CHAPTER

9

Optimizing Measurements

Vocabulary

optimization
maximum
minimum

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Strand:
Measurement and Geometry

Overall Expectations

By the end of this course, students will:

MGV.01 determine, through investigation, the optimal values of various measurements;

MGV.02 solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

MGV.03 verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Investigating the Optimal Values of Measurements

By the end of this chapter, students will:

MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;

MG1.02 determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant;

MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Chapter Problem

The Chapter Problem focusses on packaging computer supplies for shipment to customers. Have students discuss their understanding of the topic, encouraging them to suggest factors that need to be considered when designing the best package for an item. The supplies need to be packaged so that they are not likely to be damaged during the shipping process. However, we also want to avoid "over-packaging" and use recyclable materials as much as possible to protect our environment.

You may wish to have students complete the Chapter Problem questions that occur throughout the chapter. These questions incorporate acquired skills as they are introduced in the sections where they occur. They are designed to help students move toward the Chapter Problem Wrap-Up on page 519.

Alternatively, you may wish to assign the Chapter Problem Wrap-Up when students have completed the chapter. The Chapter Problem Wrap-Up may be used as a summative assessment and is open-ended in nature to allow students originality in their solutions.

Chapter 9 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 9 Opener • 15 min	474–475			
Get Ready • 80 min	476–477	• BLM 9.GR.1 Practice: Ready	• BLM 9 GR.2 Get Ready Self-Assessment Checklist	
9.1 Investigate Measurement Concepts • 80–160 min	478–483	• BLM 9.1.1 Rectangle Data Recording Table • BLM G10 Grid Paper • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.1.2 Practice: Investigate Measurement Concepts	• BLM A11 Group Work Assessment Recording Sheet • BLM A17 Teamwork Self Assessment • BLM A3 Portfolio Checklist	**Tools** • geoboards • toothpicks • elastic bands • grid paper **Technology Tools** • *The Geometer's Sketchpad*® • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.2 Perimeter and Area Relationships of a Rectangle • 80 min	484–490	• BLM G10 Grid Paper • BLM 9.2.1 Practice: Perimeter and Area Relationships of a Rectangle • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel*	• BLM 9.2.2 Achievement Check Rubric	**Tools** • toothpicks • geoboard • elastics • grid paper **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.3 Minimize the Surface Area of a Square-Based Prism • 80–160 min	491–497	• BLM 9.3.1 Square-Based Prism Data Recording Table • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.3.2 Practice: Minimize the Surface Area of a Square-Based Prims • BLM G8 Isometric Dot Paper	• BLM A5 Problem Solving Checklist	**Tools** • interlocking cubes • isometric dot paper **Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.4 Maximize the Volume of a Square-Based Prism • 80–160 min	498–503	• BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.4.1 Investigate: *The Geometer's Sketchpad*® Method • BLM 9.4.2 Practice: Maximize the Volume of a Square-Based Prism • BLM G10 Grid Paper	• BLM 9.4.3 Achievement Check Rubric • BLM A21 Opinion Piece Checklist • BLM A4 Presentation Checklist	**Tools** • grid paper **Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • *The Geometer's Sketchpad*® • computers
9.5 Maximize the Volume of a Cylinder • 80 min	504–509	• BLM 9.5.1 Cylinder Data Recording Table • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.5.2 Practice: Maximize the Volume of a Cylinder	• BLM A18 My Progress as a Problem Solver	**Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.6 Minimize the Surface Area of a Cylinder • 80 min	510–515	• BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.6.1 Practice: Minimize the Surface Area of a Cylinder	• BLM 9.6.2 Achievement Check Rubric • BLM A23 News Report Checklist • BLM A4 Presentation Checklist	**Tools** • construction paper • rulers • scissors • tape **Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
Chapter 9 Review • 80 min	516–517	• BLM 9.CR.1 Chapter 9 Review		

