CHAPTER 3

# Polynomials

**Vocabulary**

variable
algebraic expression
term
polynomial
degree of a term
degree of a polynomial
like terms
distributive property

## Curriculum Expectations

### Mathematical Process Expectations

Throughout this course, students will:

**PROBLEM SOLVING**

**MPS.01** develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

**REASONING AND PROVING**

**MPS.02** develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

**REFLECTING**

**MPS.03** demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

**SELECTING TOOLS AND COMPUTATIONAL STRATEGIES**

**MPS.04** select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

**CONNECTING**

**MPS.05** make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

**REPRESENTING**

**MPS.06** create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

**COMMUNICATING**

**MPS.07** communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

**Strand:**
Number Sense and Algebra

## Overall Expectations

By the end of this course, students will:

**NAV.01** demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions;

**NAV.02** manipulate numerical and polynomial expressions, and solve first-degree equations.

## Specific Expectations

### *Operating With Exponents*

By the end of this chapter, students will:

**NA1.01** substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and $9.8^3$ by using a calculator]);

**NA1.02** describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by $x$; area, which is two dimensional, can be represented by $(x)(x)$ or $x^2$; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or $x^3$];

**NA1.03** derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;

**NA1.04** extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.

### *Manipulating Expressions and Solving Equations*

By the end of this chapter, students will:

**NA2.03** relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations;

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $\left(3x^2y + 2xy^2\right) + \left(4x^2y - 6xy^2\right)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

**NA2.05** multiply a polynomial by a monomial involving the same variable [e.g., $2x(x + 4)$, $2x^2(3x^2 - 2x + 1)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil);

**NA2.06** expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

## Chapter Problem

The Chapter Problem is introduced in the Chapter Opener, in which a school logo is designed and manipulated in various ways. Have students discuss their understanding of the topic. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. The questions draw connections between measurement problems and the algebraic skills learned throughout the chapter. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 177. Alternatively, you may wish to assign the Chapter Problem when students have completed the chapter, as part of a summative assessment.

# Chapter 3 Planning Chart

| Section<br>Suggested Timing | Student Text<br>Page (s) | Teacher's Resource<br>Blackline Masters | Assessment | Tools |
|---|---|---|---|---|
| **Chapter 3 Opener**<br>• 15 min | 100–101 | | | |
| **Get Ready**<br>• 80–240 min | 102–103 | • BLM 3.GR.1 Practice: Get Ready<br>• BLM G1 Integer Number Lines | • BLM A6 Knowledge/ Understanding General Scoring Rubric<br>• 3.GR.2 Get Ready Self-Assessment Checklist | |
| **3.1 Build Algebraic Models Using Concrete Materials**<br>• 80 min | 104–109 | • BLM 3.1.1 Practice: Build Algebraic Models Using Concrete Materials<br>• BLM T4 *The Geometer's Sketchpad*® 3<br>• BLM T5 *The Geometer's Sketchpad*® 4 | • BLM A7 Thinking/General Scoring Rubric | **Tools**<br>• algebra tiles<br>• linking cubes<br>**Technology Tools**<br>• *The Geometer's Sketchpad*®<br>• computers |
| **3.2 Work With Exponents**<br>• 80 min | 110–118 | • BLM 3.2.1 Practice: Work With Exponents<br>• BLM G15 Placemat<br>• BLM G10 Grid Paper | • BLM A1 Assessment Recording Sheet: Work With Exponents | **Tools**<br>• calculators<br>• grid paper |
| **3.3 Discover the Exponent Laws**<br>• 160 min | 119–129 | • BLM 3.3.1 Practice: Discover the Exponent Laws<br>• BLM 3.3.3 Student Success: Exponent Law for Multiplication of Powers<br>• BLM 3.3.4 Student Success: Exponent Law for Division of Powers<br>• BLM 3.3.5 Student Success: Exponent Law for Powers of Powers | • BLM A8 Application General Scoring Rubric<br>• BLM 3.3.2 Achievement Check Rubric | **Tools**<br>• calculators |
| **3.4 Communicate With Algebra**<br>• 80 min | 130–139 | • BLM T4 *The Geometer's Sketchpad*® 3<br>• BLM T5 *The Geometer's Sketchpad*® 4<br>• BLM 3.4.1 Practice: Communicate With Algebra | • BLM A5 Problem Solving Checklist<br>• BLM A18 My Progress as a Problem Solver<br>• BLM A9 Communication General Scoring Rubric<br>• BLM A7 Thinking/General Scoring Rubric<br>• BLM 3.4.2 Achievement Check Rubric | **Tools**<br>• algebra tiles<br>**Technology Tools**<br>• *The Geometer's Sketchpad*®<br>• computers<br>• graphing calculators |
| **Use Technology: Virtual Algebra Tiles With *The Geometer's Sketchpad*®**<br>• 40–80 min | 140–143 | • BLM T4 *The Geometer's Sketchpad*® 3<br>• BLM T5 *The Geometer's Sketchpad*® 4 | • BLM A10 Observation General Scoring Rubric | **Technology Tools**<br>• *The Geometer's Sketchpad*®<br>• Algebra Tiles.gsp<br>• computers |
| **3.5 Collect Like Terms**<br>• 80 min | 144–153 | • BLM 3.5.1 Practice: Collect Like Terms | • BLM A8 Application General Scoring Rubric<br>• BLM A9 Communication General Scoring Rubric | **Tools**<br>• algebra tiles |
| **3.6 Add and Subtract Polynomials**<br>• 80 min | 154–159 | • BLM 3.6.1 Practice: Add and Subtract Polynomials<br>• BLM T4 *The Geometer's Sketchpad*® 3<br>• BLM T5 *The Geometer's Sketchpad*® 4 | • BLM A8 Application General Scoring Rubric<br>• BLM A9 Communication General Scoring Rubric | **Tools**<br>• algebra tiles<br>**Technology Tools**<br>• *The Geometer's Sketchpad*®<br>• computers |
| **3.7 The Distributive Property**<br>• 80–160 min | 160–169 | • BLM 3.7.1 Investigate Table<br>• BLM 3.7.2 Practice: The Distributive Property | • BLM 3.7.3 Achievement Check Rubric | **Tools**<br>• algebra tiles |
| **Use Technology: Computer Algebra Systems**<br>• 80 min | 170–173 | • BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator | | **Technology Tools**<br>• TI-89 calculators |

| Section<br>Suggested Timing | Student Text<br>Page (s) | Teacher's Resource<br>Blackline Masters | Assessment | Tools |
|---|---|---|---|---|
| **Chapter 3 Review**<br>• 80 min | 174–175 | • BLM 3.CR.1 Chapter 3 Review<br>• BLM G10 Grid Paper | • BLM A14 Self-Assessment Recording Sheet<br>• BLM A15 Self-Assessment Checklist | **Tools**<br>• grid paper |
| **Chapter 3 Practice Test**<br>• 60–80 min | 176–177 | | • BLM 3.PT.1 Chapter 3 Practice Test<br>• BLM 3.CT.1 Chapter 3 Test | |
| **Chapter 3 Problem Wrap-Up**<br>• 30 min | 177 | • BLM T4 *The Geometer's Sketchpad*® 3<br>• BLM T5 *The Geometer's Sketchpad*® 4<br>• BLM G10 Grid Paper | • BLM 3.CP.1 Chapter 3 Problem Wrap-Up Rubric | **Tools**<br>• algebra tiles<br>• grid paper<br>**Technology Tools**<br>• *The Geometer's Sketchpad*®<br>• computers |
| **Chapters 1 to 3 Review**<br>• 80 min | 178–179 | • BLM G10 Grid Paper | • BLM A14 Self-Assessment Recording Sheet<br>• BLM A15 Self-Assessment Checklist | |
| **Task: Electricity and Gas Costs**<br>• 20 min | 180 | • BLM G10 Grid Paper<br>• BLM T4 *The Geometer's Sketchpad*® 3<br>• BLM T5 *The Geometer's Sketchpad*® 4 | • BLM 3.T1.1 Task: Electricity and Gas Costs Rubric | **Tools**<br>• grid paper<br>**Technology Tools**<br>• graphing calculators<br>• *The Geometer's Sketchpad*®<br>• computers |
| **Task: Perimeters and Areas**<br>• 20 min | 181 | • BLM G10 Grid Paper<br>• BLM G7 Square Dot Paper<br>• BLM G8 Isometric Dot Paper | • BLM 3.T2.1 Task: Perimeters and Areas Rubric | **Tools**<br>• grid paper<br>• square dot paper<br>• isometric dot paper |
| **Task: Mind Reader**<br>• 20 min | 181 | | • BLM 3.T3.1 Task: Mind Reader Rubric | |

# Chapter 3 Blackline Masters Checklist

| | BLM | Title | Purpose |
|---|---|---|---|
| **Get Ready** | | | |
| | **BLM 3.GR.1** | Practice: Get Ready | Practice |
| | **BLM A6** | Knowledge/Understanding General Scoring Rubric | Assessment |
| | **BLM G1**<br>**BLM 3.GR.2 Get Ready Self-Assessment Checklist** | Integer Number Lines | Student Support |
| **3.1: Build Algebraic Models Using Concrete Materials** | | | |
| | **BLM 3.1.1** | Practice: Build Models Using Concrete Materials | Practice |
| | **BLM T4** | *The Geometer's Sketchpad*® 3 | Technology |
| | **BLM T5** | *The Geometer's Sketchpad*® 4 | Technology |
| | **BLM A7** | Thinking General Scoring Rubric | Assessment |
| **3.2: Work With Exponents** | | | |
| | **BLM A1** | Assessment Recording Sheet | Assessment |
| | **BLM 3.2.1** | Practice: Work With Exponents | Practice |
| | **BLM G15** | Grid Paper | Student Success |
| | **BLM G10** | Placemat | Student Support |
| **3.3: Discover the Exponent Laws** | | | |
| | **BLM 3.3.1** | Practice: Discover the Exponent Laws | Practice |
| | **BLM 3.3.2** | Achievement Check Rubric | Assessment |
| | **BLM 3.3.3** | Student Success: Exponent Law for Multiplication of Powers | Student Success |
| | **BLM 3.3.4** | Student Success: Exponent Law for Division of Powers | Student Success |
| | **BLM 3.3.5** | Student Success: Exponent Law for Powers of Powers | Student Success |
| | **BLM A8** | Application General Scoring Rubric | Assessment |

| | BLM | Title | Purpose |
|---|---|---|---|
| **3.4: Communicate with Algebra** | | | |
| | **BLM A5** | Problem Solving Checklist | Assessment |
| | **BLM A18** | My Progress as a Problem Solver | Student Self-Assessment |
| | **BLM 3.4.1** | Practice: Communicate With Algebra | Practice |
| | **BLM A9** | Communication General Scoring Rubric | Assessment |
| | **BLM A7** | Thinking General Scoring Rubric | Assessment |
| | **BLM T4** | *The Geometer's Sketchpad*® 3 | Technology |
| | **BLM T5** | *The Geometer's Sketchpad*® 4 | Technology |
| | **BLM 3.4.2** | Achievement Check Rubric | Assessment |
| **Use Technology: Virtual Algebra Tiles with *The Geometer's Sketchpad*®** | | | |
| | **BLM T4** | *The Geometer's Sketchpad*® 3 | Technology |
| | **BLM T5** | *The Geometer's Sketchpad*® 4 | Technology |
| | **BLM A10** | Observation General Scoring Rubric | Assessment |
| **3.5: Collect Like Terms** | | | |
| | **BLM 3.5.1** | Practice: Collect Like Terms | Practice |
| | **BLM A8** | Application General Scoring Rubric | Assessment |
| | **BLM A9** | Communication General Scoring Rubric | Assessment |
| **3.6: Add and Subtract Polynomials** | | | |
| | **BLM 3.6.1** | Practice: Add and Subtract Polynomials | Practice |
| | **BLM A8** | Application General Scoring Rubric | Assessment |
| | **BLM A9** | Communication General Scoring Rubric | Assessment |
| | **BLM T4** | *The Geometer's Sketchpad*® 3 | Technology |
| | **BLM T5** | *The Geometer's Sketchpad*® 4 | Technology |
| **3.7: The Distributive Property** | | | |
| | **BLM 3.7.1** | Investigate Table | Student Support |
| | **BLM 3.7.2** | Practice: The Distributive Property | Practice |
| | **BLM 3.7.3** | Achievement Check Rubric | Assessment |
| **Use Technology: Computer Algebra Systems** | | | |
| | **BLM T7** | The Computer Algebra System (CAS) on the TI-89 Calculator | Technology |

