 *Fathom*™<br>• BLM 5.4.1 Practice: Slope as a Rate of Change<br>• BLM G10 Grid Paper | • BLM A12 Group Work Assessment General Scoring Rubric<br>• BLM 5.4.2 Achievement Check Rubric | **Tools**<br>• grid paper<br>**Technology Tools**<br>• *Fathom*™<br>• computers<br>• graphing calculators |
| **5.5 First Differences**<br>• 80 min | 272–278 | • BLM G10 Grid Paper<br>• BLM 5.5.1 Practice: First Differences | • BLM A20 Learning Skills Checklist | **Tools**<br>• grid paper<br>**Technology Tools**<br>• graphing calculators |
| **5.6 Connecting Variation, Slope, and First Differences**<br>• 160 min | 279–287 | • BLM 5.6.1 Practice: Connecting Variation, Slope, and First Differences<br>• BLM 5.6.2 Student Success: The Rule of Four<br>• BLM G10 Grid Paper | • BLM 5.6.3 Achievement Check Rubric | **Tools**<br>• grid paper |
| **Chapter 5 Review**<br>• 80 min | 288–289 | • BLM G10 Grid Paper<br>• BLM 5.CR.1 Chapter 5 Review | • BLM A4 Presentation Checklist | **Tools**<br>• grid paper |
| **Chapter 5 Practice Test**<br>• 80 min | 290–291 | • BLM G10 Grid Paper | • BLM 5.PT.1 Chapter 5 Practice Test<br>• BLM 5.CT.1 Chapter 5 Test | **Tools**<br>• grid paper<br>**Technology Tools**<br>• graphing calculators |
| **Chapter 5 Problem Wrap-Up**<br>• 40–80 min | 291 | • BLM G10 Grid Paper | • BLM 5.CP.1 Chapter 5 Problem Wrap-Up Rubric | **Tools**<br>• grid paper<br>• toothpicks and/or algebra tiles<br>**Technology Tools**<br>• graphing calculators |

# Chapter 5 Blackline Masters Checklist

| | BLM | Title | Purpose |
|---|---|---|---|
| **Get Ready** | | | |
| | **BLM 5.GR.1** | Practice: Get Ready | Practice |
| | **BLM 5.GR.2** | Get Ready Self-Assessment Checklist | Student Self-Assessment |
| **5.1: Direct Variation:** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM T6** | *Fathom*™ | Technology |
| | **BLM 5.1.1** | Practice: Direct Variation | Practice |
| | **BLM A9** | Communication General Scoring Rubric | Assessment |
| **5.2: Partial Variation** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM 5.2.1** | Practice: Partial Variation | Practice |
| | **BLM 5.2.2** | Achievement Check Rubric | Assessment |
| | **BLM A7** | Thinking General Scoring Rubric | Assessment |
| **5.3: Slope** | | | |
| | **BLM G10** | Grid Paper | Student Success |
| | **BLM A16** | My Progress as a Mathematician | Student Self-Assessment |
| | **BLM 5.3.1** | Practice: Slope | Practice |
| | **BLM A23** | News Report Checklist | Assessment Literacy |
| **5.4: Slope as a Rate of Change** | | | |
| | **BLM A12** | Group Work Assessment General Scoring Rubric | Assessment Group Work |
| | **BLM T6** | *Fathom*™ | Technology |
| | **BLM 5.4.1** | Practice: Slope as a Rate of Change | Practice |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM 5.4.2** | Achievement Check Rubric | Assessment |
| **5.5: First Differences** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM A20** | Learning Skills Checklist | Assessment |
| | **BLM 5.5.1** | Practice: First Differences | Practice |
| **5.6: Connecting Variation, Slope, and First Differences** | | | |
| | **BLM 5.6.1** | Practice: Connecting Variation, Slope, and First Differences | Practice |
| | **BLM 5.6.2** | Student Success: The Rule of Four | Student Success |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM 5.6.3** | Achievement Check Rubric | Assessment |

| | BLM | Title | Purpose |
|---|---|---|---|
| **Chapter 5 Review** | | | |
| | **BLM 5.CR.1** | Chapter 5 Review | Review |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM A4** | Presentation Checklist | Assessment |
| **Chapter 5 Practice Test** | | | |
| | **BLM 5.PT.1** | Chapter 5 Practice Test | Diagnostic Assessment |
| | **BLM 5.CT.1** | Chapter 5 Test | Student Assessment |
| | **BLM G10** | Grid Paper | Student Support |
| **Chapter 5 Problem Wrap-Up** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM 5.CP.1** | Chapter 5 Problem Wrap-Up Rubric | Student Assessment |

# Get Ready

**Student Text Pages**
236 to 237

**Suggested Timing**
80 min

**Related Resources**
BLM 5.GR.1 Practice: Get Ready

BLM 5.GR. 2 Get Ready Self-Assessment Checklist

## Common Errors

- Some students may treat negative mixed fractions as the sum of a negative and a positive, for example $-2\frac{1}{3} = -2 + \frac{1}{3}$.

**$R_x$** Explain that the negative sign belongs with the entire number. Then, show them on a number line that $-2\frac{1}{3}$ is between $-2$ and $-3$, just as $2\frac{1}{3}$ is between 2 and 3.

## Accommodations

**Gifted and Enrichment**—Encourage students to learn more about mathematical puzzles, to solve them, and to create their own.

**Memory**—Encourage students to review the rules for equivalent fractions and expression ratios in simplest form.

## Teaching Suggestions

- Students need a sound understanding of rational numbers, as slopes and rates of change will involve rationals. For a more complete development of operations with rational numbers, see Chapter 1, Section 1.5.
- If students need remedial work on rationals, you may wish to use **BLM 5.GR.1 Practice: Get Ready** for remediation or extra practice.
- Refer to the Link to Get Ready in the chapter sections of the Teacher's Resource.
- All **BLMs** referred to throughout this chapter can be found on the *Principles of Mathematics 9* Teacher's Resource CD-ROM.

### Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; students can place a checkmark beside topics in the Get Ready in which they feel confident. You may wish to use **BLM 5.GR.2 Get Ready Self-Assessment Checklist** as a self-assessment for students. Remedial action can be taken in small groups or with a whole class skill review.

# 5.1 Direct Variation

**Strand:**
Linear Relations

**Strand:**
Analytic Geometry

**Student Text Pages**
238 to 245

**Suggested Timing**
80 min

**Tools**
- grid paper

**Technology Tools**
- *Fathom*™
- computers
- graphing calculators

**Related Resources**
BLM G10 Grid Paper
BLM T6 *Fathom*™
BLM 5.1.1 Practice: Direct Variation
BLM A9 Communication General Scoring Rubric

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Understanding the Characteristics of Linear Relations**

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**RE2.04** compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

**Investigating the Properties of Slope**

**AG2.03** determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, $p$, in this equation);

**Connecting Various Representations of Linear Relations**

**RE3.02** describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;

**RE3.03** determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

**RE3.04** describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).

**Using the Properties of Linear Relations to Solve Problems**

**AG3.03** describe the meaning of the slope and $y$-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

**AG3.04** identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, $C$ is the cost of holding a party in a hall and $n$ is the number of guests; $n$ is restricted to whole numbers of 100 or less, because of the size of the hall, and $C$ is consequently restricted to \$50 to \$2550);

## Link to Get Ready

The Get Ready segments, Rational Numbers, and Ratio and Proportion, provide the needed skills for this section. You may wish to have students complete Get Ready questions 1 to 6 before starting this section.

## Common Errors

- Some students may have difficulties deciding which is the independent variable.

$R_x$ Have students consider not only cause and effect, but which variable must be known before the other can be calculated. Refer students to Chapter 2, Section 2.3.

## Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

### Warm-Up

**1.** Mental Math: Multiply each of the following.

**a)** $10 \times 12$ **b)** $6 \times 9$ **c)** $11 \times 7$
**d)** $100 \times 72$ **e)** $15 \times 3$ **f)** $30 \times 5$

**2.** Divide each of the following.

**a)** $72 \div 8$ **b)** $100 \div 5$ **c)** $24 \div 6$
**d)** $60 \div 3$ **e)** $1000 \div 10$ **f)** $99 \div 11$

### Warm-Up Answers

**1. a)** 120 **b)** 54 **c)** 77 **d)** 7200 **e)** 45 **f)** 150

**2. a)** 9 **b)** 20 **c)** 4 **d)** 20 **e)** 100 **f)** 9

## Teaching Suggestions

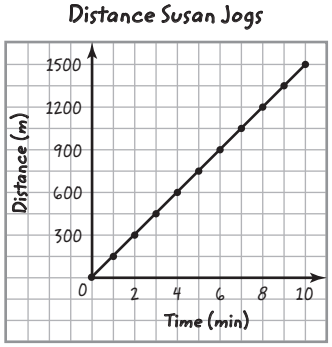
- Discuss the opening photo and what is meant by speed. Ask how speed is related to distance and time and how to write the relationship as an equation. (5 min)
- Have students work on the Investigate. At the conclusion of the Investigate, discuss the multiple representations (words, numbers, graph, equations) of the relationship, which is the key to this activity. You may wish to use **BLM G10 Grid Paper** to support this activity. (15 min)
- Discuss Examples 1 and 2 as a class, or examples similar to them, showing the multiple representations of direct variation. (20 min)
- Students will have worked with proportions in grade 7 and 8, but will not have calculated a constant of variation. Stress the importance of this constant, which is what makes the graph a straight line. Ensure that students understand that the graphs must pass through the origin. Although included in the examples, technology is not as important a tool for understanding direct variation as the multiple representations. You may wish to use **BLM T6 *Fathom*™** to support this activity.
- Review the vocabulary in this section (independent variable, dependent variable, direct variation, constant of variation) before moving on to Communicate Your Understanding. (10 min)
- Discuss the Communicate Your Understanding questions C1 and C2. (10 min)
- You may wish to use **BLM 5.1.1 Practice: Direct Variation** for remediation or extra practice.

### Investigate Answers (page 238)

**1. a)**

| Time (min) | Distance (m) |
|---|---|
| 0 | 0 |
| 1 | 150 |
| 2 | 300 |
| 3 | 450 |
| 4 | 600 |
| 5 | 750 |
| 6 | 900 |
| 7 | 1050 |
| 8 | 1200 |
| 9 | 1350 |
| 10 | 1500 |

**b)** Independent variable: time; dependent variable: distance



## Accommodations

**Visual**—Encourage students to work together with a reading buddy to receive verbal instructions for the questions in this section.

**Perceptual**—Encourage students to use visual and verbal cues to identify the independent variable and the dependent variable in linear relations where the variables are letters other than $x$ and $y$.

**Spatial**—Provide students with opportunities to use technology to relate direct variation ($y = mx$) to a linear relation with a $y$-intercept of zero.

**Motor**—Give students extra time to complete the questions in this section.

**ESL**—Allow students to work together when working through the questions in this section, and encourage them to use their dictionaries to understand the meanings of the new words.

**c)** The graph is a straight line that slopes upward, from left to right, and intersects the vertical axis at the origin, (0, 0).

**d)** $d = 150t$

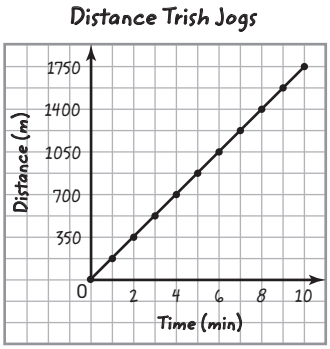
**e)** 6000 m or 6 km

**f)** The distance is doubled. The distance is tripled.

**2. a)**

| Time (min) | Distance (m) |
|---|---|
| 0 | 0 |
| 1 | 175 |
| 2 | 350 |
| 3 | 525 |
| 4 | 700 |
| 5 | 875 |
| 6 | 1050 |
| 7 | 1225 |
| 8 | 1400 |
| 9 | 1575 |
| 10 | 1750 |

**b)** time, distance




**c)** The graph is a straight line that slopes upward, from left to right, and intersects the vertical axis at the origin (0, 0).

**d)** $d = 175t$

**e)** 7000 m or 7 km

**f)** The distance is doubled. The distance is tripled.

**3.** The distance jogged is a product of the time and the average speed. For example, if the average speed is 120 m/min, then an equation for the distance, $d$, in metres, travelled in $t$ minutes would be $d = 120t$.

### Communicate Your Understanding Responses (page 242)

**C1.** $A = 2C$ is an example of direct variation, because it has the same form as $y = kx$. The variable $C$ varies directly with the variable $A$.

**C2. a)** Both graphs are straight lines that start at the origin and go upward, from left to right.

**b)** The difference between the graphs is their level of steepness. The graph of $d = 2t$ has a slope of 2 and the graph of $d = 3t$ has a slope of 3, so, the graph of $d = 3t$ is steeper.

## Practise

Students should not have difficulties with the Practise questions; they are straightforward and reinforce the Examples. You may wish to use **BLM G10 Grid Paper** to support questions 1 to 5.

## Connect and Apply

Questions 6 and 7 are straightforward. In part c) of both questions, students use the formula for extrapolation.

Questions 8c) and 9c) provide a good opportunity for students to use their reasoning and communication skills.

For question 10, you may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Questions 11, 12, and 13 provide interesting contexts for the students. You may wish to use **BLM G10 Grid Paper** to support questions 7, 9, and 12.

## Extend

Question 14 involves re-arranging an equation involving two variables and decimal coefficients. It is accessible to most level 3 and 4 students.

Question 15 discusses an interesting scientific relationship among four variables.

Questions 16 and 17 are Math Contest questions.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–6, 8 |
| Typical | 1–10, 11, 13 |
| Extension | 14—17 |

# 5.2 Partial Variation

**Strand:**
Linear Relations

**Strand:**
Analytic Geometry

**Student Text Pages**
246 to 253

**Suggested Timing**
80 min

**Tools**
- grid paper

**Related Resources**
BLM G10 Grid Paper
BLM 5.2.1 Practice: Partial Variation
BLM 5.2.2 Achievement Check Rubric
BLM A7 Thinking General Scoring Rubric

**Mathematical Process Expectations Emphasis**

- [x] Problem Solving
- [ ] Reasoning and Proving
- [ ] Reflecting
- [ ] Selecting Tools and Computational Strategies
- [x] Connecting
- [x] Representing
- [x] Communicating

## Specific Expectations

**Understanding the Characteristics of Linear Relations**

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**RE2.04** compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

**Connecting Various Representations of Linear Relations**

**RE3.02** describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;

**RE3.03** determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

**RE3.04** describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).

**Investigating the Properties of Slope**

**AG2.03** determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, $p$, in this equation);

**Using the Properties of Linear Relations to Solve Problems**

**AG3.03** describe the meaning of the slope and $y$-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

**AG3.04** identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, $C$ is the cost of holding a party in a hall and $n$ is the number of guests; $n$ is restricted to whole numbers of 100 or less, because of the size of the hall, and $C$ is consequently restricted to \$50 to \$2550).

**Common Errors**

- Some students may have difficulty seeing the constant of proportionality, since it is not simply multiplied by the independent variable.

**$R_x$** Have students graph the data in the problem. Draw right triangles between successive points illustrating the steps while going through the relation. Alternatively, add another column to their data chart and ask them to subtract successive values of the dependent variable.

**Ongoing Assessment**

- Use Achievement Check question 12 to monitor student success. See Achievement Check Answers and **BLM 5.2.2 Achievement Check Rubric**.
- Chapter Problem question 8 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

## Link to Get Ready

The Get Ready segment Percents provides the needed skills for this section. You may wish to have students complete or review Get Ready questions 7 and 8 before starting this section.