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 9 Practice Test • 60 min	518–519		• BLM 9.PT.1 Chapter 9 Practice Test • BLM 9.CT.1 Chapter 9 Test	
Chapter 9 Problem Wrap-Up • 30–60 min	519		• BLM 9.CP.1 Chapter 9 Problem Wrap-Up Rubric	
Chapters 7 to 9 Review • 80 min	520–521		• BLM A14 Self-Assessment Recording Sheet • BLM A15 Self-Assessment Checklist	
Task: The Horse Barn • 20 min	522		• BLM 9.T1.1 Task: The Horse Barn Rubric	
Task: The Ice Rink • 20 min	523	• BLM G10 Grid Paper	• BLM 9.T2.1 Task The Ice Rink Rubric	**Tools** • grid paper
Task: Packing Compressed Air • 20 min	523		• BLM 9.T3.1 Task: Packing Compressed Air Rubric	

Chapter 9 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 9.GR.1	Practice: Get Ready	Practice
	BLM 9.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
9.1 Investigate Measurement Concepts			
	BLM G10	Grid Paper	Student Support
	BLM 9.1.1	Rectangle Data Recording Table	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.1.2	Practice: Investigate Measurement Concepts	Practice
	BLM A11	Group Work Assessment Recording Sheet	Assessment Group Work
	BLM A17	Teamwork Self Assessment	Assessment Group Work
	BLM A3	Portfolio Checklist	Assessment
9.2 Perimeter and Area Relationships of a Rectangle			
	BLM G10	Grid Paper	Student Support
	BLM 9.2.1	Practice: Perimeter and Area Relationships of a Rectangle	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.2.2	Achievement Check Rubric	Assessment
9.3 Minimize the Surface Area of a Square-Based Prism			
	BLM 9.3.1	Square-Based Prism Data Recording Table	Student Support
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.3.1	Practice: Minimize the Surface Area of a Square-Based Prism	Practice
	BLM G8	Isometric Dot Paper	Student Support
	BLM A5	Problem Solving Checkist	Assessment

	BLM	Title	Purpose
9.4 Maximize the Volume of a Square-Based Prism			
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.4.1	Investigate: *The Geometer's Sketchpad*® Method	Student Support Technology
	BLM 9.4.2	Practice: Maximize the Volume of a Square-Based Prism	Practice
	BLM G10	Grid Paper	Student Support
	BLM 9.4.3	Achievement Check Rubric	Assessment
	BLM A21	Opinion Piece Checklist	Assessment Literacy
	BLM A4	Presentation Checklist	Assessment
9.5 Maximize the Volume of a Cylinder			
	BLM 9.5.1	Cylinder Data Recording Sheet	Student Support
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM 9.5.2	Practice: Maximize the Volume of a Cylinder	Practice
9.6 Minimize the Surface Area of a Cylinder			
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.6.1	Practice: Minimize the Surface Area of a Cylinder	Practice
	BLM 9.6.2	Achievement Check Rubric	Assessment
	BLM A23	News Report Checklist	Assessment Literacy
	BLM A4	Presentation Checklist	Assessment
Chapter 9 Review			
	BLM 9.CR.1	Chapter 9 Review	Review
Chapter 9 Practice Test			
	BLM 9.PT.1	Chapter 9 Practice Test	Diagnostic Assessment
	BLM 9.CT.1	Chapter 9 Test	Summative Assessment
Chapter 9 Problem Wrap-Up			
	BLM 9.CP.1	Chapter 9 Problem Wrap-Up Rubric	Summative Assessment

	BLM	Title	Purpose
Chapters 7 to 9 Review			
	BLM A14	Self-Assessment Recording Sheet	Student Self-Assessment
	BLM A15	Student Self-Assessment Checklist	Student Self-Assessment
Task: The Horse Barn			
	BLM 9.T1.1	Task: The Horse Barn Rubric	Summative Assessment
Task: The Ice Rink			
	BLM G10	Grid Paper	Student Support
	BLM 9.T2.1	Task: The Ice Rink Rubric	Summative Assessment
Task: Packing Compressed Air			
	BLM 9.T3.1	Task: Packing Compressed Air Rubric	Summative Assessment

Get Ready

Student Text Pages
476 to 477

Suggested Timing
80 min

Related Resources
BLM 9.GR.1 Practice: Get Ready
BLM 9.GR.2 Get Ready Self-Assessment Checklist

Common Errors

- Some students may use improper units in their answers.