| | BLM | Title | Purpose |
|---|---|---|---|
| **Chapter 3 Review** | | | |
| | **BLM 3.CR.1** | Chapter 3 Review | Review |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM A14** | Self-Assessment Recording Sheet | Student Self-Assessment |
| | **BLM A15** | Self-Assessment Checklist | Student Self-Assessment |
| **Chapter 3 Practice Test** | | | |
| | **BLM 3.PT.1** | Chapter 3 Practice Test | Diagnostic Assessment |
| | **BLM 3.CT.1** | Chapter 3 Test | Summative Assessment |
| **Chapter 3 Problem Wrap-Up** | | | |
| | **BLM T4** | *The Geometer's Sketchpad*® 3 | Technology |
| | **BLM T5** | *The Geometer's Sketchpad*® 4 | Technology |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM 3.CP.1** | Chapter 3 Problem Wrap-Up Rubric | Assessment |
| **Chapters 1 to 3 Review** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM A14** | Self-Assessment Recording Sheet | Student Self-Assessment |
| | **BLM A15** | Self-Assessment Checklist | Student Self-Assessment |
| **Task: Electricity and Gas Costs** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM T4** | *The Geometer's Sketchpad*® 3 | Technology |
| | **BLM T5** | *The Geometer's Sketchpad*® 4 | Technology |
| | **BLM 3.T1.1** | Task: Electricity and Gas Costs Rubric | Assessment |
| **Task: Perimeters and Areas** | | | |
| | **BLM G10** | Grid Paper | Student Support |
| | **BLM G7** | Square Dot Paper | Student Support |
| | **BLM G8** | Isometric Dot Paper | Student Support |
| | **BLM 3.T2.1** | Task: Perimeters and Areas Rubric | Assessment |
| **Task: Mind Reader** | | | |
| | **BLM 3.T3.1** | Task: Mind Reader Rubric | Assessment |

# Get Ready

**Student Text Pages**
102 to 103

**Suggested Timing**
80–240 min

**Related Resources**
BLM 3.GR.1 Practice: Get Ready Practise

BLM A6 Knowledge/ Understanding General Scoring Rubric

BLM G1 Integer Number Lines

BLM 3.GR.2 Get Ready Self-Assessment Checklist

## Common Errors

- Some students may misapply rules for adding, multiplying, or dividing integers.

**$R_x$** Have students work with manipulatives (e.g., integer tiles) to correct fundamental understanding of operations. Then, follow up with practice exercises in the form of worksheets. You may wish to use **BLM G1 Integer Number Lines**. Use **BLM 3.GR.1 Practice: Get Ready** for remediation.

- Some students may struggle with signs when multiplying rational numbers.

**$R_x$** Consolidate their understanding of multiplication of integers and multiplication of positive fractions first, before mixing the two concepts. Then, have students consider the sign of the result independently of the fraction work when multiplying positive and negative rational numbers.

## Accommodations

**Gifted and Enrichment**—Encourage students to research formulas used in other disciplines, such as science, and to rearrange each formula in as many different ways as they can.

**Perceptual**—Encourage students to used calculators to check their answers for the questions in this section.

**ESL**—Let students work with partners when using a graphing calculator in this section.

## Teaching Suggestions

- Students' needs may vary significantly for these topics from class to class and from student to student.
- A solid grounding in operations, involving integers and rational numbers in particular, is critical for students' success in this chapter.
- Use the Get Ready as a diagnostic assessment, with additional remediation provided as needed. You may wish to use **BLM 3.GR.1 Practice: Get Ready** as remediation or extra practice.
- Refer to the Link to Get Ready in the chapters sections of the Teacher's Resource
- You may wish to use **BLM A6 Knowledge/Understanding General Scoring Rubric** to assist you in assessing your students.
- All **BLMs** referred to throughout this chapter can be found on the *Principles of Mathematics 9* Teacher's Resource CD-ROM.

### Assessment

Assess student readiness to proceed by informal observation as students are working on the questions. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; students can place a checkmark beside topics in the Get Ready in which they feel confident with the necessary skills. You may wish to use **BLM 3.GR.2 Get Ready Self-Assessment Checklist** as a self-assessment for students. Remedial action can be taken in small groups or with a whole class skill review.

# 3.1 Build Algebraic Models Using Concrete Materials

**Strand:**
Number Sense and Algebra

**Student Text Pages**
104 to 109

**Suggested Timing**
80 min

**Tools**
- algebra tiles
- linking cubes

**Technology Tools**
- *The Geometer's Sketchpad®*
- computers

**Related Resources**
BLM 3.1.1 Practice: Build Algebraic Models Using Concrete Materials
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM A7 Thinking General Scoring Rubric

**Mathematical Process Expectations Emphasis**

- ☐ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Operating with Exponents**

**NA1.01** substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and $9.8^3$ by using a calculator]);

**NA1.02** describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by $x$; area, which is two dimensional, can be represented by $(x)(x)$ or $x^2$; volume, which is three dimensional, can be represented by $(x)(x)(x)$; $(x^2)(x)$, or $x^3$];

**Manipulating Expressions and Solving Equations**

**NA2.03** relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

### Warm-Up

**1.** How can you find the area of a square?

**2.** Find the area, including proper units, of a square having a side length of:
**a)** 3 cm **b)** 5 mm

**3.** How can you find the volume of a cube?

**4.** Find the volume, including proper units, of a cube having a side length of:
**a)** 2 m **b)** 4 cm

**Warm-Up Answers**

**1.** Multiply the length by the width, or square one dimension.

**2. a)** $9\ cm^2$ **b)** $25\ cm^2$

**3.** Multiply the height, width, and length, or cube one dimension.

**4. a)** $8\ m^3$ **b)** $64\ cm^3$

## Teaching Suggestions

- Have students complete the Warm-Up, working independently. (5–10 min)
- Have students work in pairs or small groups for the Investigate. (30–40 min)
- To prepare for the Investigate, hand out the algebra tiles, and let students examine them briefly before beginning the Investigate, especially if this is their first encounter with algebra tiles.
- Conduct a class discussion. Some prompting questions to get students familiar with the tiles include:
  - *What shapes do the tiles come in?*
  - *Are there any relationships between the dimensions of different tiles? What are they?*
  - *How many colours do the tiles come in? Why might this be so?*

## Common Errors

- Some students may interpret the $x$-tile (or $x^2$-tile) as having a fixed length of five units.

**$R_x$** Remind students that the $x$-tile represents a variable length. Use virtual tiles in a software program, such as *The Geometer's Sketchpad®*, to demonstrate. Virtual tiles have an advantage in that you can click and drag their lengths to change them (you can not click and drag the length of a virtual unit tile, however). This can provide a good opportunity to discuss the distinction between a variable quantity and a constant quantity. You may wish to use **BLM T4 *The Geometer's Sketchpad®* 3 or BLM T5 *The Geometer's Sketchpad®* 4** if you are using virtual tiles.

## Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Have students work in pairs or small groups on Part A of the investigation. Then, have the class discuss how unit tiles and $x$-tiles can be used to connect visual and algebraic representations of length.
- Note that most tile sets are such that the length of the $x$-tile is equal to the length of five unit tiles. It is important to realize that the $x$-tile represents a *variable* length, and generally is not equal to five. This is simply a limitation of the physical model.
- Have students move on to Part B, and discuss how $x^2$-tiles can be used to relate visual and algebraic representations of area. Clarify that these tiles are intended to represent variable area quantities (not $5 \times 5$ areas). However, the relationship between the length of an $x$-tile and the side length of an $x^2$-tile is important.
- To extend into dimensional relationships of volume, have students continue with Part C, using linking cubes. It is important for students to recognize through these investigations why units of measure depend on the dimensional factors that comprise them, for example,

$$\begin{aligned} \text{length} &= 5 \text{ cm} \\ \text{area} &= 5 \text{ cm} \times 5 \text{ cm} \\ &= 5 \times 5 \text{ cm} \times \text{cm} \\ &= 25 \text{ cm}^2 \\ \text{volume} &= 5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm} \\ &= 5 \times 5 \times 5 \text{ cm} \times \text{cm} \times \text{cm} \\ &= 125 \text{ cm}^3 \end{aligned}$$

- Ensure that students understand the Key Concepts before assigning the Communicate Your Understanding questions. (5–10 min)
- Assign the Practise as independent work. (balance of period)
- Use **BLM 3.1.1 Practice: Build Algebraic Models Using Concrete Materials** if additional practice or remediation is required.

### Investigate Answers (pages 104–106)

**A.**

**1. a)**  **b)** 

**c)**  **d)** 

**2. a)** 

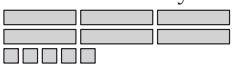
**b)** 

**c)** 

**d)** 

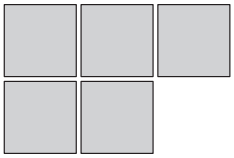
**3. a)** 4 **b)** $x + 1$ **c)** $3x$ **d)** $5x + 3$

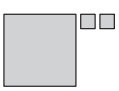
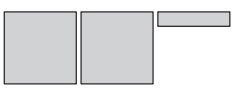
**4.** Answers will vary. $6x + 5$



**B.**

**5. a)**  **b)** 

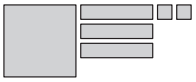
**c)**  **d)** 

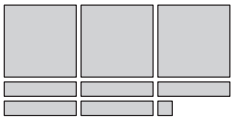
**6. a)**  **b)** 

**Accommodations**

**Gifted and Enrichment**—Challenge students to research famous mathematicians in the library or on the Internet who have researched exponents. Have students present their findings to the class.

**Spatial**—Encourage students to use algebra tiles and linking cubes to give a visual representation of the questions in this section.

**c)** 

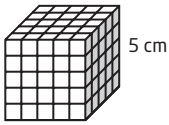
**d)** 

**C.**

**7. a)** 


**b)** 27 $cm^3$

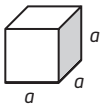
**c)** $3^3$ $cm^3$

**8. a)** 


**b)** 125 $cm^3$

**c)** $5^3$ $cm^3$

**9.** Answers will vary.

**10. a)** 


**b)** $x^2$ **c)** $x^3$

**11.** Answers will vary. For example, linking cubes and algebra tiles can be used to build algebra models. Unit tiles can be used to represent constants, *x*-tiles to represent unknown lengths, and cubes of various sizes can be used to represent volume.

**Communicate Your Understanding Responses (page 107)**

**C1. a)** length: 1 unit, width: 1 unit

**b)** length: *x*, width: 1 unit

**c)** length: *x*, width: *x*

**C2. a)** The width of an *x* tile is same as the length and width of a unit tile.

**b)** The length and width of an $x^2$ tile is same as the length of an *x* tile.

**C3. a)** Answers will vary.

**b)** Answers will vary.

**c)** Answers will vary.

**C4.** Answers will vary.

**a)** By lining up a series of tiles, students may better understand the concept of length.

**b)** By arranging tiles in a pattern and then counting them, students may better understand the concept of area.

**c)** By making a three-dimensional figure using linking cubes and counting them, students may better understand the concept of area.

## Practise

These initial questions provide a good opportunity to lay the groundwork for future discussions of coefficients and variables. For example, to model $x^2 + 3x$, students need to use one $x^2$-tile and three *x*-tiles. This will be helpful later in understanding why the "invisible" coefficient of the squared term is one, and not zero.

## Connect and Apply

For question 5, have some students bring their tile models to the front, and display them on the overhead. Ask the class to identify each expression.

Questions 6 to 8 require students to identify more clearly the relationship between volume and area. It may be counterintuitive to some that you can discuss area as it relates to one or more faces of a three-dimensional object. Cubes, three-dimensional models, and drawn or computer-generated nets are useful tools for demonstrating such relationships. You may wish to use **BLM A7 Thinking General Scoring Rubric** to assist you in assessing your students.

## Extend

Students should be familiar building area models using multiplication from their work in elementary school, for example, modelling $2 \times 3 = 6$ using a rectangular array of tiles. The new skill here is applying variable length measures within the dimensions. This is a useful skill as area models using tiles are applied later in the chapter when investigating the distributive property, and in grade 10 when factoring polynomials.