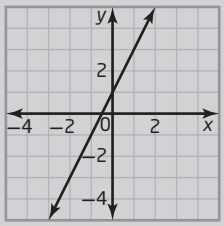
### Warm-Up

**1.** Mental Math: Continue each of the following number patterns, stating the next two terms.

**a)** 2, 5, 8, 11 **b)** 15, 10, 5, 0 **c)** 6, 8, 10, 12 **d)** 1, 11, 21, 31

**2.** Which of the following represent direct variation?

**a)**




**b)** The page numbering in the student's textbook versus the number of pieces of paper.

**c)** (0, 0), (2, 4), (4, 8), (6, 12) **d)** $y = 8x$ **e)** $C = 2n + 5$

**Warm-Up Answers**

**1. a)** 14, 17 **b)** −5, −10 **c)** 14, 16 **d)** 41, 51

**2. a)** No **b)** Yes **c)** Yes **d)** Yes

**e)** No; The graph does not pass through the origin.

# Teaching Suggestions

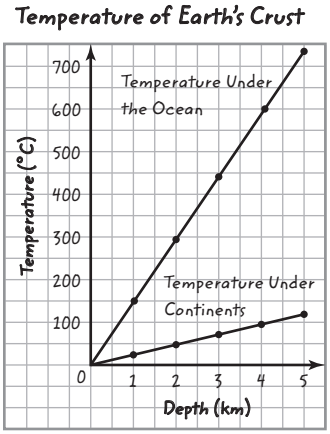
- Assign the Warm-Up. (2 min)
- Discuss the opening paragraph and what the students need to do in the Investigate.
- After students complete the Investigate, relate the multiple representations of partial variation numerically, verbally, graphically, and algebraically. Discuss how the equation is developed, for example, by looking at the fixed, initial, or starting value, and adding the variable term, which is formed by multiplying the constant of variation by the independent variable. You may wish to use **BLM G10 Grid Paper** to support this activity. (25 min)
- Present Examples 1 and 2, or similar examples that illustrate the multiple representations of relations. It is equally important that students experience abstract examples, such as Example 1 and applications such as Example 2. When looking at data, such as in Example 1, students may need assistance in finding the constant of variation. Some students divide each difference between successive $y$-values by the corresponding difference between successive $x$-values. Others may try to see a pattern in the terms. Both methods are valid. (20 min)
- Discuss the questions from Communicate Your Understanding. (10 min)
- Assign and discuss Practise questions 1 and 2. (10 min)
- You may wish to use **BLM 5.2.1 Practice: Partial Variation** for remediation or extra practice.

### Investigate Answers (page 246)

**1.**

| Depth (km) | Temperature Under Oceans (°C) | Temperature Under Continents (°C) |
|---|---|---|
| 0 | 10 | 10.00 |
| 1 | 155 | 31.75 |
| 2 | 300 | 53.50 |
| 3 | 445 | 75.25 |
| 4 | 590 | 97.00 |
| 5 | 735 | 118.75 |

**2. a) and b)**




**3.** Both graphs start at 10 on the temperature axis and go in a straight line upward, from left to right. The "Temperature Under Oceans" graph rises more steeply than the other graph does.

**4.** They differ in that they do not start at (0, 0) on the vertical axis; they start at (0, 10).

**5. a)** 155, 300, 445, 590, 735; they are the same.

**b)** This equation works in a similar way to how an equation for direct variation works. There is a constant of variation that represents the constant average increase in temperature. The difference is that a value is added to the equation.

**c)** $T = 21.75d + 10$, where $T$ represents the temperature, in degrees Celsius, and $d$ represents the depth, in kilometres, under the continents.

**6.** For $T = 145d + 10$, $T$ represents the temperature, in degrees Celsius, at a depth of $d$ kilometres under the oceans, 145 is the constant of variation, and 10 is the starting temperature. For $T = 21.75d + 10$, $T$ represents the temperature, in degrees Celsius, at a depth of $d$ kilometres under the continents, 21.75 is the constant of variation, and 10 is the starting temperature. The $d$ is the "depth" column in the table and the $T$ is the temperature column. Both equations are of the form of a direct variation equation $(y = kx)$, except there is a value added on to them $(y = kx + \text{value})$.

### Communicate Your Understanding Responses (page 250)

**C1. a)** Partial variation; it is a straight line that does not pass through the origin.

**b)** Neither; it is not a straight line.

**c)** Direct variation; it is a straight line that passes through the origin.

**C2.** The initial value of the cost of repair occurs when the number of hours is 0. The initial value of the cost of repair is \$50. The constant of variation is 40. This relationship is a partial variation, so, its equation will be of the form $c = mh + b$, where $c$ is the cost of repair, $m$ is the constant of variation, $h$ is the number of hours, and $b$ is the constant value. Substitute $b = 50$ and $m = 40$ to get the equation $c = 40h + 50$. As the number of hours changes from 0 to 1, the cost of repair changes from \$50 to \$90. Therefore, the cost of repair increases by \$40 as the number of hours increase by 1.

**C3.** As $x$ changes from 0 to 1, $y$ changes from 10 to 14. Therefore, $y$ increases by 4 as $x$ increases by 1. The initial value of $y$ occurs when $x$ is 0, so, the initial value of $y$ is 10. So, the constant of variation is 4. Substitute $b = 10$ and $m = 4$ into the equation $y = mx + b$ to get $y = 4x + 10$.

## Practise

In questions 2 and 3, students may not see the pattern in the growth of the $y$-values. Suggest that they count by 5s for each question. You may wish to use **BLM G10 Grid Paper** to support questions 2 and 3.

## Connect and Apply

Students may find question 7 challenging, since it is the first time the independent variable does not increase by 1. Ask them what the increase in cost is per 100 and to convert to a rate.

Question 8 refers to the Chapter Problem. Note to students that the first toothpick pattern has been coloured red. This will help students see the initial, or fixed, value of the relation, since this is the first example where the independent variable begins at 1 instead of 0. Encourage students to make a chart to help them see the pattern of numbers.

You may wish to use **BLM G10 Grid Paper** to support questions 6, 11, and 12.

## Accommodations

**Perceptual**—Allow students to use words instead of variables to identify the independent variable and the dependent variable.

**Spatial**—Provide students with opportunities to use technology to relate direct variation ($y = mx + b$) to a linear relation with a fixed initial amount or $y$-intercept of $b$.

**Memory**—Encourage students to colour code the slope and $y$-intercept to relate the variable cost to the slope and the fixed cost to the slope.

**ESL**—Give students extra time to interpret and complete the questions in this section.

## Student Success

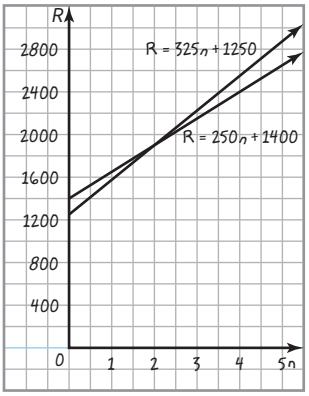
Give each student a different direct or partial variation problem to solve. Then, use an inside/outside circle to have stdents present their problem and solution to others.

### Achievement Check Answers (page 253)

**12. a)** The equation for *Cats* is a partial variation:
$R = 325n + 1250$, where $R$ is the royalty fee, in dollars, and $n$ is the number of shows performed.
The equation for *Fame* is a partial variation:
$R = 250n + 1400$, where $R$ is the royalty fee, in dollars, and $n$ is the number of shows performed.