R_x Ensure that students have a clear understanding of the difference among linear units (m, cm) square units (m^2, cm^2), and cubic units (m^3, cm^3). Use a visual representation to clarify the difference in these units. For example, use interlocking cubes, and discuss the length of an edge, the area of a face, and the volume of a shape.

- Some students may need to be reminded to use proper form in their solutions.

R_x Have students show the formula they are using, in the next line show the values they are substituting into the formula, and then the rest of their solution following the proper order of operations. Encourage students to use equal signs appropriately and show their solution flowing vertically as they move from line to line. Concluding statements should be made, especially if there is a context provided in the question.

Accommodations

Visual—Provide visual clues for the students such as colour-coding the variables in the formulas to match the related dimensions on a diagram of the shape.

Perceptual—Allow students to use a scientific calculator when calculating answers.

Memory—Let students use the "formula cue-cards" that they have created while working their way through the textbook.

Teaching Suggestions

- Have students work with a partner or in a small group.
- The Get Ready segment Measurement Concepts: Perimeter, Circumference, Area, Surface Area, and Volume reviews the measurement skills developed in Chapter 8. This chapter incorporates these same skills in optimization problems, so it is important that skills from Chapter 8 have been properly developed.
- The questions in this section could be assigned on an individual basis as required. You may also wish to use **BLM 9.GR.1 Practice: Get Ready** for remediation or extra practice.
- If you are confident that your students have the necessary skills from Chapter 8, you may wish to assign the Compare Figures segment. This section provides a preview of the rest of this chapter. Students calculate the surface area and volume of two containers and make comparisons. After students have completed questions 5 and 6, ask students which container they would recommend and why. A discussion is a perfect lead-in to this chapter.
- All **BLMs** referred to throughout this chapter can be found in the Principles of Mathematics 9 Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation while students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 Curriculum for this chapter. Student self-assessment is also an effective technique. Using **BLM 9.GR.2 Get Ready Self-Assessment Checklist**, have students place a checkmark beside topics in the Get Ready in which they feel confident with the necessary skills. Take remedial action in small groups or with a class skill review.

9.1 Investigate Measurement Concepts

Strand:
Measurement and Geometry

Student Text Pages
478 to 483

Suggested Timing
80 to 160

Tools
- geoboard
- toothpicks
- elastic bands
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- Corel® *Quattro Pro®*
- Microsoft® *Excel*
- computers

Related Resources
BLM G10 Grid Paper
BLM 9.1.1 Rectangle Data Recording Table
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM T1 Corel® *Quattro Pro®* 8
BLM T2 Corel® *Quattro Pro®* 10
BLM T3 Microsoft® *Excel*
BLM 9.1.2 Practice: Investigate Measurement Concepts
BLM A11 Group Work Assessment Recording Sheet
BLM A17 Teamwork Self Assessment
BLM A3 Portfolio Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements
MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;
MG1.02 determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant;
MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

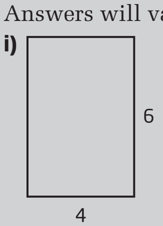
This section requires skills with perimeter and area of two-dimensional figures. Assign questions 1 and 2.

Warm-Up

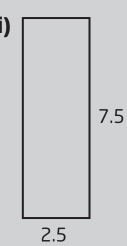
1. a) On grid paper, draw different rectangles that have a perimeter of 20 units.
b) Calculate the area of each rectangle in part a).

Warm-Up Answers

1. a) Answers will vary. Possible answers include:

i)


ii)


iii)


b) i) 24 **ii)** 21 **iii)** 18.75

Teaching Suggestions

- You may wish to use **BLM G10 Grid Paper** for the Warm-Up.
- Introduce the Investigate. Ensure that students understand that the perimeter is fixed. Have students complete the Investigate A, Method 1: Use a Geoboard.
- You may wish to have some students share their results with the class. Use an overhead geoboard to have students illustrate the different rectangles that are possible. (5–10 min)

Common Errors

- Some students may consider the conclusion to an investigation more important than the process.