## Literacy Connections

**Variables**

Explain to students that, in mathematics, we often use the letters $x$, $y$, and $z$ to represent variables, put in place of values that can change or vary. One common use of these variables is as the coordinates of points on a Cartesian plane. For example, the point (2, 3) is a particular point when $x$ is 2 and $y$ is 3. The point $(x, y)$ represents many different points on a Cartesian plane, depending on the values chosen for $x$ and $y$.

**Expressions and Equations**

Explain to students that when we use the word *expression* in English, we are referring to a short part of a sentence. In mathematics, however, an expression refers to a series of numbers and letters joined by addition and/or subtraction—a small part of an equation.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–4, 6 a), b) |
| Typical | 1–8 |
| Extension | 9–14 |

# 3.2 Work With Exponents

**Strand:**
Number Sense and Algebra

**Student Text Pages**
110 to 118

**Suggested Timing**
80 min

**Tools**
- grid paper

**Related Resources**
BLM A1 Assessment Recording Sheet
BLM 3.2.1 Practice: Work With Exponents
BLM G15 Placemat
BLM G10 Grid Paper

**Technology**
- graphing calculator
- scientific calculator

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Operating with Exponents**

**NA1.01** substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and $9.8^3$ by using a calculator]);

**NA1.02** describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by $x$; area, which is two dimensional, can be represented by $(x)(x)$ or $x^2$; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or $x^3$];

**Manipulating Expressions and Solving Equations**

**NA2.03** relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

## Link to Get Ready

Students will multiply integers and rational numbers in this section. Have students complete questions 3, 5, and 6 of the Get Ready before beginning this section.

### Warm-Up

**1.** Evaluate.

**a)** $(-3)(-3)$ **b)** $(-2)(-2)(-2)$

**2.** Write in expanded form, then evaluate.

**a)** $5^2$ **b)** $2^4$ **c)** $3^3$ **d)** $10^5$

**3.** Evaluate.

**a)** $\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$ **b)** $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

**Warm-Up Answers**

**1. a)** 9 **b)** $-8$

**2. a)** $5 \times 5 = 25$ **b)** $2 \times 2 \times 2 \times 2 = 16$

**c)** $3 \times 3 \times 3 = 27$ **d)** $10 \times 10 \times 10 \times 10 \times 10 = 10\,000$

**3. a)** $\frac{9}{16}$ **b)** $\frac{1}{8}$

## Teaching Suggestions

- Assign the Warm-Up as independent work. (5 min)
- The cartoon and the Investigate that follows are designed to illustrate exponential growth and how it can occur. Have students complete the activity in pairs or small groups. (10–15 min)
- Then, discuss other areas in which such growth can occur, such as, population growth, investments, contagious diseases, rumours, and e-mail chain letters.

## Common Errors

- Some students may think that the base is multiplied by the exponent, for example, $2^3 = 6$.

**$R_x$** Students associate powers with multiplication, which is why this error occurs. Have students practise writing out powers in expanded form, for example, $2^3 = 2 \times 2 \times 2$, before evaluating them. Once students have demonstrated clear understanding, allow them to skip this step.

- Some students may improperly interpret and evaluate powers having negative bases.

**$R_x$** Use several comparative examples, such as those in Example 1b) and c) to reinforce proper form for identifying whether a negative is part of the base of a power. Have students practise writing out such questions in expanded form before evaluating so that they can clearly see the number of negative factors involved.

## Ongoing Assessment

The Chapter Problem question 13 can be used as an assessment tool. You may wish to use **BLM A1 Assessment Recording Sheet**.

- Discuss limitations of the model in the cartoon, for example, some friends may hear about the restaurant more than once, some may tell more or less than two friends, not everyone will try the restaurant, etc.
- Discuss the Examples. (10–15 min)
- Example 1 illustrates most of the mechanical skills that students need in order to evaluate powers with rational bases. Remind students to pay special attention when negative bases are present and in the placement of brackets (see parts b, c, and d).
- Example 2 uses a contextual problem to draw connections between powers, algebra, and measurement. Prior knowledge required includes the formulas for finding the volume of a cube and the volume of a cylinder.
- When the exponents are small, encourage students to use mental math skills to evaluate the expression. When the exponents are large or the base contains a decimal number, encourage students to use a calculator.
- Discuss the Communicate Your Understanding questions. (5 min)
- Have students work independently to complete the questions. (balance of period)
- Use **BLM 3.2.1 Practice: Work With Exponents** for extra practice if needed.

### Investigate Answers (page 111)

**1.**

| Day | New Customers | Expanded Form | Power |
|---|---|---|---|
| 1 | 2 | 2 | $2^1$ |
| 2 | 4 | $2 \times 2$ | $2^2$ |
| 3 | 8 | $2 \times 2 \times 2$ | $2^3$ |
| 4 | 16 | $2 \times 2 \times 2 \times 2$ | $2^4$ |

**2.** $2^7 = 128$. Each day, the number of new customers increases by a factor of 2. Therefore, based upon this model, on day 7, Barney should expect $2^7$ new customers.

**3.** $2^{14} = 16\,384$. Using the model, Barney should expect 16 384 new customers on day 14. This is not realistic. The model has limitations; the same pattern cannot continue indefinitely because there is no guarantee that each customer that visits Barney's will also bring two new customers the next day, each of whom in turn will bring another two customers the day after, and so on. The pattern will slow down.

**4.** Answers will vary. Possible answer: In a school of 500 students, all students will know about Barney's in 9 days. $2^9 = 512$

**5. a)** $3^2$, or 9 **b)** $3^4$, or 81
**c)** For the school with 500 students, all students will know in 6 days. $3^6 = 729$

**6.** Answers will vary. Exponents can be used to show some growth patterns, when the pattern is changing by a multiple of the same factor.

### Communicate Your Understanding Responses (page 113)

**C1. a)** base: 3, exponent: 4
**b)** base: $\frac{1}{2}$, exponent: 4
**c)** base: $-2$, exponent: 6
**d)** base: 2, exponent: 6
**e)** base: 1.2, exponent: 2

**C2. a)** 81, $\frac{1}{16}$, 64, $-64$, 1.44
**b)** The answers in part c) and d) are different due to the presence of brackets, which changes the order in which the operations are performed.

**C3.** The next step is to square the value of $r$.

**C4.** Answers may vary. I would use a calculator to evaluate the expressions in parts c) and e) because they involve higher powers and, in part c) a decimal base.

## Accommodations

**Visual**—Provide students with photocopies of the charts to be completed in this section instead of copying the information in their notebooks.

**Perceptual**—Allow students to use patterning to understand the exponents of negative numbers.

**Language**—Encourage students to use their dictionaries and research the new words in this section on the Internet.

**ESL**—Encourage students to use their dictionaries or translators to understand the meanings of the new words in this section.

## Student Success

Working in groups of four, have students complete a placemat activity. Instruct each student to list examples of powers or exponents used in formulas or real-life situations in their area of the placemat. Then, have each group place a summary list of examples in the centre of the placemat. You may wish to use **BLM G15 Placemat** to support this activity.

## Practise

Use questions 3 to 5 to emphasize proper form when writing powers. In particular, negative bases should be surrounded by brackets in both power form and expanded form. A single negative sign in front of a power (see question 5c), for example), means that the base of the power is positive, and that the entire expression is negative. Use of a calculator is recommended for evaluating powers involving decimals (see question 3b), for example).

Review BEDMAS for question 6.

For question 7, remind students to use brackets when substituting into expressions.

## Connect and Apply

Questions 9 to 10 are good examples of real non-linear relations. Discuss the implications of such growth patterns as they compare to linear growth.

Students interested in music may appreciate the context in question 11.

For question 14, note to students that exponential decay is the counterpart to exponential growth. Have students explore this type of curve on a graphing calculator and see what happens as the amount approaches (but never reaches) zero. This type of thinking will be expanded upon in future mathematics courses when students explore asymptotes and limits. You may wish to use **BLM G10 Grid Paper** to support questions 9 and 14.

## Extend

Question 15 mentions **scientific notation**, a concept only explicitly mentioned in the science curriculum. However, students may benefit from some exposure to it in this section to consolidate the skills and appreciate how the two disciplines of study are related. Special attention to entering and interpreting numbers in scientific notation using a scientific calculator can be quite helpful, particularly if a surprising result appears in this form. Students should have some basic awareness of how to work with such values.

## Literacy Connections

**Literal and Numerical Coefficients**

Explain to students that a *literal coefficient* is a letter, and a *numerical coefficient* is a number. Remind students that the terms used in mathematics make a lot of sense when they examine what they actually mean and where the words come from. Use this discussion to lead into a discussion on prefixes.

**Prefixes**

Explain that many of the prefixes used in mathematics also make a lot of sense. For example, a *monomial* has only one term. Ask students to think of other examples in real life where the *mono-* prefix is used. (For example, monocle, monologue, monolingual, monochromatic, monochrome, monogamy, monolith, etc.) Tell students that binomials and trinomials are special polynomials and that they are used so often that they are referred to by their specific names. The word *polynomial* refers to the whole group of expressions with two or more terms. Ask students to think of other examples where we use the prefix *bi* and *tri*. (For example, bicycle, bicarbonate, biceps, bicultural, bicuspid, etc., and tricycle, triangle, triad, triangulate, trilateral, triceps, etc.)

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–5, 6a), c), e), 7a), b), c), 8 |
| Typical | 1–12, 14 |
| Extension | 15–20 |

# 3.3 Discover the Exponent Laws

**Strand:**
Number Sense and Algebra

**Student Text Pages**
119 to 129

**Suggested Timing**
160 min

**Tools**
- calculators

**Related Resources**
BLM 3.3.1 Practice: Discover the Exponent Laws
BLM A8 Application General Scoring Rubric
BLM 3.3.2 Achievement Check Rubric
BLM 3.3.3 Student Success: Exponent Law for Multiplication of Powers
BLM 3.3.4 Student Success: Exponent Law for Division of Powers
BLM 3.3.5 Student Success: Exponent Law for Powers of Powers

**Mathematical Process Expectations Emphasis**

- ☐ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Operating with Exponents**

**NA1.03** derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;

**Manipulating Expressions and Solving Equations**

**NA1.04** extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.

## Link to Get Ready

Most of the concepts in the Get Ready section are needed here. Ensure that students have completed all of the Get Ready questions prior to working on this section.

### Warm-Up

**1.** Identify the base and exponent of each power.
**a)** $5^3$ **b)** $10^5$ **c)** $(-2)^4$ **d)** $(-1)^7$

**2.** Write each power in question 1 in expanded form.

**3.** Evaluate each power in question 1.

**Warm-Up Answers**

**1. a)** base: 5, exponent: 3
**b)** base: 10, exponent: 5
**c)** base: −2, exponent: 4
**d)** base: −1, exponent: 7

**2. a)** $5 \times 5 \times 5$
**b)** $10 \times 10 \times 10 \times 10 \times 10$
**c)** $-2 \times -2 \times -2 \times -2$
**d)** $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1$

**3. a)** 125 **b)** 100 000 **c)** 16 **d)** −1

## Teaching Suggestions

- Consider teaching this lesson over two periods.

**Day 1: Product and Quotient Rule**

- Assign the Warm-Up. (5 min)
- Have students complete Investigate A Parts A, B, and C in pairs or small groups. (30–35 min)
- Follow Examples 1 and 2 with a discussion on the product and quotient rules. (10–15 min)
- Assign question C1 of Communicate Your Understanding. Support this activity with a class discussion. (5 min)
- Assign questions 1 to 4 and 11 to 13 as independent work. (balance of period)

## Common Errors

- Some students may change the base when applying exponent laws, for example, $2^3 \times 2^4 = 4^7$.

**Rx** Use expanded form (as in the Investigates) to illustrate to students why the base does not change. Have students verify with a calculator or by hand that the two expressions above are not equal, but that $2^3 \times 2^4 = 2^7$.

- Some students may confuse the product rule and the power of a power rule, for example, $3^2 \times 3^4 = 3^8$ and $(4^2)^3 = 4^5$.

**Rx** Use expanded form (as in the Investigates) to justify and reinforce the proper exponent laws to apply for each type of situation. Have students verify equality/non-equality of results with a calculator or by hand by evaluating initial and final expressions.

## Ongoing Assessment

- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 3.3.2 Achievement Check Rubric** to assist you in assessing your students.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills (see **BLM 3.3.2 Achievement Check Rubric** for levels.)

- Depending on how the class is progressing on the first day, you may wish to move Quotient of Powers to the second day.