**b)**

| Number of Shows | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Royalty for *Cats* ($) | 1250 | 1575 | 1900 | 2225 | 2550 |
| Royalty for *Fame* ($) | 1400 | 1650 | 1900 | 2150 | 2400 |




**c)** The company pays the same amount for each show, $1900, if there are two performances.

**d)** Answers will vary. Points to consider:
- If direct variation is used, the cost for theatre groups performing more than one or two shows will be high. For example, if the direct variation is based on royalties of $1900 after two shows then four shows would cost $3800 compared with the $2550 and $2400 for partial variation.
- The company needs a fixed price to ensure a reasonable income from groups that perform only one or two shows. In the direct variation example above one show would cost $950 compared to the $1575 and $1650 of the partial, variation model. Hence, partial variation is used to set a rate that is more equitable for performers and royalty rights holders.

## Extend

Question 13 asks students to solve the equation for the dependent variable. It is accessible to level 3 and 4 students. You may wish to use **BLM A7 Thinking General Scoring Rubric** for question 13 to assist you in assessing your students.

Question 14 is a good example of a piecewise linear function. Students should graph each "piece" separately, but on the same grid. You may wish to use **BLM G10 Grid Paper** to support question 14.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–6, 10 |
| Typical | 1–7, 9, 10, 11 |
| Extension | 13, 14 |

# 5.3 Slope

**Strand:**
Linear Relations

**Strand:**
Analytic Geometry

**Student Text Pages**
254 to 265

**Suggested Timing**
80 min

**Tools**
- grid paper

**Related Resources**
BLM G10 Grid Paper
BLM A16 My Progress as a Mathematician
BLM 5.3.1 Practice: Slope
BLM A23 News Report Checklist

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Understanding the Characteristics of Linear Relations**

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**Connecting Various Representations of Linear Relations**

**RE3.01** determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;

**Investigating the Properties of Slope**

**AG2.01** determine, through investigation, various formulas for the slope of a line segment or to determine the slope of a line segment or a line.

## Link to Get Ready

The Get Ready segment Ratio and Proportion provides the needed skills for this section. You may wish to have students complete Get Ready questions 4 to 6 before starting this section.

### Warm-Up

**1.** Mental Math: Express in lowest terms.

**a)** $\frac{8}{6}$ **b)** $\frac{50}{75}$ **c)** $\frac{7}{14}$ **d)** $-\frac{9}{12}$

**2.** Change to decimal form.

**a)** $\frac{1}{2}$ **b)** $\frac{3}{8}$ **c)** $\frac{-2}{9}$ **d)** $\frac{-68}{10}$

**Warm-Up Answers**

**1. a)** $\frac{4}{3}$ **b)** $\frac{2}{3}$ **c)** $\frac{1}{2}$ **d)** $-\frac{3}{4}$

**2. a)** $0.5$ **b)** $0.375$ **c)** $-0.22\overline{2}$ **d)** $-6.8$

## Teaching Suggestions

- For Investigate A, through a class discussion, introduce the concept of steepness, without introducing positive or negative. (5 min)
- Investigate B introduces the concept of slope, one of the most important concepts in understanding relations. Ensure that students understand that graphs are read from left to right and that the slope is the rate of change of the rise (vertical) with respect to the run (horizontal). Negative slope occurs when travelling downward while reading from left to right. Clear, accurate graphs on grid paper allow the students to visualize this abstract concept. You may wish to use **BLM G10 Grid Paper** to support this activity. (15 min)
- Discuss the Examples. (30 min)
- Example 1 illustrates slope in an applied situation, as a rate of change. Note to students that the slope is a decimal value related to a unit change in horizontal distance.

## Common Errors

- Some students may reverse the numerator and denominator of slope because they are used to writing $x$ before $y$.

**R$_x$** Have students graph (2, 4) and (7, 5) or any two points one square apart horizontally. Extend the line through the points to illustrate that the slope is constant. Calculate the slope using the given points and repeat using any two points that are more than one square apart horizontally. Illustrate that the slope is a rate of change with respect to $x$. You may wish to use **BLM G10 Grid Paper**.

## Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.)

- Example 2 calculates positive and negative slopes and extends to horizontal and vertical lines. You may wish to provide contextual examples of these. For a horizontal example, plot a graph of the temperature of the interior of an oven (dependent) while it is heating up (time = independent), then reaching its cooking temperature. For a vertical example, plot a graph of volume of gasoline in the gas tank (dependent) versus driving distance (independent) with a stop at the gas station along the way.
- For Example 3, students may have difficulties with where to place the negative sign. With slope, by convention it should be placed in the numerator. Discuss with the students that there is an infinite number of possibilities for point B. The one shown is simply the neighbouring one.
- Discuss the vocabulary (slope, rise, run) before discussing questions from Communicate Your Understanding. (2 min)
- You may wish to use **BLM A16 My Progress as a Mathematician** as a self-assessment for students at any time during this section.
- You may wish to use **BLM 5.3.1 Practice: Slope** for remediation or extra practice.

### Investigate Answers (page 254)

**A.**

**1.** middle hill, hill on right, hill on left

**2.** The steepness of the first hill is $2 \div 8 = 0.25$. The steepness of the second hill is $4 \div 10 = 0.4$. The second hill is steeper, since 0.4 is greater than 0.25.

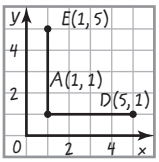
**3.** Divide the height of the hill by the length of the hill. The greater the result is, the steeper the slope will be.

**B.**

**1. a)** The slope is positive because it rises upward, from left to right.
**b)** rise 3, run 2
**c)** $\frac{3}{2}$

**2. a)** The slope is negative because it goes downward, from left to right.
**b)** rise $-2$, run 3
**c)** $-\frac{2}{3}$

**3. a)**




**b)** rise 0, run 4
**c)** slope $= 0 \div 4$
$= 0$
The segment is flat so there is no slope.

**4. a)** See answer to question 3a).
**b)** rise 4, run 0
**c)** slope $= 4 \div 0$; division by 0 is undefined, so, it is not possible to calculate the slope of a vertical line segment.

**5.** To calculate the slope of a line segment, divide its rise (vertical height) by its run (horizontal distance). If the line segment rises upward, from left to right, then its slope is positive. If it goes downward, from left to right, then its slope is negative. If the line segment is horizontal, then it has a slope of 0, or no slope. If the line segment is vertical, then its slope is undefined.

### Communicate Your Understanding Responses (page 258)

**C1.** Her conclusion is correct but her reasoning is incorrect. It is possible for two points with negative coordinates to form a positive slope. For example, a line segment with endpoints P(1, −8) and Q(4, −1) has a positive slope, even though some of the coordinates are negative. The slope in this graph is negative because it goes downward, from left to right.

**C2. a)** increase the rise **b)** decrease the run

## Accommodations

**Visual**—Encourage students to draw visual representations of slopes, such as the slope of a roof, in order to visualize the rise and the run, to use to calculate the slope, $m = \frac{\text{rise}}{\text{run}}$.

**Perceptual**—Have students use visual cues, such as highlighting or colour coding to remember the different forms used to calculate the slope when given two points.

**Spatial**—Let students work with visual representations of lines with positive, negative, and zero slopes and match them to numerical values.

**Language**—Give students verbal instructions (for example, down 2, right 3) to find a second point when given a point and a slope of a line, instead of (+3, −2).

**ESL**—Let students use their translators to understand the meanings of the new words in this section.

## Student Success

Question 21 can be turned into a larger investigation involving the entire class. Have students collect data, then analyse the data using a graphing calculator. Have students do an internet search for safety or building code restrictions on the maximum slopes for stairs.

## Practise

The Practise questions are straightforward and support the skills learned in the Examples. Some students may have difficulties with negatives.

## Connect and Apply

Question 10 may be challenging to some students. With a slope of $\frac{3}{5}$ and a horizontal spacing of 1 m, the height would be $\frac{3}{5}$ m or 0.6 m.

For question 11, students need to review their understanding of percents. Review Get Ready, Percents, or use **BLM 5.3.1 Practice: Slope** as remediation or a skills review. The slope would be $3\% = \frac{3}{100}$.