R_x Stress the importance of the investigation itself. Value the process, not just the conclusion in all class discussions. Encourage students to keep clear records of the results of an investigation (e.g., tables, diagrams, computer sketches, etc.). Treat the investigations like an experiment in a science class. Encourage students to keep their investigations organized so that they may be referred to later. For example, instruct students to keep all of their investigations in a portfolio, and collect and assess their work at the end of the chapter. You may wish to use **BLM A3 Portfolio Checklist** to assist you in assessing student portfolios.

Ongoing Assessment

Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- You may wish to use **BLM 9.1.1 Rectangle Data Recording Table** for Investigate A and B.
- As an alternate strategy to Investigate A, have students use toothpicks instead of a geoboard.
- If the class has access to computers, you may wish to use Method 2: Use *The Geometer's Sketchpad*®. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity. Have students work with a partner. (10 min)
- For another approach, use an overhead geoboard to introduce the Investigate. Have students form different rectangles with an elastic on the overhead geoboard and then discuss the areas of the rectangles. After some discussion, students could complete *The Geometer's Sketchpad*® Method 2. (15–20 min)
- Follow this Investigate with a class discussion. The play area should be square to result in the largest area.
- The OSAPAC Committee (Ontario Software Acquisition Program Advisory Committee) has licensed the student edition of *The Geometer's Sketchpad*® for students' home use. Make students aware of this opportunity.
- Investigate B involves finding the perimeter of rectangles with a fixed area. This can be done on grid paper using a paper and pencil technique. Use **BLM G10 Grid Paper** for this activity.
- Alternatively, if there is computer access, have students use a spreadsheet program. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- Follow up with a class discussion. The pet exercise area should be square to use the least amount of fencing.
- The Investigate skills are the focus of this section. Encourage students to use any manipulatives that might suit an Investigate. Have geoboards and elastics, toothpicks, grid paper, interlocking cubes, etc., readily available in the classroom. Where possible, provide access to computers so that students can use spreadsheets or *The Geometer's Sketchpad*® to complete the Investigates.
- You may wish to use **BLM 9.1.1 Practice: Investigate Measurement Concepts** for remediation or extra practice.

Investigate Answers (page 478)

A: Method 1

1. b)

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m)
1	1	5	12	5
2	2	4	12	8
3	3	3	12	9

c) 3 rectangles

2. a) Rectangle 1 has the least area; its dimensions are 1×5; its shape is a narrow rectangle.

b) Rectangle 3 has the greatest area; its dimensions are 3×3; its shape is a square.

3. Answers will vary. Sample answer: I would choose rectangle 3 (a square) because for a given length of rope, 12 m, greatest area can be enclosed.

Method 2

8. Use *The Geometer's Sketchpad*® to draw graphs.

15. Answers will vary. Sample answer: I would choose rectangle 3 (a square) because for a given length of rope, 12 m, greatest area can be enclosed.

Accommodations

Gifted and Enrichment—Challenge students to use toothpicks to build enclosures with the greatest area using other types of quadrilaterals and other shapes.

Visual—Provide verbal instructions for the steps for students to use *The Geometer's Sketchpad®* to complete the Investigate.

Motor—Provide students with enlarged photocopies of the tables in this section for them to fill in the results of the Investigate. If students complete the Investigate using *The Geometer's Sketchpad®*, they should work together with a partner, or in groups.

Language—Let students give an oral report of the findings of the Investigate rather than a written report.

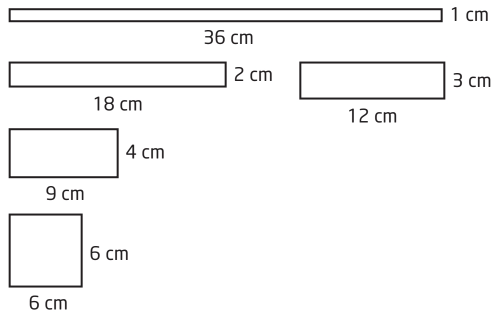
Memory—Review with the students the steps required to use a spreadsheet to complete the Investigate.

Student Success

Use **jigsaw** to have students learn and teach the investigations in this section using multiple methods (pencil and paper, spreadsheet, manipulatives, graphing calculators).