**Day 2: Power of a Power and Simplifying Algebraic Expressions**

- Have students work on Investigate B with partners or in small groups. (5–10 min)
- Work through the remaining examples as a class. (15–20 min)
- Assign Communicate Your Understanding questions C2 and C3, and follow up with a class discussion. (5 min)
- Assign questions 6 to 13 as independent work. (balance of period)
- Use **BLM 3.3.1 Practice: Discover the Exponent Laws** if extra practice is required.

### Investigate Answers (page 119)

**A.**

**Part A**

**1.** 1 km = 1000 m; $10^3$ m

**2.**

| Unit | Number of these in 1 m | Power of 10 |
|---|---|---|
| decimetre | 10 | $10^1$ |
| centimetre | 100 | $10^2$ |
| millimetre | 1000 | $10^3$ |

**3. a)** 100 000 cm in 1 km **b)** $10^2 \times 10^3 = 10^5$

**4. a)** 1 000 000 mm = 1 km **b)** $10^1 \times 10^2 \times 10^3 = 10^6$

**5.** Answers will vary. The exponents increase by one.

**Part B**

**6.**

| Product | Expanded Form | Single Power |
|---|---|---|
| $3^2 \times 3^4$ | $(3 \times 3) \times (3 \times 3 \times 3 \times 3)$ | $3^6$ |
| $4^3 \times 4^3$ | $(4 \times 4 \times 4) \times (4 \times 4 \times 4)$ | $4^6$ |
| $6^4 \times 6^1$ | $(6 \times 6 \times 6 \times 6) \times (6)$ | $6^5$ |
| $2^4 \times 2^2 \times 2^3$ | $(2 \times 2 \times 2 \times 2) \times (2 \times 2) \times (2 \times 2 \times 2)$ | $2^9$ |
| $k^3 \times k^5$ | $(k \times k \times k) \times (k \times k \times k \times k \times k)$ | $k^8$ |

**7.** The bases of the powers in each product are the same.

**8.** The sum of the exponents in the first column is equal to the exponent in the last column.

**9.** You can write a product of powers with the same base by showing a sum in the exponent.

**10.** $x^a \times x^b = x^{a+b}$

**Part C**

**11.**

| Quotient | Expanded Form | Single Power |
|---|---|---|
| $5^5 \div 5^3$ | $\frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5}}$ | $5^2$ |
| $7^4 \div 7^1$ | $\frac{\cancel{7} \times 7 \times 7 \times 7}{\cancel{7}}$ | $7^3$ |
| $10^6 \div 10^4$ | $\frac{10 \times 10 \times \cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10}}$ | $10^2$ |
| $2^7 \div 2^6$ | $\frac{2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}$ | $2^1$ |
| $p^8 \div p^5$ | $\frac{p \times p \times p \times \cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p}}{\cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p}}$ | $p^3$ |
| $A^4 \div A^2$ | $\frac{\cancel{A} \times \cancel{A} \times A \times A}{\cancel{A} \times \cancel{A}}$ | $A^2$ |

## Accommodations

**Gifted and Enrichment**—Challenge students to investigate patterns in an exponent table and the Binomial Expansion (Pascal's Triangle).

**Memory**—Encourage students to use cue cards to memorize the exponent laws.

## Student Success

Have students start off in "home triads" (groups of 3). Assign an exponent law to each home triad, and have them work on the Investigates in these triads. Then, split up the home triads and have one expert for each exponent law per new group. Have the new groups work on the Examples. Finally, have students return to their home triads, and teach the new exponent law they learned to the others in the triad. You may wish to use **BLM 3.3.3 Student Success: Exponent Law for Multiplication of Powers, BLM 3.3.4 Student Success: Exponent Law for Division of Powers, and BLM 3.3.5 Student Success: Exponent Law for Powers of Powers** to support this activity.

**12.** The bases of the powers in each quotient are the same.

**13.** If you subtract the exponents in the first column you get the exponent of the single power in the last column.

**14.** You can write a quotient of two powers with the same base using one power by showing a subtraction symbol between the exponents. For example, $4^7 \div 4^5 = 4^{7-5} = 4^2$.

**15.** $x^a \div x^b = x^{a-b}$

**B.**

**1.**

| Power of a Power | Expanded Form | Single Power |
|---|---|---|
| $(2^2)^3$ | $(2^2) \times (2^2) \times (2^2)$<br>$= (2 \times 2) \times (2 \times 2) \times (2 \times 2)$ | $2^6$ |
| $(5^3)^4$ | $(5^3) \times (5^3) \times (5^3) \times (5^3)$<br>$= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times$<br>$(5 \times 5 \times 5)$ | $5^{12}$ |
| $(10^4)^2$ | $(10^4) \times (10^4)$<br>$= (10 \times 10 \times 10 \times 10) \times (10 \times 10 \times 10 \times 10)$ | $10^8$ |
| $(3^3)^3$ | $(3^3) \times (3^3) \times (3^3)$<br>$= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \times (3 \times 3 \times 3)$ | $3^9$ |

**2.** The exponent in the last column is equal to the product of the exponents in the first column.

**3.** You can write the power of a power as a single power by keeping the same base with exponent equal to the product of the two exponents. For example,
$(4^3)^2 = 4^{3 \times 2}$
$= 4^6$

**4.** $(x^a)^b = x^{ab}$

### Communicate Your Understanding Responses (page 126)

**C1. a)** quotient rule **b)** power rule
**c)** no law applies, the bases are different **d)** product rule
**e)** quotient rule **f)** no law applies, the bases are different

**C2.** Answers will vary. For example:
**a)** $y^5 \times y^4 = y^9$ **b)** $(y^5)^4 = y^{20}$ **c)** $\frac{y^5}{y^2} = y^3$

**C3. a)** Substitute into the simplified expression because it is easier to solve. It requires less arithmetic.
**b)** 6

## Practise

When applying exponent laws, it is important for students to realize that:
- the base of each power must be identical
- the base does not change

Watch for these key things as students begin to apply exponent laws in the early practise questions. Ensure that students put brackets around bases containing negative signs and/or fractions. Omitting the brackets changes the meaning of the expression, for example,

$$\left(\frac{2}{5}\right)^6 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$
$$= \frac{64}{15625}$$
$$\text{but } \frac{2}{5}^6 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{5}$$
$$= \frac{64}{5}$$

$$
\begin{aligned}
(-3)^6 &= (-3)(-3)(-3)(-3)(-3)(-3) \\
&= 729 \\
\text{but } -3^6 &= -3 \times 3 \times 3 \times 3 \times 3 \times 3 \\
&= -729
\end{aligned}
$$

Students may be confused by the use of nested brackets (for example, question 6b), c), and d)). Discuss the need for these: the round brackets are used to simply identify the base of the power, and the square brackets denote a power of that power.

## Connect and Apply

Questions 10 and 14 are useful in illustrating how application of exponent laws can reduce the complexity level of an expression, and the calculations involved when substituting. Question 11 draws connections between measurement and application of exponent laws. Students will need to recall the formula for the volume of a prism. You may wish to use **BLM A8 Application General Scoring Rubric** for questions 11 to assist you in assessing your students.

Questions 12 and 13 draw connections between probability and exponents. Have students draw a tree diagram to illustrate why the probability of flipping heads three times in a row is $\left(\frac{1}{2}\right)^3$.

**Achievement Check Answers (page 128)**

**14. a)** Substitute $m = 4$, $n = -3$, which gives $\frac{(-3)(16)(-27)(64)(9)}{(4)(16)(81)(3)(4)(-3)} = \frac{-16 \times 9}{9}$
$= -16$

**b)** $\frac{-3m^2n^3 \times 4m^3n^2}{(2mn^2)^2 \times 3mn} = \frac{(-12)(m^5)(n^5)}{4(m^2)(n^4)(3)(m)(n)}$
$= \frac{-12(m^5)(n^5)}{12(m^3)(n^5)}$
$= -m^2$

**c)** Substitute $m = 4$, value is $(-4 \times 4) = -16$

**d)** Method b) is the better solution as it involves substituting in a simpler expression.

**e)** Josie forgot to add the exponent 3 on the $m$ in the numerator of the expression and she forgot to add the exponent 2 on the $m$ inside brackets in the denominator. Since Josie's expression reduces to the same value as the original, i.e., $-m^2$, the solutions will be the same.

## Extend

For questions 15 to 18, scientific notation is explicitly mentioned in the science curriculum only, however some exposure to application of exponent laws to numbers in scientific notation may be beneficial. Assign these questions only if students had a chance to do question 15 on introduction to scientific notation in Section 3.2.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–7, 8, 10, 11 |
| Typical | 1, 14 |
| Extension | 15–20 |

# 3.4 Communicate With Algebra

**Strand:**
Number Sense and Algebra

**Student Text Pages**
130 to 139

**Suggested Timing**
80 min

**Tools**
- algebra tiles

**Technology Tools**
- *The Geometer's Sketchpad®*
- computers
- graphing calculators

**Related Resources**
BLM A5 Problem Solving Checklist
BLM A18 My Progress as a Problem Solver
BLM 3.4.1 Practice: Communicate With Algebra
BLM A9 Communication General Scoring Rubric
BLM A7 Thinking General Scoring Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 3.4.2 Achievement Check Rubric
BLM 3.4.3 Student Success: Crossword

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

### Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of the Get Ready questions prior to working on this section.

## Teaching Suggestions

- Example 1 highlights the distinction between coefficients and variables, using contextual references. This understanding is critical as it prepares students for algebraic operations, such as collecting like terms. The specific examples have been carefully selected to address some of the common misunderstandings.
- Use the comments column to discuss these issues with the class, and provide additional examples to illustrate, as needed.
- The focus of Examples 2, 3, and 4 is on classifying terms and polynomials. Understanding the degree of a polynomial is important when working with linear and non-linear relations, which students will do later in this course and in future mathematics courses.
- Example 5 applies basic algebraic concepts to model and solve a contextual problem. A general algebraic expression is set up, into which values are substituted in order to solve for a particular case. Point out to students that that the algebraic expression is useful in a number of particular situations. Explain that these expressions are sometimes referred to as "formulas." You may wish to use **BLM A5 Problem Solving Checklist** to help you in assessing your students. Alternatively, have students use **BLM A18 My Progress as a Problem Solver** as a self-assessment tool.
- You may wish to use **BLM 3.4.1 Practice: Communicate With Algebra** for extra practice or remediation.

**Communicate Your Understanding Responses (page 134)**

**C1. a)** Answers will vary. A sample answer: $5a$; $8t$
**b)** Answers will vary. A sample answer: $13d + 7$; $x - 5$
**c)** Answers will vary. A sample answer: $7s^2 + 3s + 2$; $x^2 + 3x - 1$
**d)** Answers will vary. A sample answer: $6x^3 + 3x^2 + 8x + 1$; $a^3 + 2a^2 - a^2 + 3$

**C2.** No, the variable is $x^2$, the coefficient is 1. The degree of the polynomial is 2.

**C3. a)** Yes, the expressions are the same. The coefficient 1 does not have to be written if it goes with a variable.
**b)** No, the expressions are not equivalent. The second expression has a constant of 1 added to the term $3x$.
**c)** You must write the coefficient 1 when it is a constant but not when it is part of a term.

## Common Errors

- Some students may misidentify a missing coefficient as zero, instead of 1.

**$R_x$** Use algebra tiles to model expressions such as $x$. Remind students to think: there is one tile, so the coefficient is 1.

- Some students may misidentify a missing exponent on a variable as zero, instead of 1.

**$R_x$** Use a pattern of powers in expanded form to illustrate that the exponent of $x$ is 1:
$x^3 = x \times x \times x$
$x^2 = x \times x$
$x^1 = x$

- Some students may confuse monomial, binomial, trinomial, etc.

**$R_x$** Have students look for addition and subtraction operators, which separate terms. Refer to the margin tip in Example 2.

- Some students may struggle in setting up algebraic expressions to model given information.

**$R_x$** Have students consider whether a part of an expression is fixed or can vary. This will suggest whether or not a variable is required. Looking for key words such as total, less, etc., can suggest whether terms should be added or subtracted, etc. Have students work in pairs on some of the Connect and Apply questions. Consider providing remedial work on writing algebraic expressions. Use **BLM 3.4.1 Practice: Communicate With Algebra.**

## Ongoing Assessment

- Use Achievement Check question 17 to monitor student success. See Achievement Check Answers and **BLM 3.4.2 Achievement Check Rubric**.
- Chapter Problem question 16 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills (see **BLM 3.4.2 Achievement Check Rubric** for levels).