Question 12 is an excellent real-life job application of slope.

Questions 16 and 17 offer two good examples of the use of slope in architecture. You may wish to use **BLM G10 Grid Paper** for any of these questions.

## Extend

Question 18 is an interesting problem as it applies to ski hills, but asks the students to manipulate the slope formula. It is accessible to level 3 and 4 students.

Question 19 involves working with three dimensions and will require students to use the Pythagorean theorem.

## Literacy Connections

**News Report**

Have students look at the section opener picture, and imagine that the headline reads, "Downhill Skier is Top in the Field." Instruct students to write a news report about the photo and the given headline. Remind them to include a link between the photo and the headline. Explain to students that they should try to answer the questions who, what, where, when, why, and how in their news reports. Remind them to write in the third person. You may wish to use **BLM A23 News Report Checklist** to assist you in assessing your students.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–9 |
| Typical | 1–17 |
| Extension | 18–23 |

# 5.4 Slope as a Rate of Change

**Strand:**
Linear Relations

**Strand:**
Analytic Geometry

**Student Text Pages**
264 to 271

**Suggested Timing**
80 min

**Tools**
- grid paper

**Technology Tools**
- *Fathom*™
- computers
- graphing calculators

**Related Resources**
BLM A12 Group Work Assessment Rubric
BLM T6 *Fathom*™
BLM 5.4.1 Practice: Slope as a Rate of Change
BLM G10 Grid Paper
BLM 5.4.2 Achievement Check Rubric

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Understanding Characteristics of Linear Relations**

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**Investigating the Properties of Slope**

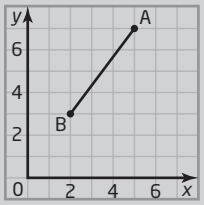
**AG2.01** determine, through investigation, various formulas for the slope of a line segment or to determine the slope of a line segment or a line;

**AG2.03** determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is $50, plus $5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, $p$, in this equation);

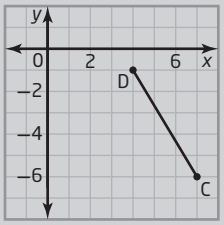
### Warm-Up

**1.** Calculate the slope of each line segment.

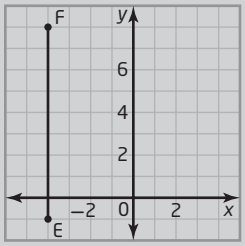
**a)**



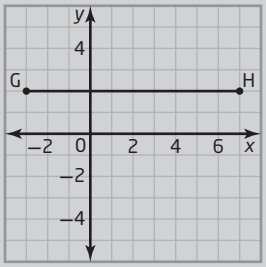

**b)**




**c)**




**d)**




**Warm-Up Answers**

**1. a)** rise $= 5 - 2$, or 3, run $= 7 - 3$, or 4, slope $= \frac{3}{4}$

**b)** rise $= -6 - (-1)$, or $-5$, run $= 9 - 4$, or 5, slope $= -\frac{5}{5}$, or $-1$

**c)** rise $= 8 - (-1)$, or 9, run $= -4 - (-4)$, or 0, slope $= \frac{9}{0}$ or undefined

**d)** rise $= 2 - 2$, or 0, run $= 7 - (-4)$, or 10, slope $= \frac{0}{10}$, or 0

## Teaching Suggestions

- Read the opening paragraph aloud, and have a short class discussion on the meaning of rate of change. Some examples include: speed, temperature change per hour, and change in volume of gasoline per kilometre travelled. (5 min)

## Common Errors

- Some students may divide by the *x*-value instead of the change in *x*.

$R_x$ Remind students that, since it is a rate of *change*, both the numerator and denominator need to represent change.

- Some students may invert the slope/rate of change formula

$R_x$ Discuss the meaning of the term *per* and how it is frequently represented by a slash, meaning, "divided by."

## Ongoing Assessment

- Use Achievement Check question 18 to monitor student success. See Achievement Check Answers and **BLM 5.4.2 Achievement Check Rubric**.
- Chapter Problem question 9 also can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

## Accommodations

**Visual**—Allow students to work in groups to complete specific questions in this section and do group presentations of their solutions to their classmates.

**Perceptual**—Encourage students to use visual cues to relate rate of change to the slope of a line.

**Memory**—Encourage students to use cue cards to remember the formulas for slope and rate of change.

- Have students work with a partner or in small groups on the Investigate. You may wish to use **BLM A12 Group Work Assessment Rubric** to assist you in assessing your students. Extend the earlier discussion on rate of change to include slope of a graph. (5 min)
- Discuss Examples 1 and 2, or similar examples that show multiple representations of rates of change and slope, numerically, graphically, verbally, and with the slope formula. Stress to students that, although most rates of change are relative to time, this is not always the case. (15 min)
- Discuss Communicate Your Understanding C1 and C2. Question C2 will provide a good indicator of a student's level of understanding of rates of change. (15 min)
- If students will be using technology for the graphing in this section, you may wish to use **BLM T6 *Fathom*™** and/or refer them to the Technology Appendix in their textbook to support these activities.
- Assign and take up Practise questions 1 to 5. (15 min)
- You may wish to use **BLM 5.4.1 Practice: Slope as a Rate of Change** for remediation or extra practice.

### Investigate Answers (page 264)

**1.** The distance travelled by the person or animal in a given time.

**2.** sprinter 10.2, polar bear 11.1, alligator 15.5, cyclist 16.5, cheetah 31.1

**3.** sprinter 10.2 m/s, polar bear 11.1 m/s, alligator 15.5 m/s, cyclist 16.5 m/s, cheetah 31.1 m/s

**4.** The slope represents the rate of change. The greater, or faster, a rate of change is, the steeper the graph of the relation will be.

### Communicate Your Understanding Responses (page 267)

**C1.** He divided the run by the rise ($5 \div 400$). He should have divided the rise by the run ($400 \div 5$).

**C2. a)** C **b)** A **c)** D **d)** B

## Practise

The Practise questions are straightforward and consolidate the examples. Students may have difficulties using appropriate words in their interpretations.

## Connect and Apply

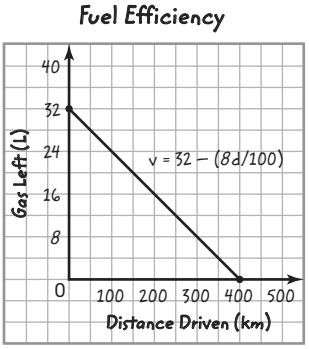
The questions provide interesting contexts for rate of change and are accessible to most students. Question 8 should provide some interesting discussion. Level 4 students may convert to a percent rate of change before discussing whether it was a popular piece of software.

Question 9 refers to the Chapter Problem. You may wish to assign this problem, but not take it up until the end of the chapter.

Students may find question 18 challenging because the rate is per 100 km, rather than a unit rate. Question 18, the Achievement Check, is a good communication question. You may wish to use **BLM G10 Grid Paper** for these activities.

**Achievement Check Answers (page 271)**

**18. a)**




| Distance (km) | 0 | 100 | 200 | 300 | 400 |
|---|---|---|---|---|---|
| Volume of Gasoline Remaining (L) | 32 | 24 | 16 | 8 | 0 |

**b)** slope $= \frac{\text{rise}}{\text{run}}$

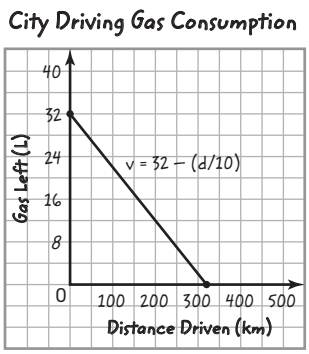
$= \frac{-8}{100}$

$= -0.08$

This means that Kim's car uses 0.08 L of gas for each 1 km driven on the highway. The negative sign means that there is less gas in the fuel tank.