B:

1. a)



b) Answers will vary. The values in the Width (m) and the Length (m) columns can be interchanged.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	36	74	36
2	2	18	40	36
3	3	12	30	36
4	4	9	26	36
5	6	6	24	36

2. a) 6 m × 6 m **b)** 1 m × 36 m

3. As can be seen from the table above, all five rectangles enclose 36 m^2 of land but require different lengths of fencing. I would use Rectangle 5 (a square: 6 m × 6 m) for the pet exercise area as it requires the least amount of fencing to enclose the same area.

Communicate Your Understanding Responses (page 482)

C1. Answers will vary. Let each square on the grid paper represent 1 unit and draw different rectangles with a perimeter of 40 units. Once the rectangles have been drawn, measure the dimensions of each rectangle and record them. Then, use the formula for the area of a rectangle (Area = length × width) to determine and compare the areas of various rectangles with a perimeter of 40 units.

C2. Answers will vary. Let the space between 2 pins equal 1 unit and use an elastic band to make different rectangles with a perimeter of 15 units.

Practise

Questions 1 and 2 are similar to Investigate A and use grid paper (question 1) and toothpicks (question 2). Use **BLM G10 Grid Paper** for question 1.

Question 3 is similar to Investigate B, but uses a geoboard.

Connect and Apply

Question 4 has a fixed area and a geoboard is used.

Question 5 has a fixed perimeter and uses *The Geometer's Sketchpad®*. Use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** for question 5.

Question 6 has a fixed perimeter and uses either grid paper or a spreadsheet. Use **BLM G10 Grid Paper or BLM T1 Corel® *Quattro Pro®* 8**, **BLM T2 Corel® *Quattro Pro®* 10**, or **BLM T3 Microsoft® *Excel*** for question 6.

Give students opportunities to use a variety of strategies and tools for conducting investigations. The strategies for the questions can be interchanged.

Extend

Have students work in small groups for question 7. Have each group submit a brief report. You may wish to use **BLM A11 Group Work Assessment Recording Sheet** to assist you in assessing your students. Or, use **BLM A17 Teamwork Self Assessment** as a self-assessment. Students will handle the circular enclosure in a variety of ways. Some may use a circumference of 36, solve for the radius, and then calculate the area. Others may try to calculate the area of the 36-sided regular polygon! Any appropriate technique would be acceptable. Follow with a class discussion. The circular enclosure would have the largest area and student answers should illustrate this trend.

The Math Contest question is a variation on the fixed perimeter scenario. The fact that the three fields are adjoined will mean that a square is not the optimal shape. Again, students should be encouraged to use different manipulatives to conduct this investigation.

Exercise Guide

Category	Question Number
Minimum	1–4
Typical	1–6
Extension	7

9.2 Perimeter and Area Relationships of a Rectangle

Strand:
Measurement and Geometry

Student Text Pages
484 to 490

Suggested Timing
80 minutes

Tools
- toothpicks
- grid paper
- geoboard
- elastics

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- Corel® *Quattro Pro®*
- Microsoft® *Excel*
- computers

Related Resources
BLM G10 Grid Paper
BLM 9.2.1 Practice: Perimeter and Area Relationships of a Rectangle
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM T1 Corel® *Quattro Pro®* 8
BLM T2 Corel® *Quattro Pro®* 10
BLM T3 Microsoft® *Excel*
BLM 9.2.2 Achievement Check Rubric

Mathematical Process Expectations Emphasis
- ✓ Problem Solving
- ✓ Reasoning and Proving
- ✓ Reflecting
- ✓ Selecting Tools and Computational Strategies
- ✓ Connecting
- ✓ Representing
- ✓ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;
MG1.02 determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant;
MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

This section involves perimeter and area of rectangles. Assign Get Ready question 1 before starting this section.

Warm-Up

Have students construct a rectangle with a perimeter of 24 units, using grid paper, a geoboard, and elastics or toothpicks. Use **BLM G10 Grid Paper** for this activity. Have students determine the area of each rectangle they construct.