## Practise

Caution students about the presence of "invisible 1s," for example:
- $x^2$ has a coefficient of 1, not zero
- $3y$ has degree 1, not zero
- however, $-4$ has degree zero, not 1.

## Connect and Apply

For question 9, remind students to use brackets when substituting into algebraic expressions to ensure that proper operations are applied.

For question 10, remind students how to convert a percent to a decimal.

Make algebra tiles available for question 14. Some students may benefit from starting with the tiles and developing the expression from them.

Questions 10–13, 16, and 17 require students to set up algebraic expressions based on given information in prose form. Many students struggle with this skill. Consider having students work with a partner on some of these questions, and/or discuss one or two questions as a class to help build confidence.

For question 15, you may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

For question 17, you may wish to use **BLM A7 Thinking/General Scoring Rubric** to assist you in assessing your students.

**Achievement Check Answers (page 137)**

**17. a)** $3w + 2l + t$

**b)** $w$ represents the number of wins; $l$ represents the number of losses; and $t$ represents the number of ties

**c)** $3(5) + 2(2) + 3 = 22$

**d)** The Falcons could have had 6 wins, 0 losses, and 4 ties. $3(6) + 2(0) + 4 = 22$

## Extend

For question 18, the trinomial will involve fractions, and so may look intimidating. Use the swim time as an example for generating the rest of the expression. After substituting values in part d), students need to divide each distance by speed to find the times for each leg of the race, before adding. Have students use *The Geometer's Sketchpad*® for question 20, which is a particularly useful tool when examining part c). Use **BLM T4 T*he Geometer's Sketchpad*® 3 or BLM T5 *The Geometer's Sketchpad*® 4** for this activity. For enrichment, capable students can generate and examine the functional relationship between distance walked and total travel time, which can be easily produced using *The Geometer's Sketchpad*®. A graphing calculator can also be used, but lacks click-and-drag capabilities. Optimization (in this case finding the minimum travel time) is an important skill that will continue to be developed right into university level calculus courses.

Question 21 requires the use of multi-variable (for example, $x$ and $y$) tiles. Alternatively, students can adjust the representation of their existing tile set (for example, let the $x$-tile represent the value of the plane, and let the $x^2$-tile represent the number of seats). This idea of creatively bending an existing model to address a particular problem should be encouraged and extended. For example, ask students, *What tools are available that you can use to build a model to suit your purpose?*

## Accommodations

**Language**—Encourage students to work together when working through the questions where algebraic expressions are used to communicate mathematical ideas.

**ESL**—Allow students to use their dictionaries or translators to understand the new words in this section.

## Student Success

Use **BLM 3.4.3 Student Success: Crossword** to help students with the vocabulary in this chapter.

## Literacy Connections

There is a heavy emphasis on mathematical literacy in this section. There are a number of effective strategies that can be used to help students achieve clearer understanding of the various terms and concepts, such as Frayer models and word walls. For more information on these and other good strategies, refer to the Ontario Association for Mathematics Education's *Think Literacy: Mathematics Approaches Grades 7–12*, available for order or download at: http://oame.on.ca/main/index1.php?lang=en&code=ThinkLit.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–12 |
| Typical | 1-4, 5a), c), e), 6a), c), e), 7-16 |
| Extension | 18–24 |

# Use Technology

# Virtual Algebra Tiles With *The Geometer's Sketchpad®*

**Strand:**
Number Sense and Algebra

**Student Text Pages**
140 to 143

**Suggested Timing**
40–80 min

**Technology Tools**
- *The Geometer's Sketchpad®*
- computers
- Algebra Tiles.gsp

**Related Resources**
BLM T4 *The Geometer's Sketchpad®* 3

BLM T5 *The Geometer's Sketchpad®* 4

BLM A10 Observation Assessment General Scoring Rubric

**Accommodations**

**Motor**—Give students extra time to work through this section. Encourage students to practise using *The Geometer's Sketchpad®* on their home computers.

**ESL**—Encourage students to work in pairs to understand the instructions to use *The Geometer's Sketchpad®*.

## Specific Expectations

**Operating With Exponents**

**NA1.02** describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by $x$; area, which is two dimensional, can be represented by $(x)(x)$ or $x^2$; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or $x^3$];

**Manipulating Expressions and Solving Equations**

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

### Warm-Up

Review and/or make available physical algebra tiles (unit tiles, $x$-tiles, and $x^2$-tiles) for comparison with virtual tiles.

## Teaching Suggestions

- Virtual algebra tiles work in basically the same way as physical tiles, except you can actually vary the length of the variable tiles. Conduct a class discussion on the relative advantages and disadvantages of the various models.
- Allow students the opportunity to see as many different representations of algebraic concepts as possible, for example, physical tiles, virtual tiles, symbolic representation, Computer Algebra System representation. Algebra is one of the most important, and yet most abstract, branches of secondary mathematics. By exposing students with diverse learning styles to a multitude of representations, there is a better chance that they will understand key concepts.
- In questions 7 and 8, the rectangles that students are to build are useful conceptually later in the chapter, and in future mathematics courses, when students explore the distributive property and polynomial factorization.
- You may wish to use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** for this activity.
- You may wish to use **BLM A10 Observation Assessment General Scoring Rubric** to assist you in assessing your students.

# 3.5 Collect Like Terms

**Strand:**
Number Sense and Algebra

**Student Text Pages**
144 to 153

**Suggested Timing**
80 min

**Tools**
- algebra tiles

**Related Resources**
BLM 3.5.1 Practice: Collect Like Terms
BLM A8 Application General Scoring Rubric
BLM A9 Communication General Scoring Rubric

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

### Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of the Get Ready questions prior to working on this section.

### Warm-Up

Distribute algebra tiles, and have students create the following models with them:

**a)** 3 **b)** $5x$ **c)** $2x^2$
**d)** $3x + 2$ **e)** $4x^2 + x$ **f)** $x^2 + 5x + 6$

**Warm-Up Answers**

**a)** 3 unit tiles
**b)** 5 $x$-tiles
**c)** 2 $x^2$-tiles
**d)** 3 $x$-tiles and 2 unit squares
**e)** 4 $x^2$-tiles and 1 $x$-tile
**f)** 1 $x^2$-tile, 5 $x$-tiles, and 6 unit squares

## Teaching Suggestions

- The focus of this section is on identifying and collecting like terms. Have students use algebra tiles to help them in identifying likeness. Algebra tiles illustrate likeness properties effectively, for example, $x$-tiles look distinctly different from unit tiles.
- Example 1 shows how tiles can be used to model a problem that requires collecting two sets of like terms, before the algebraic terminology is formally introduced. Unit tiles represent fixed costs and $x$-tiles represent variable costs. The flexibility of the model is showcased as each $x$-tile actually represents $100. Draw connections to future work in Chapter 5, Section 5.2 Partial Variation.
- Example 2 focuses on identifying like terms. Use tiles in cases involving constants, $x$-terms and $x^2$-terms. Then, extend the concept into cases involving alternative and/or more complex variables.
- In Example 3, algebra tiles are useful in showing how to combine groups of like objects in order to add like terms. The tiles simply serve as counters representing different types of objects.

## Common Errors

- Some students may misidentify terms with the same variables but different exponents as like terms.

**R$_x$** Have students write out each variable part to see why the terms are unlike. Use tiles as an example to distinguish between $x$ and $x^2$.

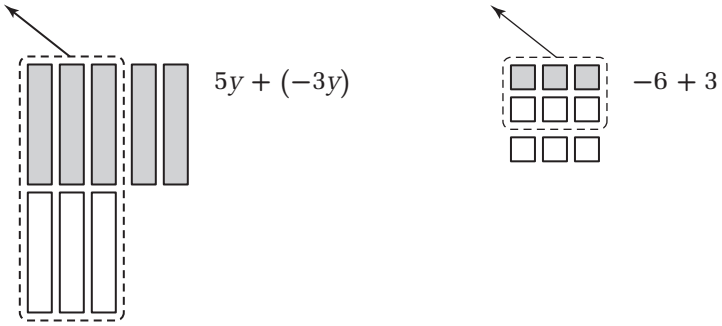
- Some students may change the variable part when collecting like terms, for example, $2x + 3x = 5x^2$.

**R$_x$** Use tiles to illustrate that combining two groups of the same type of tile does not change the type of tile, only the number of them. Use substitution of values into both expressions above to verify that they are not generally equal (do not use $x = 0$).

## Ongoing Assessment

- Chapter Problem question 14 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Example 4 has students subtract like terms using algebra tiles. The Take Away method is intuitively appealing when subtracting a lesser quantity, however, more sophisticated techniques are required when subtracting more than you start with. Students need a sound grasp of alternative representations of zero, using zero pairs (one or more pairs consisting of positive and negative unit or $x$-tiles).




- Students should be familiar with the concept of zero pairs from their work with integer tiles in elementary school, however depending on prior experiences with your students, you may wish to invest some time in consolidating understanding of this fundamental concept. You may wish to use **BLM 3.5.1 Practice: Collect Like Terms**.
- Example 5 uses a mixture of adding and subtracting techniques with manipulatives to simplify polynomial expressions, including those involving decimal coefficients. The intent is to consolidate understanding of these techniques in situations of increasing complexity, and to begin to prepare students for approaches requiring strictly symbolic reasoning.
- Example 6 moves students from concrete to abstract reasoning. By this point, students should realize that to collect like terms, they must add and/or subtract their coefficients. Various methods for identifying, reorganizing, and collecting like terms symbolically are illustrated here. A strong foundation in the addition and subtraction of integers is vital for student success here and in later algebraic work. You may wish to use **BLM 3.5.1 Practice: Collect Like Terms** as a remediation tool or for extra practice.
- Although algebra tiles are quite useful in developing initial conceptual understanding, the ultimate aim for the academic student is to be able to collect like terms efficiently without the use of manipulatives. However, it is important that the manipulatives not be treated or viewed as a crutch. Algebra tiles should be presented as one of many tools, neither inferior nor juvenile.

### Communicate Your Understanding Responses (page 150)

**C1. a)** Like terms have exactly the same variable or variables. Unlike terms have different variables or different powers of the same variable.

**b)** Answers will vary. Like terms: $4g$ and $7g$, $x^2$ and $5x^2$; Unlike terms: $5x$ and $3y$, $8r^2$ and $3r$.

**C2. a)** They forgot to add one of the coefficients, the one $x$. The right side should be $6x$.

**b)** The sum of the coefficients should be the coefficient in front of the variable in the sum not an exponent. The right side should be $3y$.

**c)** The negative sign was omitted in the answer. The right side should be $-3m$.

**d)** $-2x - 2x = -4x$ not 0.

**e)** $x + x = 2x$; $x \times x$ would give $x^2$.

**f)** $3ab - 2b$ cannot be simplified as the terms are not like terms.

**Accommodations**

**Visual**—Encourage students to use different colours when adding like terms. For example: $4x + 3y + 3x + 5y = 7x + 8y$.

**Perceptual**—Provide students with algebra tiles to use when working through the questions.

**Spatial**—Allow students to use algebraic number lines when adding and subtracting like terms.

**Memory**—Remind students to use concrete examples, such as money, when adding like terms. For example $1 + $2 = $3.

## Practise

Some of the early Practise questions require students to recognize that exponents must be identical on variables of like terms. As students conduct independent work, watch for this, since this is an important concept that is often misunderstood. Consider taking up the first couple of questions after students have had a few minutes to work on them. This should identify and correct misconceptions early on.

## Connect and Apply

Provide students with algebra tiles. These may be particularly helpful when students build their algebraic models/expressions. Students might struggle in setting up algebraic models and expressions. Have students work with a partner for some of the questions.

For question 10, you may wish to use **BLM A8 Application General Scoring Rubric** to assist you in assessing your students.

You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students in question 11.

## Extend

Connections to measurement, geometry, and the Pythagorean theorem are required for questions 15 and 16. Students will need to recall properties of equilateral, isosceles, and right triangles. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students for questions 15 and 16.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1–6, 7a), c), e), 8a), c), e), 9a), 10 |
| Typical | 1–13 |
| Extension | 15–18 |

# 3.6 Add and Subtract Polynomials

**Strand:**
Number Sense and Algebra

**Student Text Pages**
154 to 159

**Suggested Timing**
80 min

**Tools**
- algebra tiles

**Technology Tools**
- *The Geometer's Sketchpad®*
- computers

**Related Resources**
BLM 3.6.1 Practice: Add and Subtract Polynomials
BLM A8 Application General Scoring Rubric
BLM A9 Communication General Scoring Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

### Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of the Get Ready questions prior to working on this section.