**c)** Since Kim's car uses 25% more gas in city driving, it will use 25% of 8 L, or 2 L more for each 100 km driven in the city.

| Distance (km) | 0 | 100 | 200 | 300 | 320 |
|---|---|---|---|---|---|
| Volume of Gasoline Remaining (L) | 32 | 22 | 12 | 2 | 0 |




slope $= \frac{\text{rise}}{\text{run}}$

$= \frac{-10}{100}$

$= -0.10$

This means that Kim's car uses 0.10 L of gas for each 1 km driven on the highway. Again, the negative sign means that there is less gas in the fuel tank.

## Extend

Question 19 is a challenging problem as it uses percent change. This creates a non-linear (exponential) graph. Encourage students to explore other graphs created through percent decline or growth.

Question 20 shows different rates of change in a piecewise graph.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–8 |
| Typical | 1–8, 10, 13–16 |
| Extension | 19, 20 |

# 5.5 First Differences

**Strand:**
Linear Relations

**Strand:**
Analytic Geometry

**Student Text Pages**
272 to 278

**Suggested Timing**
80 min

**Tools**
- grid paper

**Technology Tools**
- graphing calculators

**Related Resources**
BLM G10 Grid Paper
BLM A20 Learning Skills Checklist
BLM 5.5.1 Practice: First Differences

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Understanding Characteristics of Linear Relations**

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**RE2.02** construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

**RE2.03** identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;

**Connecting Various Representations of Linear Relations**

**RE3.03** determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

**Investigating the Relationship Between the Equation of a Relation and the Shape of its Graph**

**AG1.01** determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of nonlinear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation).

## Teaching Suggestions

- First differences are best learned through an investigative approach. Have students investigate the first differences of data in charts and compare the results to their graphs. You may wish to use **BLM G10 Grid Paper** to support this activity.
- Graphing calculators are not essential, since students will quickly pick up on the concept while graphing by hand. However, the graphing calculator will make it easier to quickly graph quadratic and exponential relations. The key is for students to recognize first differences and how they apply to linear and non-linear relations, and to be able to recognize the equations of linear and non-linear relations. (20 min)
- Assign and take up Communicate Your Understanding C1 and C2 and Practise questions 1 to 3. (30 min)
- You may wish to use **BLM A20 Learning Skills Checklist** at any time during this section to assist you in assessing your students.
- You may wish to use **BLM 5.5.1 Practice: First Differences** for remediation or extra practice.

## Common Errors

- Some students may subtract the smaller value from the greater value, always getting a positive first difference.

**R$_x$** Have students draw small arrows pointing upward between successive data values as a reminder to subtract "upward."

## Ongoing Assessment

- Chapter Problem question 6 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

## Accommodations

**Gifted and Enrichment**—Challenge students to create and investigate tables of values with third differences or fourth differences.

**Perceptual**—Encourage students to use scientific calculators to calculate first differences.

**Spatial**—Let students use graphing calculators to determine if functions are linear or non-linear.

**Motor**—Provide students with photocopied exercise pages to complete the columns when determining missing values in tables of values.

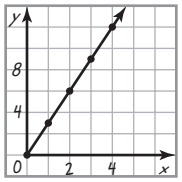
### Investigate Answers (page 272)

**A.**

**1. a)**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |

**b)**




**c)** The relation is linear.

**2. a)** They increase by 1 each time.

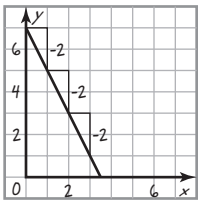
**b)**

| $x$ | $y$ | First Differences |
|---|---|---|
| 0 | 0 | |
| 1 | 3 | 3 |
| 2 | 6 | 3 |
| 3 | 9 | 3 |
| 4 | 12 | 3 |

**c)** Both values are 3.

**3. a)**

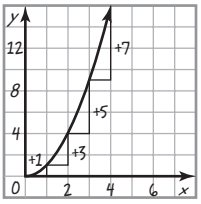
| $x$ | $y$ | First Differences |
|---|---|---|
| 0 | 7 | |
| 1 | 5 | −2 |
| 2 | 3 | −2 |
| 3 | 1 | −2 |
| 4 | −1 | −2 |




linear; $x$-values increase by 1; first differences are the same, −2

**b)**

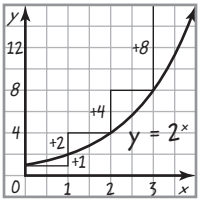
| $x$ | $y$ | First Differences |
|---|---|---|
| 0 | 0 | |
| 1 | 1 | 1 |
| 2 | 4 | 3 |
| 3 | 9 | 5 |
| 4 | 16 | 7 |




non-linear; x-values increase by 1; first differences are not the same

**c)**

| $x$ | $y$ | First Differences |
|---|---|---|
| 0 | 1 | |
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 3 | 8 | 4 |
| 4 | 16 | 8 |




non-linear; $x$-values increase by 1; first differences are not the same

**4.** The first differences of linear relations are the same.

**5. a)** linear **b)** non-linear **c)** linear

**6.** If the first differences of a relation are the same, then the relation is linear. If the first differences are not the same, then the relation is non-linear.

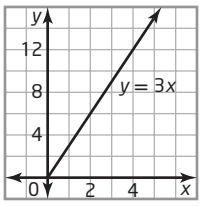
**7.** If an equation is of the form of a direct variation $(y = kx)$ or a partial variation $(y = mx + b)$, then the relation is linear.

**B.**

**1.**

| $y = 3x$ | |
|---|---|
| $x$ | $y$ |
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 5 | 15 |

**2.**

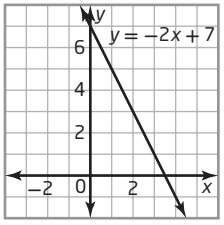



The graph is linear.

**3. a)** They increase by 1. **b)** 3
**c)** They are all the same, 3.

**4. a)**

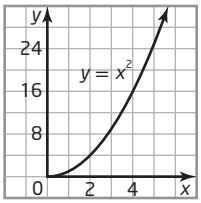
| $y = -2x + 7$ | | First Differences |
|---|---|---|
| $x$ | $y$ | |
| 0 | 7 | |
| 1 | 5 | −2 |
| 2 | 3 | −2 |
| 3 | 1 | −2 |
| 4 | −1 | −2 |
| 5 | −3 | −2 |




linear; $x$-values increase by 1; first differences are the same

**b)**

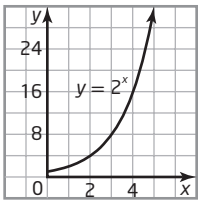
| $y = x^2$ | | First Differences |
|---|---|---|
| $x$ | $y$ | |
| 0 | 0 | |
| 1 | 1 | 1 |
| 2 | 4 | 3 |
| 3 | 9 | 5 |
| 4 | 16 | 7 |
| 5 | 25 | 9 |




non-linear; $x$-values increase by 1; first differences are not the same

**c)**

| $y = 2^x$ | | First Differences |
|---|---|---|
| $x$ | $y$ | |
| 0 | 1 | |
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 3 | 8 | 4 |
| 4 | 16 | 8 |
| 5 | 32 | 16 |




non-linear; $x$-values increase by 1; first differences are not the same

**5.** The first differences of linear relations are the same.

**6. a)** linear **b)** non-linear **c)** linear

**7.** If the first differences of a relation are the same, then the relation is linear. If the first differences are not the same, then the relation is not linear.

**8.** If an equation is of the form of a direct variation $(y = kx)$ or a partial variation $(y = mx + b)$, then the relation is linear.

### Communicate Your Understanding Responses (page 275)

**C1. a)** It is possible, because $x$-values are evenly spaced.
**b)** It is not possible, because $x$-values are not evenly spaced.

**C2.** Jacob's hourly wage is the same as the first differences.

## Practise

The Practise questions are straightforward and consolidate the Investigate.