### Warm-Up

Review collecting like terms.

**1. a)** What are like terms? **b)** Give an example of two like terms.

**2.** State two like terms for each term given.
**a)** $3x$ **b)** $-2y$ **c)** $7x^2$ **d)** $2ab^2$

**3.** Simplify, by collecting like terms.
**a)** $2x + 5x$ **b)** $7y - y$ **c)** $4x^2 - 6x^2$
**d)** $3m + 2 + 4m - 3$

**Warm-Up Answers**

**1. a)** Like terms have the same variable or variables.
**b)** $3x$ and $5x$, $4y^2$ and $7y^2$

**2. a)** $5x$, $-8x$ **b)** $-5y$, $11y$ **c)** $15x^2$, $4x^2$ **d)** $-9ab^2$, $ab^2$

**3. a)** $7x$ **b)** $6y$ **c)** $-2x$ **d)** $7m - 1$

## Teaching Suggestions

- Use the introduction to generate a brief class discussion about how different people are paid. Let the students come up with the examples, however, here are some prompting examples, if needed:
  - professional athletes
  - car salespersons
  - recording musicians
- Most of these people are paid some sort of partial commission that depends on job performance. Example 4 returns to this context.
- Example 1 deals with addition of polynomials. Students who are still struggling with symbolic methods of collecting like terms may benefit from working further with algebra tiles at this point. The desired approach here is to remove brackets and collect like terms.

## Common Errors

- Some students may only reverse the sign of the first term when subtracting polynomials, for example,
  $(5x + 4) - (3x - 2)$
  $= 5x + 4 - 3x - 2$.

**$R_x$** Use algebra tiles to illustrate that the opposite polynomial must contain the opposite of all terms. Another approach to overcoming this error is to think of distributing $-1$, for example,
$(5x + 4) - (3x - 2)$
$= (5x + 4) - 1(3x - 2)$

- Some students may mix up integer signs when simplifying double signs, for example,
  $(3y + 4) - (-2y + 7)$
  $= 3y + 4 - 2y - 7$.

**$R_x$** Have students rewrite the subtraction statement as an addition statement, using the opposite polynomial, before attempting to simplify integer signs.

## Ongoing Assessment

Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

## Accommodations

**Perceptual**—Encourage students to colour code the like terms in polynomial expressions, for example, $3x + 5y + 2x + 4y = 5x + 9y$.

- In part c), point out to students to simplify integer signs. Illustrating both methods, collecting like terms and vertical alignment, may help students to better visualize the operations involved. A strong command of integer operations is clearly required by this point.
- Example 2 provides a segue into polynomial subtraction. The focus of this example is on identifying and writing opposite polynomials. Algebra tiles are useful in providing a visual illustration. Symbolically, students should realize that to form an opposite polynomial, you must reverse the sign of each term in the polynomial.
- Example 3 addresses polynomial subtraction. The method is to add the opposite polynomial. Ensure that students take special care in managing integer signs. After rewriting the expression as an addition statement, ensure that students remove the brackets and collect like terms.
- In Example 4, the royalty context is revisited. The first step of the solution is to develop an algebraic expression to model the problem, consisting of the sum of several polynomials. In part a), this expression is then simplified to provide a general solution. Part b) is solved by substituting given information.
- Use **BLM 3.6.1 Practice: Add and Subtract Polynomials** for remediation or extra practice.

**Communicate Your Understanding Responses (page 157)**

**C1. a)** Collect any like terms together and use integer rules to find their sum. For example: $5x + 9 + 3x + 4 = 5x + 3x + 9 + 4 = 8x + 13$.

**b)** Collect any like terms together and use integer rules to subtract them. For example: $13x - 2 - 8x - 7 = 13x - 8x - 2 - 7 = -5x - 9$.

**C2. a)** The constants were not subtracted correctly: $-3 - 2 = -5$

**b)** The error occurred when removing the brackets: $-(2y - 5) = -2y + 5$. The final answer should be $2y - 2$.

## Practise

When identifying like terms in long expressions, some of the following techniques may be helpful:

- use different colours to underline each group of like terms;
- use underline, double-underline, squiggly underline, etc.;
- always extend the underline to include the sign to the left of the term.

Make sure that students do not begin to identify and collect like terms before removing brackets. This is especially important when subtracting polynomials; have students rewrite these as addition statements first.

## Connect and Apply

Students sometimes struggle with setting up algebraic models. Question 6 provides additional scaffolding for this type of problem, because the terms are all given. Ensure that students combine the information to form the algebraic expression. You may wish to use **BLM A8 Application General Scoring Rubric** for question 6 to assist you in assessing your students.

Questions 9 and 10 draw connections between measurement and algebra. Remind students how to find the perimeter. You may wish to use **BLM A9 Communication General Scoring Rubric** for question 10 to assist you in assessing your students.

## Extend

Students may wish to investigate question 11 using *The Geometer's Sketchpad®*. Use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** to support this activity.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1, 2a), c), e), 3, 4a), c), e), 5a), c), e), g), 6, 7, 8 |
| Typical | 1–10 |
| Extension | 11, 12 |

# 3.7 The Distributive Property

**Strand:**
Number Sense and Algebra

**Student Text Pages**
160 to 169

**Suggested Timing**
80–160 min

**Tools**
- algebra tiles

**Related Resources**
BLM 3.7.1 Investigate Table
BLM 3.7.2 Practice: The Distributive Property
BLM 3.7.3 Achievement Check Rubric

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.05** multiply a polynomial by a monomial involving the same variable [e.g., $2x(x + 4)$, $2x^2(3x^2 - 2x + 1)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil);

**NA2.06** expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

### Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of the Get Ready questions prior to working on this section.

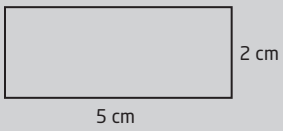
### Warm-Up

**1.** Draw a rectangle having the width and length given.
**a)** 2 cm × 5 cm **b)** 3 cm × 4 cm

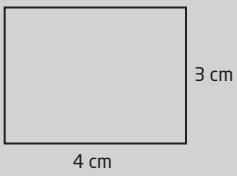
**2.** Find the area of each rectangle in #1.

**Warm-Up Answers**

**1. a)**



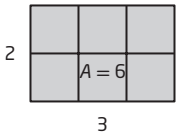

**b)**




**2. a)** 10 $cm^2$
**b)** 12 $cm^2$

## Teaching Suggestions

- Assign the Warm-Up and have students work independently. (5 min)
- For the Investigate, have students work with a partner or in small groups. (10–15 min) Distribute copies of **BLM 3.7.1 Investigate Table.** Students should be familiar with rectangular area models as a means to illustrate multiplication statements, based on their learning experiences in elementary school, e.g.,




  In this example, the two factors of the product, 2 and 3, represent the dimensions of a rectangle, and the resultant, 6, represents the area of the rectangle. Algebra tiles are used in the Investigate to extend this geometric concept to the multiplication of algebraic expressions. The distributive property is discovered by examining the initial factors and the algebraic resultant.
- After having students work through the Investigate, debrief the results as a class. Then, use the nickel and dime example following the Investigate to consolidate the idea.

## Common Errors

- Some students may distribute incompletely, for example, $2(x + 3) = 2x + 3$.

**$R_x$** Have students work with algebra tiles using simple examples. Have students build a rectangular area model and examine the area.

- Some students may distribute negative terms improperly, particularly in the middle of a long expression, for example, $2(x + y) - 3(x - y) = 2x + 2y - 3x - 3y$.

**$R_x$** Remind students that the sign to the left of the coefficient gets distributed with the coefficient. Review integer operations, as needed.

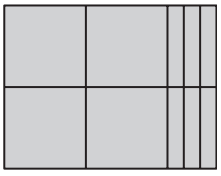
## Ongoing Assessment

- Use Achievement Check question 17 to monitor student success. See the Achievement Check Answers and **BLM 3.7.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Use Examples 1 and 2 for a class discussion. (10–15 min)
- These examples illustrate the distributive property. Algebra tiles may be helpful in providing a visual/geometric connection, but use them with caution: Tiles are not recommended when negative coefficients are involved, or when distributing a variable. Use the tiles in the Investigate, and perhaps for Example 1a), and then move directly into symbolic reasoning.
- Assign Communicate Your Understanding questions C1 and C2 and follow up with a class discussion. (5 min)
- Assign questions 1 to 4 and 6 (omit 2c), 6d) and f)). (10 min)
- Debrief answers with a class discussion. (5 min)
- Have students work on Examples 3 and 4, and consolidate with a class discussion. (10–15 min)
- In Example 3, the distributive property is applied in conjunction with other algebraic skills, for example, collecting like terms. The distribution of monomials that consist of both a constant and variable part is also introduced. In part c), students must apply operations involving rational numbers. The ability to recall and apply a disparate array of operational skills, as needed, is critically important for the academic student.
- Nested brackets are introduced in Example 4. Caution students to use careful placement and removal of brackets. When simplifying expressions of this type, encourage students to work systematically from the inside (round brackets) out [square brackets]. Remind students that as round brackets are removed, they can replace the square brackets with round brackets.
- Assign Communicate Your Understanding questions C3 and C4. Follow up with a class discussion. (5 min)
- Assign the remaining questions as independent work. (balance of period)
- Depending on the progress of the class, you may wish to extend this section to 1.5 or 2 periods.
- You may wish to use **BLM 3.7.2 Practice: The Distributive Property** as a remediation tool.

### Investigate Answers (page 160)

**1.**

| Rectangle | Width | Length | Area | Equation $w \times \ell = A$ |
|---|---|---|---|---|
|  | 2 | $x + 1$ | $2x + 2$ | $2 \times (x + 1) = 2x + 2$ |
|  | $x$ | $x + 2$ | $x^2 + 2x$ | $x \times (x + 2) = x^2 + 2x$ |
|  | $x$ | $2x + 5$ | $2x^2 + 5x$ | $x \times (2x + 5) = 2x^2 + 5x$ |
|  | $2x$ | $2x + 3$ | $4x^2 + 6x$ | $2x \times (2x + 3) = 4x^2 + 6x$ |
|  | $x$ | $x + 3$ | $x^2 + 3x$ | $x \times (x + 3) = x^2 + 3x$ |

**Accommodations**

**Memory**—Provide students with visual or verbal clues to remember the steps involved when using the distributive property.

**2.** Take each term in the width and multiply it with each term in the length. Then collect all the like terms and add/subtract them.

**3. a)** $4x + 12$ **b)** $2x^2 + 7x$
**c)** $3x^2 + 6x$

**4.** Answers will vary. For example, $7(2y - 5)$. First multiply $7 \times 2y = 14y$. Then, multiply $7 \times (-5) = -35$. The result is $7(2y - 5) = 14y - 35$.

### Communicate Your Understanding Responses (page 165)

**C1.** The terms in the brackets are unlike terms and therefore cannot be added. The 3 in front of the bracket is understood to multiply the bracket. The distributive property does this.

**C2.** Distributive property is used to multiply a polynomial by a monomial. To apply the distributive property, multiply each term in the polynomial by the monomial. For example:

$$\begin{aligned} & 4a(3b + 4c + 1) \\ &= 4a(3b) + 4a(4c) + 4a(1) \\ &= 12ab + 16ac + 4a \end{aligned}$$

**C3. a)** In the second line, Dmitri did not multiply the term outside of the brackets ($3x$) by every term in the brackets ($x + 2$). The second line should read: $3x^2 + 6x$, which is the answer.

**b)** Verify by substituting a value for $x$ into the original expression and into Dmitri's answer. When simplified, the value of the original expression and the simplified expression will not be the same.

**C4.** The distributive property, which should be used to simplify this expression, has not been used correctly. $-5$ should be multiplied by $x^2$, $-3x$, and 1, and then each product should be simplified. The correct answer is $-5x^2 + 15x - 5$.

**C5.**

| | |
|---|---|
| $3(x + 2(x - 1))$ | Apply the distributive property to the innermost set of brackets. |
| $= 3(x + 2x - 2)$ | Simplify the expression inside the brackets. |
| $= 3(3x - 2)$ | Apply the distributive property again. |
| $= 9x - 6$ | |

## Practise

Students may struggle with distributing terms with negative coefficients, for example, in question 3c) and d). Review and consolidate integer operations with students.

## Connect and Apply

For question 10, make a connection with Chapter 5, Section 5.2 Partial Variation.

Question 12 makes a connection between measurement and algebra. This question also justifies the need to apply the distributive property to rational terms.

Algebra tiles may be helpful for question 13.

In question 15, have students distribute into all sets of brackets simultaneously. Remind students to take care as to which terms get distributed into which brackets. Ensure they also indicate the signs of the results when distributing terms with negative coefficients.

For question 16, distribution of rational numbers requires careful work operating with both fractions and integers. Remind students to take extra care to clearly show all steps to their solutions, to more easily troubleshoot any problem areas.

Connections to perimeter are made in question 17. To solve and part c), set length = width and solve.

**Achievement Check Answers (page 169)**

**17. a)** Perimeter $= x + (2x - 1) + (2x + 3)$
$= 5x + 2$

Perimeter $= 2(2x + 1) + 2x$
$= 6x + 2$

Perimeter $= (2x + 1 + 2x - 1 + 3x + 1 + 20 - 4x)$
$= 3x + 21$

Perimeter $= 3(2w + 3) + 3(3w - 2)$
$= 6w + 9 + 9w - 6$
$= 15w + 3$

**b)** Perimeter $= 2(2x - 1) + 2(8 - 2x)$
$= 4x - 2 + 16 - 4x$
$= 14.$

So the perimeter is always 14, BUT the smallest value for $x$ is $\frac{1}{2}$, otherwise the length is negative and the largest value is 4, otherwise the width is negative. In both of these cases, the rectangle degenerates into a line.

**c)** If the rectangle is also a square, then the length = the width, so $2x - 1 = 8 - 2x$ or $4x = 9$ and $x = 2.25$.

## Extend

The type of reasoning in question 19 will be applied in grade 10 when students will learn how to multiply two binomials.

Question 20 is a natural extension.

## Exercise Guide

| Category | Question Number |
|---|---|
| Minimum (essential questions for all students to cover the expectations) | 1, 3a), c), e), 4a) c), 5a), c), 6a), c), e), 7a), c), e), 8a), c), 10, 12, 15a), c), e) |
| Typical | 1, 3–16a), c) |
| Extension | 18–21 |

# Use Technology

# Computer Algebra Systems

**Strand:**
Number Sense and Algebra

**Student Text Pages**
170 to 173

**Suggested Timing**
80 min

**Technology Tools**
• TI-89 calculators

**Related Resources**
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $\left(3x^2y + 2xy^2\right) + \left(4x^2y - 6xy^2\right)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

**Operating with Exponents**

**NA1.03** derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;

**NA1.04** extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.

### Warm-Up

- Hand out the TI-89 calculators, and go over review some basics with the students, such as how to:
  - turn the calculator on and off
  - get to the Home Screen
  - find certain important keys (operation keys, ENTER, etc.)
  - perform basic operations
  - clean up the home screen

## Teaching Suggestions

- Some of the TI-89 functions may not be intuitive to students. Have them work with a partner, but encourage them to take turns entering commands. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this section.
- Some of the layout features and functions are similar to the TI-83+/84+; students with prior experience with them may be able to transfer some working knowledge to the new TI-89 environment. For example, leading negatives have a special key, the keyboard layouts are similar, the second ENTER command brings you backward through previously entered commands, etc.
- Most students will have had some experience working in a Windows-like computer environment. The TI-89 has similar Cut-Copy-Paste commands, which are very useful when entering lengthy repeated expressions.
- For Investigate A, selecting NewProb also clears the variables to which a previous user may have assigned values.
- Investigate C could be assigned as a performance task.
- Investigate D, question 5 would be a very good activity for gifted students. Have students give demonstrations to the class.
- Use discretion in assigning question 5 if you are concerned about students getting lost, or changing modal settings on the calculator.

# Chapter 3 Review

**Student Text Pages**
174 to 175

**Suggested Timing**
80 min

**Tools**
- grid paper

**Related Resources**
BLM 3.CR.1 Chapter 3 Review
BLM G10 Grid Paper
BLM A14 Self-Assessment Recording Sheet
BLM A15 Self-Assessment Checklist

**Ongoing Assessment**

- Upon completing the Chapter Review, students can also answer questions, such as,
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related example in the text to help you with the questions? For which questions?*

## Using the Chapter Review

Each question in the **BLM 3.CR.1 Chapter 3 Review** reviews different skills and concepts. The students might work independently to complete the Chapter Review, then in pairs to compare solutions. Alternatively, the Chapter Review could be assigned for reinforcing skills and concepts in preparation for the Practice Test. You may wish to use **BLM G10 Grid Paper**. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult.

After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content. You may wish to use **BLM A14 Self-Assessment Recording Sheet** or **BLM A15 Self-Assessment Checklist** to help students assess their understanding.

# Chapter 3 Practice Test

**Student Text Pages**
176 to 177

**Suggested Timing**
60–80 min

**Related Resources**
BLM 3.PT.1 Chapter 3 Practice Test

BLM 3.CT.1 Chapter 3 Test

**Summative Assessment**

- After students complete **BLM 3.PT.1 Chapter 3 Practice Test**, you may wish to use **BLM 3.CT.1 Chapter 3 Test** as a summative assessment.

**Accommodations**

**Language**—Allow students to give oral responses to the review questions.

## Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

| Question | Section(s) | Refer to |
|---|---|---|
| 1 | 3.2 | Example 1f) (page 112) |
| 2 | 3.3 | Example 4a) (page 125) |
| 3 | 3.3 | Example 2c) (page 122) |
| 4 | 3.3 | Example 3a) (page 124) |
| 5 | 3.5 | Example 2 (page 145) |
| 6 | 3.4 | Example 2 (page 131) |
| 7 | 3.4 | Example 3 (page 132) |
| 8 | 3.5 | Example 5 (page 148) |
| 9 | 3.7 | Example 3b) (page 164) |
| 10a) | 3.3 | Example 1b), c) (page 121) |
| 10b) | 3.3 | Example 4d) (page 125) |
| 11 | 3.3 | Example 4 (page 125) |
| 12a) | 3.6 | Example 1 (page 154) |
| 12b) | 3.6 | Example 3 (page 155) |
| 13 | 3.7 | Example 3 (page 164) |
| 14 | 3.7 | Investigate (page 160) |
| 15 | 3.2 | Investigate (page 111) |
| 16 | 3.5 | Example 4 (page 146) |

### Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students. Can students do each of the following?

- build visual models to represent length, area and volume, using concrete materials
- expand and evaluate powers
- apply exponent laws to simplify and evaluate expressions
- identify coefficients and variables within a term
- classify terms by degree
- classify polynomials by name and degree
- identify like terms
- collect like terms
- add and subtract polynomials
- apply the distributive property to expand and simplify algebraic expressions
- construct algebraic models in order to solve problems

# Chapter 3 Problem Wrap-Up

**Student Text Pages**
177

**Suggested Timing**
60 min

**Tools**
- grid paper

**Technology Tools**
- *The Geometer's Sketchpad*®
- computers

**Related Resources**
BLM T4 *The Geometer's Sketchpad*® 3

BLM T5 *The Geometer's Sketchpad*® 4

BLM G10 Grid Paper

BLM 3.CP.1 Chapter 3 Problem Wrap-Up Rubric

**Summative Assessment**

- Use **BLM 3.CP.1 Chapter 3 Problem Wrap-Up Rubric** to assess student achievement.

## Using the Chapter Problem

- Students should be ready to perform this activity at the end of the chapter, especially if they have been working on the Chapter Problem components throughout.
- The Chapter Problem can be done individually or in groups.
- Suggest to students that their design must not be so complex that the measurements and calculations become too difficult.
- You may wish to have students use computer software such as *The Geometer's Sketchpad*® when designing their logo. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4**. This could be used as part of a summative assessment activity.
- If computer software is unavailable, use **BLM G10 Grid Paper** so that students can create accurate drawings of their ideas.
- This problem is quite open-ended. Students can provide simple solutions or very complex ones. Encourage students to reflect on their work and consider revising to provide a more sophisticated set of solutions.
- Provide sufficient time for the revision process. Students will create higher quality work if they have an opportunity for constructive feedback and time to incorporate suggestions and additional ideas into their project.
- This would be a good project for students to include in their portfolios.

### Level 3 Notes

Look for the following:
- A complete logo that is in the form of a polygon
- All polygon segments are labelled with variables
- Variables are used in the dimensions in a relevant way
- The logo is related to the team name
- Problems created are accurate and have simple solutions
- Algebraic calculations are mostly correct, with only minor errors

### What Distinguishes Level 2

At this level, look for the following:
- A logo is created but it may not be in the form of a polygon and hence the remaining parts of the question are made difficult
- Most polygon segments are labelled
- A different variable may label each straight side or the variables may be a reflection of measurements (e.g., only one variable $x$ is used, and 5 cm becomes $5x$)
- Logo may not be related to the team name (e.g., the student chooses a logo that interests them)
- Problems created may not involve any algebraic skills or may contain minor errors
- Algebraic calculations may have some significant errors

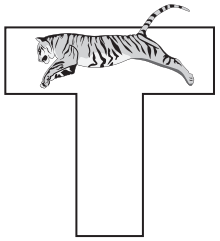
### What Distinguishes Level 4

At this level, look for the following:
- A complete logo that may involve multiple polygons
- All polygon sides are labelled with variables so the scale of the diagram parts is obvious
- The choice of variables is related to the actual measurements of the dimensions of the diagram
- Logo is a creative blend of team name, graphics, and mathematical requirements

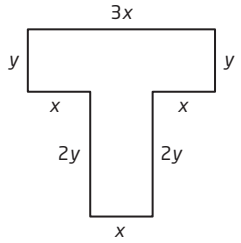
- Problems created are sophisticated yet related to the context
- Students may provide a sample of their logo created on card, fabric, etc.
- Algebraic calculations are flawless

**Level 3 Sample Response**

**a)** I designed this logo for my team, the Tigers



**b)** I labelled the dimensions of the T with the variables $x$ and $y$ as shown.




**c)** The perimeter of my logo is:

$P = x + 2y + x + y + 3x + y + x + 2y$
$= 6x + 6y$

**d)** The area of my logo can be found by dividing the letter into two rectangles

$A = (3x)(y) + (x)(2y)$
$= 3xy + 2xy$
$= 5xy$

**e)** Problem 1: Sarah wants to make a large logo for the gym wall. She notes that $y = 0.8x$. If the overall logo is to be 3 m tall, how wide will it be?

Solution 1: The height is $3y = 3$ m. Therefore, $y = 1$ m. The width is $3x$. Therefore, we need the value of $x$.

$y = 0.8x \rightarrow 1 = 0.8x \rightarrow \frac{1}{0.8} = x \rightarrow x = 1.25$

The width is 3.75 m.

Problem 2: If $x = 10$ cm and $y = 8$ cm, what is the perimeter and area of the logo?

Solution 2: $P = 6x + 6y$
$= 6(10) + 6(8)$
$= 108$

The perimeter is 108 cm.

$A = 5xy$
$= 5(10)(8)$
$= 400$

The area is 400 $cm^2$.

# Chapters 1 to 3 Review

**Student Text Pages**
178 to 179

**Suggested Timing**
80 min

**Tools**
- grid paper

**Related Resources**
BLM G10 Grid Paper
BLM A14 Self-Assessment Recording Sheet
BLM A15 Self-Assessment Checklist

**Ongoing Assessment**

- This is an opportunity for students to assess themselves by completing selected questions and checking their answers against the answers in the back of the student text. They can then revisit any questions with which they had difficulty.
- Upon completing the Chapters 1 to 3 Review, students can also answer questions such as the following:
  - Did you work by yourself or with others?
  - What questions did you find easy? Difficult? Why?
  - How often did you have to check the related example in the text to help you with the questions? For which questions?

## Using the Chapters 1 to 3 Review

Each question reviews different skills and concepts. Have students work independently to complete the Chapters 1 to 3 Review, then with a partner to compare solutions. Alternatively, assign the Chapters 1 to 3 Review to reinforce skills and concepts in preparation for the specific chapter Practice Test. You may wish to use **BLM G10 Grid Paper**. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult.

After they complete the Chapters 1 to 3 Review, provide an opportunity for students to discuss any questions, consider alternative strategies, and ask about questions with features they find difficult. You may wish to use **BLM A14 Self-Assessment Recording Sheet** or **BLM A15 Self-Assessment Checklist** to help students assess their understanding.