## Connect and Apply

Students will enjoy the toothpick patterns in question 6, the Chapter Problem.

## Extend

In question 8, students need to generate their own data from the situation. You may have to remind students of the formula for the area of a triangle.

Question 10 requires proportional reasoning to generate the data.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–3 |
| Typical | 1–5, 7 |
| Extension | 8–10 |

# 5.6 Connecting Variation, Slope, and First Differences

**Strand:**
Linear Relations

**Strand:**
Analytic Geometry

**Student Text Pages**
279 to 287

**Suggested Timing**
160 min

**Related Resources**
BLM 5.6.1 Practice: Connecting Variation, Slope, and First Difference
BLM5.6.2 Rule of Four
BLM G10 Grid Paper
BLM 5.6.3 Achievement Check Rubric

**Tools**
- grid paper
- pencils
- paper

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Understanding Characteristics of Linear Relations**

**RE2.01** construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

**RE2.02** construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

**RE2.05** determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

**Connecting Various Representations of Linear Relations**

**RE3.03** determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

**Investigating the Properties of Slope**

**AG2.01** determine, through investigation, various formulas for the slope of a line segment or to determine the slope of a line segment or a line;

**AG2.03** determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, $p$, in this equation).

## Link to Get Ready

You may wish to have students review their skills with rational numbers by completing Get Ready questions 1 and 2.

## Teaching Suggestions

- You may wish to teach this concept over two days, leaving Examples 2 and 3 to the second day. Concentrate on the comparisons on the first day, and add some abstractions on the second day.
- You may wish to use **BLM 5.6.1 Practice: Connecting Variation, Slope, and First Differences** for remediation or extra practice.

**Day 1**

- Have students work through the Investigate independently or with a partner. Follow up with a class discussion of their findings. Allow students to state their findings in their own words, using formal language to summarize at the end. (20 min)
- Describe the symbolic language used for slope:

$$m = \frac{\text{rise}}{\text{run}} \text{ or } \frac{\text{change in } y}{\text{change in } x} \text{ or } \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{\Delta y}{\Delta x}$$

Explain that each is acceptable to use in a given situation. (10 min)

## Common Errors

- Some students may not be able to make the connection between variation, slope, and rate of change.

**$R_x$** As many students are visual learners, have them graph each data relationship before considering the connections. Ask them to calculate the slope of the line formed by the data and to interpret it in words, relative to the problem at hand. Prompt them with terms such as *per* or *for each*.

## Ongoing Assessment

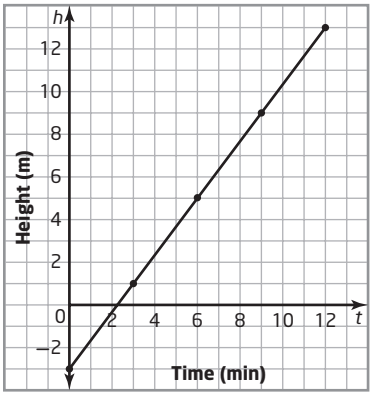
- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 5.6.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Make the connection between an initial, or constant, value of a relation and the *y*-intercept of a graph, *b*, and how this then forms the linear relation $y = mx + b$.
- Discuss Example 1 to consolidate the connection between variation, slope and rate of change. Stress the Rule of Four, which formalizes the multiple representations of relations. You may wish to use **BLM 5.6.2 Student Success: The Rule of Four** to help students consolidate the learning. (15 min)
- You may wish to use **BLM G10 Grid Paper**.
- Discuss Communicate Your Understanding questions C1 and C2 before assigning and taking up Practise questions 1 and 2. (10 min)

### Day 2

- Review the Rule of Four with students, and discuss Example 2 as a class, or a similar example that begins with a word description and proceeds to develop a table of values, a graph, and an equation. You may wish use an enlarged, completed copy of **BLM 5.6.2 Student Success: The Rule of Four** and use as a poster, or have students produce their own poster. Follow up with Example 3, or similar abstract example, that begins with an equation of a line in $y = mx + b$ form, and proceeds to complete the Rule of Four. (30 min)
- Discuss Communicate Your Understanding question C3. (10 min)
- Before assigning the questions, summarize the different representations of slope, the meaning of the *y*-intercept, and the Rule of Four. (5 min)

### Investigate Answers (page 279)

**1.** 

It is a partial variation.

**2.** *t*-values increase by 3; since first differences are the same, 4, the relation is linear.

**3.** $\frac{4}{3}$

**4.** The slope is the value of the first differences divided by the amount that the *t*-values increase.

**5.** −3 m

**6.** $h = \frac{4}{3}t - 3$

**7.** In a linear relation, the slope is equal to the value of the first differences of the dependent variable divided by the amount that the independent variable increases. The slope is the constant of variation. The initial value of the dependent variable is the same as the fixed value of a partial variation.

### Communicate Your Understanding Responses (page 284)

**C1.** They both tell how one quantity changes as another quantity changes.

**C2.** For example, calculate the rise over the run, the first differences divided by the increment in *x*-values in a table of values, or $\frac{y_2 - y_1}{x_2 - x_1}$ for two points $(x_1, y_1)$ and $(x_2, y_2)$ on the graph.

**C3.** The line has a rise of 3 and a run of 2, so, the slope is $\frac{3}{2}$ or 1.5.

**Accommodations**

**Perceptual**—Encourage students to create visual representations using either pencils and paper or technology to relate the constant of variation to slope to first differences.

**Memory**—Allow students to use words instead of variables when working with rates of change.

**Student Success**

Use the **timed retell** strategy to ensure that all students understand the relationship between slope, direct variation, and first diiferences.

## Practise

The Practise questions are straightforward and consolidate the Examples. In questions 4, 5, and 6, ensure the students complete the Rule of Four fully and in clear language. If you have not used **BLM 5.6.2 Student Success: The Rule of Four** previously, you may wish to have students complete it now.

## Connect and Apply

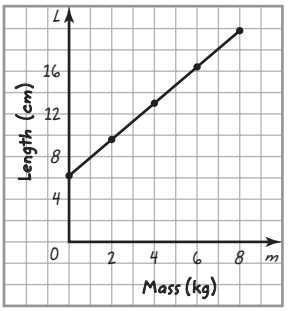
Most questions are straightforward and will be accessible for most students. Level 1 and 2 students may find the abstractions in questions 9 to 12 challenging. In all questions, the key is the multiple representations of the relation.

Students also need to demonstrate skills in interpolation, such as in question 13e) and with extrapolation in question 14e). You may wish to use **BLM G10 Grid Paper** to support these activities.

For question 14, you may wish students to use technology for graphing. Refer students to the Technology Appendix at the back of their textbook.

**Achievement Check Answers (page 287)**

**14. a)**




**b)** The 6.2 cm represents the starting length of the elastic band when no mass is attached.

**c)** slope $= \frac{9.6 - 6.2}{2 - 0}$ or slope $= \frac{19.8 - 9.6}{8 - 2}$

$= 1.7$ $= 1.7$

The various calculations show that the slope is constant.
The slope represents the additional stretch of 1.7 cm that occurs for each 1 kg of mass that is added.

**d)** $L = 6.2 + 1.7m$ or $y = 1.7x + 6.2$

**e)** $L = 6.2 + 1.7(10) = 23.2$
The elastic band will be 23.2 cm long.

**f)** The graph would no longer be linear. This might indicate that the elastic band is near its maximum stretching capacity. Therefore, the length for 10 kg might be 19.2 cm or else the band might break.