# Task: Electricity and Gas Costs

**Strand:**
Linear Relations

**Student Text Page**
180

**Suggested Timing**
20 min

**Related Resources**
BLM G10 Grid Paper
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 3.T1.1 Task: Electricity and Gas Costs Rubric

**Tools**
- grid paper
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Using Data Management to Investigate**

**RE1.02** pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;

**RE1.04** describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

**Understanding Characteristics of Linear Relations**

**RE2.02** construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

**RE2.03** identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear.

## Teaching Suggestions

- Discuss the task as a class so that students are clear on what is expected. The objective of such discussions is to clarify the problem, brainstorm possible hypotheses, and discuss how to plan a strategy for solving the Task.
- Use **BLM G10 Grid Paper** or **BLM T4 *The Geometer's Sketchpad®* 3 BLM T5 *The Geometer's Sketchpad®* 4**, if students will be using graphing software.
- As extension activities, have students work with a partner to draw scatter plots of other possible pairs of variables and prepare a report on their results. Instruct them to draw lines of best fit where the data are not linear and discuss ways in which the set of data is related (e.g., seasons, temperature) and ways in which it is not related.
- You may wish to add questions on interpolation and extrapolation using the data and scatter plots (e.g., If Tico's business were to increase its monthly use of natural gas to 1000 $m^3$, find the monthly cost).

### Prompts for Getting Started

The following list of questions will help students continue with their investigation. Ask students,

- *What does the data show?*
- *How many different variables are there in the data?*
- *How will you draw the scatter plots?*

### Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the

## Accommodations

**Motor**—Allow students extra time to complete the questions in this section. Have students work with a partner when creating graphs using technology.

## Ongoing Assessment

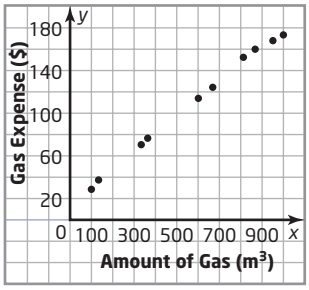
- Use **BLM 3.T1.1 Task: Electricity and Gas Costs Rubric** to assess student achievement.

following questions:

- How much assistance did the student need to get started with the graphing?
- How much assistance did the student need to complete the task?
- Does the student present scatter plots that are clearly labelled?
- Did the student make reasonable conjectures and then test them?
- What parts of the task did the student complete/not complete?
- Does the student show an understanding of the differences between linear and non-linear relationships?

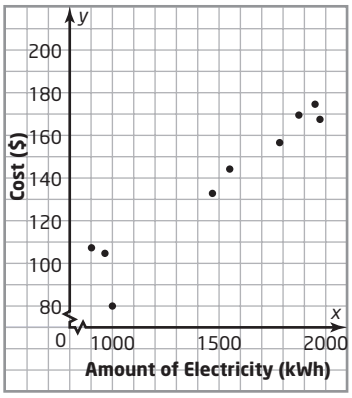
### Level 3 Sample Response

**a)** Scatter plot:

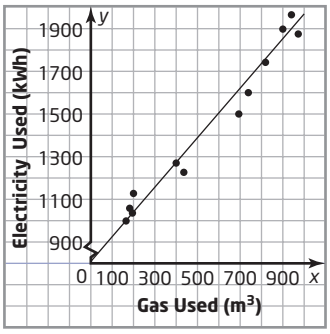



**b)** The relationship is linear. The amount of gas used multiplied by the cost of 1 $m^3$ ($0.18) gives the monthly cost.

**c)** The relationship between amount of amount of electricity used (in kWh) and the cost is also linear. The cost is 8.5 cents per kWh of electricity used. This scatter plot confirms the linear nature of the relationship.




**d)** Answers will vary. Any other two variables should not give an exact linear relationship. For example comparing the amount of gas and the amount of electricity used by month gives the following scatter plot. It shows that there is a similar pattern between these uses (i.e., in winter the usage of each increases at almost the same rate) for which a line of best fit can be found.




Electricity Used = 1.11 Gas Used + 830

**Level 3 Notes**

Look for the following:

- Complete scatter plots
- Evidence that the data table is understood
- Use of tools
- Line of best fit present, may be hand drawn but will smoothly fit the data
- Clear report using good form and correct mathematical notation
- Evidence of correct inference

**What Distinguishes Level 2**

Look for the following:

- Scatter plots may contain several errors
- Evidence that the data table is only partially understood
- Errors in the use of tools
- Line of best fit may be absent or inaccurately drawn
- Somewhat clear report, with some lack of form and mathematical notation
- Some evidence of correct inference

**What Distinguishes Level 4**

Look for the following:

- Complete detailed scatter plots
- Very appropriate use of tools
- Line of best fit included and analysis/justification of its implications
- Very clear report using good form and correct mathematical notation
- Very clear evidence of correct inferential conclusions

# Task: Perimeters and Areas

**Strand:**
Number Sense and Algebra

**Student Text Page**
181

**Suggested Timing**
20 min

**Tools**
- grid paper
- square dot paper
- isometric dot paper

**Related Resources**
BLM G10 Grid Paper
BLM G7 Square Dot Paper
BLM G8 Isometric Dot Paper
BLM 3.T2.1 Task: Perimeters and Areas Rubric

**Mathematical Process Expectations Emphasis**

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.03** relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations;

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

**NA2.05** multiply a polynomial by a monomial involving the same variable [e.g., $2x(x + 4)$, $2x^2(3x^2 - 2x + 1)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil).

## Teaching Suggestions

- Most students should find the first two parts of this Task to be straightforward. Parts c) and d) are quite challenging and require detailed analysis of all possible combined configurations of 16 congruent small rectangles.
- Give the students 3 min to complete a) and b), and think about c). Follow with a brief discussion of what is being asked in c), possibly displaying a visual aid (e.g., 16 rectangular blocks or a sketch). Then, have the students complete the Task.
- You may wish to use **BLM G10 Grid Paper**, **BLM G7 Square Dot Paper**, and/or **BLM G8 Isometric Dot Paper**.
- As an extension, have students make up a similar question and provide a solution.

### Hints for Evaluating a Response

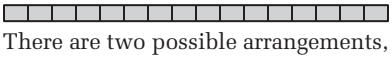
Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

- How much assistance did the student need to get started with the task?
- How much assistance did the student need to complete the task?
- Does the student show a clear understanding of the difference between perimeter and area?
- How many different ways did the student find to combine the rectangles?
- Does the student perform arithmetic calculations correctly?
- What parts of the task did the student complete/not complete?

**Level 3 Sample Response**

**a)** $2(x) + 2(x + 2) = 4x + 4$ **b)** $(x)(x + 2) = x^2 + 2x$

**c)** There are five possible configurations: Two for each of 16 × 1 and 8 × 2 small rectangles, and one for 4 × 4 small rectangles.

**16 × 1:** 

There are two possible arrangements, which give:

$P = 32(x) + 2(x + 2)$ **OR** $P = 32(x + 2) + 2(x)$
$= 34x + 4$ $= 34x + 64$

**8 × 2:** 

Again there are two possible arrangements.

$P = 16(x) + 4(x + 2)$ **OR** $P = 16(x + 2) + 4(x)$
$= 20x + 8$ $= 20x + 32$

## Accommodations

**Visual**—Have students use normal grid or dot paper, isometric graph or dot paper, and/or blocks to help them visualize this activity.

## Ongoing Assessment

Use **BLM 3.T2.1 Task: Perimeters and Areas Rubric** to assess student achievement.

**4 × 4:** 

There is only one arrangement:

$P = 8(x) + 8(x + 2)$

$= 16x + 16.$

**d)** Substituting $x = 1$, 2, and 3 in each case above, shows that:

For $x = 1$, $P = 20(1) + 8$ or 28 units is the smallest perimeter

For $x = 2$, $P = 20(2) + 8$ or 48 and $P = 16(2) + 16$ or 48. So, both configurations have the same perimeter.

For $x = 3$, $P = 16(3) + 16$ or 64 units is the smallest perimeter.

For $x > 2$, the $4 \times 4$ combination will always have the least perimeter.

**e)** Since all the areas are formed from 16 small rectangles of area $(x^2 + 2x)$, the total area is $16(x^2 + 2x) = 16x^2 + 32x$.

### Level 3 Notes

Look for the following:

- Clear understanding of area and perimeter concepts
- Clear understanding of how to add polynomials and multiply a monomial and a binomial
- Evidence of knowledge of all possible large rectangular configurations
- Mostly accurate calculations of polynomial expressions for perimeter and area
- Mostly accurate substitution of values for $x$
- Use of good form and correct mathematical notation

### What Distinguishes Level 2

At this level, look for the following:

- Some understanding of area and perimeter concepts
- Some understanding of how to add polynomials and multiply a monomial and a binomial
- Evidence of knowledge of some of the possible large rectangular configurations
- Somewhat accurate calculations of polynomial expressions for perimeter and area
- Somewhat accurate substitution of values for $x$
- Some use of good form and correct mathematical notation

### What Distinguishes Level 4

At this level, look for the following:

- Very clear understanding of area and perimeter concepts
- Very clear understanding of how to add polynomials and multiply a monomial and a binomial
- Evidence of knowledge of all possible large rectangular configurations
- Accurate calculations of polynomial expressions for perimeter and area
- Accurate substitution of values for $x$, and observation that for $x = 2$ two of the configurations have the same perimeter
- Use of very good form and correct mathematical notation; possibly inclusion of sketches of the two configurations with minimal perimeter for $x = 2$

# Task — Mind Reader

**Strand:**
Number Sense and Algebra

**Student Text Pages**
181

**Suggested Timing**
20 min

**Related Resources**
BLM 3.T3.1 Task: Mind Reader Rubric

**Mathematical Processes Expectations Emphasis**

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

## Specific Expectations

**Manipulating Expressions and Solving Equations**

**NA2.04** add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

**NA2.06** expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

**NA2.08** rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement).

## Teaching Suggestions

- Introduce the Task to the class using a simple number trick to start students thinking. Emphasize that students are expected to explain the inner workings of the number challenge, and provide a symbolic representation if they can.
- Allow students to work in pairs if that is appropriate.
- If modelling the EQAO task, assign Parts 1 and 2. (20 min).
- For extension activities, ask students to design or analyse each other's number tricks or analyse a similar or more complex trick, using symbols.
- To extend this Task as a performance assessment, have students complete the following assignments:
  - Design a similar (or more challenging) Mind Reader number trick, and provide a symbolic explanation of the way it works.
  - Switch the number tricks and challenge each other to solve them.

### Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following:

- students' level of understanding and ability to translate the word instructions into symbols and operations
- students' level of reasoning and explanation of the way in which the operations in the trick work

**Level 3 Sample Response**

**Part 1**

**a)** 5 $\quad 5 + 20 = 25 \quad 5 \times 25 = 125 \quad 2 \times 125 = 250$
$250 - 200 = 50 \quad 50 \div 5 = 10$

**b)** Steps 1 to 6 give:
$x \quad x + 20 \quad 5x + 100 \quad 10x + 200 \quad 10x \quad 10$

**Part 2**

**a)** Answers will vary. Possible answer: For birth date March 28, 1992, the answer would be 280392.

**b)** Steps 1 to 9 give for day/month/year, e.g., *x/y/z*
$x \quad 5x \quad 5x + 4 \quad 100x + 80 \quad 100x + 80 + y$
$10\,000x + 8000 + 100y \quad 10\,000x + 8000 + 100y + z \quad 10000x + 100y + z$

In the case of 15 April 1994, this will be 150494.

**Ongoing Assessment**

Use **BLM 3.T3.1 Task: Mind Reader Rubric** to assess student achievement.

**Level 3 Notes**

Look for the following:

- Considerable understanding of word instructions
- Considerable ability to translate word instructions into symbols and operations
- Considerable understanding of how to add polynomials and multiply a polynomial by a number
- Considerable use of good form and correct mathematical notation
- Provides an essentially correct reasoned analysis of the number tricks

**What Distinguishes Level 2**

At this level, look for the following:

- Some understanding of word instructions
- Some ability to translate word instructions into symbols and operations
- Some understanding of how to add polynomials and multiply a polynomial by a number
- Some use of good form and correct mathematical notation
- Provides an analysis of the number tricks with some correct reasoning

**What Distinguishes Level 4**

At this level, look for the following:

- Very clear understanding of word instructions
- A high degree of ability to translate word instructions into symbols and operations
- Very clear understanding of how to add polynomials and multiply a polynomial by a number
- Very good form and correct mathematical notation
- Provides a detailed and correctly reasoned analysis of the number tricks