## Extend

Question 15 requires students to multiply by percents greater than 100 and then make a comparison to a given relationship. This question is accessible to level 3 and 4 students.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–8, 11, 12 |
| Typical | 1–13 |
| Extension | 15, 16 |

# Chapter 5 Review

**Student Text Pages**
288 to 289

**Suggested Timing**
80 min

**Tools**
• grid paper

**Related Resources**
BLM 5.CR.1 Chapter 5 Review
BLM G10 Grid Paper
BLM A4 Presentation Checklist

**Ongoing Assessment**

- Upon completing the Chapter Review, students can also answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related Example in the text to help you with the questions? For which questions?*

## Using the Chapter Review

Each question in **BLM 5.CR.1 Chapter 5 Review** reviews different skills and concepts. Have students work independently to complete the Chapter Review, then with a partner to compare solutions. Alternatively, the Chapter Review could be assigned for reinforcing skills and concepts in preparation for the Practice Test. You may wish use **BLM G10 Grid Paper**. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult. Having students present their solutions to the entire class is an excellent opportunity for students to practise their communication skills and to discuss alternative ways of presenting their answers, especially with multiple representations being one of the key concepts in this unit. You may wish to use **BLM A4 Presentation Checklist** to assist you in assessing students' presentation skills.

After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content.

# Chapter 5 Practice Test

**Student Text Pages**
290 to 291

**Suggested Timing**
80 min

**Related Resources**
BLM 5.PT.1 Chapter 5 Practice Test
BLM 5.CT.1 Chapter 5 Test
BLM G10 Grid Paper

**Tools**
• grid paper

**Technology Tools**
• graphing calculators

**Summative Assessment**

• After students complete **BLM 5.PT.1 Practice Test**, you may wish to use **BLM 5.CT.1 Chapter Test** as a summative assessment.

**Accommodations**

**Motor** —Give students extra time to complete these questions or allow them to do fewer questions, including examples of each type of question.

**Memory**—Encourage students to use technology such as a graphing calculator when working through the questions.

**ESL**—Allow students to use a dictionary or translator when completing the questions in the Chapter Review and Chapter Test.

## Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

| Question | Section(s) | Refer to |
|---|---|---|
| 1 | 5.2 | Example 1 (page 248) |
| 2 | 5.1 | Example 1 (page 239) |
| 3 | 5.3 | Example 1 (page 255) |
| 4 | 5.6 | Investigate (page 279) |
| 5 | 5.6 | Example 1, 2 (pages 280, 281) |
| 6 | 5.1 | Example 1, 2 (pages 239, 240) |
| 7 | 5.5 | Investigate (page 272) |
| 8 | 5.2 | Example 2 (page 249) |
| 9 | 5.4 | Example 2 (page 266) |

### Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?

- Discern between direct and partial variation
- Calculate the constant of variation
- Calculate slope
- Relate slope and rate of change
- Find first differences
- Interpret first differences to determine if a relation is linear or non-linear
- Represent relations verbally, numerically, graphically, and algebraically
- Write a linear relation in the form $y = mx + b$, given an initial value or $y$-intercept, and the slope or constant rate of change
- Communicate their understanding of the concepts
- Solve problems involving these concepts
- Choose the appropriate tools and calculations when solving problems
- Reflect on their findings and adjust their solutions appropriately

# Chapter 5 Problem Wrap-Up

**Student Text Pages**
291

**Suggested Timing**
40–70 min

**Related Resources**
BLM G10 Grid Paper
BLM 5.CP.1 Chapter 5 Problem Wrap-Up Rubric

**Tools**
- grid paper
- toothpicks and/or algebra tiles

**Technology Tools**
- graphing calculators

**Summative Assessment**

- Use **BLM 5.CP.1 Chapter 5 Problem Wrap-Up Rubric** to assess student achievement.

## Using the Chapter Problem

If the Chapter Problem questions have been taken up in class, plan on 40 min of class time. Otherwise, plan on 30 min to discuss the Chapter Problem questions and 40 min for the Chapter Problem Wrap-Up.

When discussing the toothpick patterns throughout this chapter, have students visualize the patterns by seeing how and by how much they grow on each step. Have them organize their thoughts by placing the data into a chart. They should use the tools of this chapter to calculate the slope, analyse the rate of change, and investigate first differences. You may also wish students to extend their learning by investigating other toothpick patterns or designing their own to determine what makes a pattern linear versus non-linear.

The Chapter Problem Wrap-Up question appears to be straightforward. However, the second part of the question is quite challenging and requires some creativity on the part of students. The first part of the problem can be completed in 15 min, but the second half may require an extended period of time. The second half might even be assigned as a take-home problem. Conduct a short review of the Chapter Problem questions from the sections, reviewing the equation creation skills that were used.

Many students will benefit from having a supply of toothpicks or similar manipulatives available for this problem. Grid paper and graphing calculators should also be available for students to use. You may wish to use **BLM G10 Grid Paper** to support this activity.

### Level 3 Sample Response

I made a chart for the first pattern after drawing a few more diagrams in the pattern:

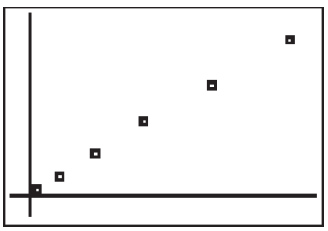
| Number of Regions ($R$) | Number of Toothpicks ($T$) |
|---|---|
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |
| 6 | 19 |

I think this pattern is linear because the first differences are always 3.
To make an equation, I chose the variables $R$ and $T$. The common first difference should be the slope of the equation.
The equation is of the form $T = 3R + b$
To find $b$, substitute values for $R$ and $T$. $b = 1$
Therefore, the linear equation is $T = 3R + 1$

I made a similar chart for the second pattern after drawing three more patterns.

| Number of Regions ($R$) | Number of Toothpicks ($T$) |
|---|---|
| 1 | 3 |
| 4 | 9 |
| 9 | 18 |
| 16 | 30 |
| 25 | 45 |
| 36 | 63 |

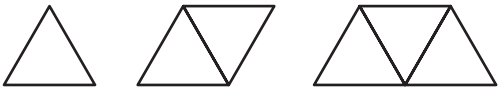
I don't think this is linear. However, I can't do a first differences test because the $R$ variable does not increase in constant manner. I tried doing a scatter plot in my calculator, and it seemed almost linear but not quite. See my screen below.



I could try this another way. Let's try calculating slope.
The slope from (1, 3) to (4, 9) is 2. The slope from (4, 9) to (9, 18) is 1.8. Clearly, this is not linear.
I don't know how to change the diagrams to make the pattern linear.

**Level 4 Sample Response**

For a linear pattern, you have to add the same number of toothpicks each time and create an additional shape. One way to do this is as follows:



The chart for this pattern is:

| Number of Regions ($R$) | Number of Toothpicks ($T$) |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |

The related equation for this pattern is of the form $T = 2R + b$.
Substituting the pair (1, 3), we find that $b = 1$
Therefore, the equation is $T = 3R + 1$

**Level 3 Notes**

Look for the following:

- Complete error-free charts for each pattern based on the given diagrams
- Correct use of slope or first differences calculations in both patterns
- An equation is developed for the first pattern
- The second pattern may not be modified or have an equation provided
- Justifications for each response

**What Distinguishes Level 2**

At this level, look for the following:

- Partial charts for each pattern with possible errors; not all diagrams may be used
- Chart headings for the second pattern may reflect pattern number rather than number of closed regions
- Some errors in slope or first differences calculations in both patterns
- An incomplete equation provided for the first pattern, for example, students may only write $y = mx + b$
- There may be little or no attempt at a solution for the second pattern
- Limited justification for responses

**What Distinguishes Level 4**

At this level, look for the following:

- Additional diagram will be drawn to extend the patterns
- Complete error-free charts of each pattern that include data from additional diagrams in the patterns
- Correct use of slope or first differences calculations in both patterns with recognition that first differences cannot be used in the second pattern
- One or more suggested modifications for the second pattern with appropriate equations
- Possible references to the fact that these patterns are examples of partial variation relationships
- Use of diagrams, graphs, or technology to support that the second pattern is not linear
- Clear and detailed justification for responses