Teaching Suggestions

- Review graphing using the lists on a graphing calculator and by entering a function. Use a linear example that would review skills in previous chapters, or a non-linear example that will be similar to the quadratic results in the Investigate.
- Have students work with a partner in or small groups on the Investigate A: Method 1. (5–10 min)
- In Method 2, students enter the width and the area of the rectangles into L1 and L2 on a graphing calculator. The resulting graph of area versus width is a quadratic relationship. (10 min)
- Investigate B involves a rectangle enclosed on just three sides. Have students work with a partner. This Investigate illustrates that a square will not necessarily enclose the maximum area for a given perimeter. (10 min)
- Follow up with a class discussion.
- Complete the Communicate Your Understanding questions as a class.
- You may wish to use **BLM 9.2.1 Practice: Perimeter and Area Relationships of a Rectangle** for remediation or extra practice.

Common Errors

- Some students may think that a square will always produce the maximum area for a given perimeter.

R_x It is important that all students carry out investigations where the shape is not enclosed on all sides. Investigate B should be done by all students for this reason. It should be pointed out that the optimal shape depends on how many sides are enclosed.

Ongoing Assessment

- Use Achievement Check question 12 to monitor student success. See the suggested solutions and **BLM 9.2.2 Achievement Check Rubric**.
- Chapter Problem question 7 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Investigate Answers (page 484)

A: Method 1

1. See measurements in table in Q.2.

2.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	15	32	15
2	2	14	32	28
3	3	13	32	39
4	4	12	32	48
5	5	11	32	55
6	6	10	32	60
7	7	9	32	63
8	8	8	32	64

3. a) 8 m × 8 m **b)** 64 m^2 **c)** square

4. a) Answers will vary. Possible answer: 10 m × 10 m
c) Answers will vary. Possible answer: The result matches the prediction.

5. Answers will vary. Divide the perimeter by 4 to get the length of each side.
$\frac{\text{Perimeter}}{4} = \text{side length}$

6. a) $\frac{60}{4}$, or 15 m
b) $\frac{30}{4} = 7.5$. In this case, we have a decimal number. To create a rectangle with this side length, two toothpicks will have to be broken into halves.

A: Method 2

3. a) $P = 2 \times (\ell + w) = 32$ m
$(\ell + w) = 16$
$\ell = 16 - w$
Let $w = x$
$\ell = (16 - x)$
b) $A = w \times \ell$
$A = x \times (16 - x)$
$A = 16x - x^2$

4. The graph of $x(16 - x)$ overlaps the scatter plot graph created in question 1. However, in this graph, x also assumes negative values and passes through origin.

5. a) $x = 8$, $y = 64$
b) width is 8 m. This point is at the top or the peak of the curve.

6. a) length is now $(20 - x)$. The point of maximum area: (10, 100)
b) Square; Yes; 10 m × 10 m

7. a) Answers will vary. 11.25 m × 11.25 m
b) Answers will vary. Yes.

8. Divide the perimeter by 4 to get the length of each side and square the number to get the area.
$\frac{\text{Perimeter}}{4} = \text{side length}$
$\text{Area} = (\text{side length})^2$

B:

1. a) Answers will vary. He will be able to enclose a greater area.
b) Answers will vary. The shape will be a rectangle.
c) Answers will vary. The dimension of the side opposite the hedge will be twice as long as the other two sides.

Accommodations

Gifted and Enrichment—Challenge students to create extra Math Contest questions and to extend question 15 to find the dimensions of other shapes of maximum area that can be inscribed in a circle of radius 10 cm.

Visual—Let students use technology to complete the questions in this section.

Perceptual—Encourage students to draw a diagram when solving the questions in this section.

Memory—Review with the students the steps required to create a scatter plot using a graphing calculator.

Student Success

Have students do an **Internet search** for real-life examples of where the concepts of this section are/could be used. Students can then share their information using the **timed retell** strategy.

2.

Rectangle	Width (m)	Length (m)	Sum of Length of Three Sides (m)	Area (m^2)
1	1	30	32	30
2	2	28	32	56
3	3	26	32	78
4	4	24	32	96
5	5	22	32	110
6	6	20	32	120
7	7	18	32	126
8	8	16	32	128
9	9	14	32	126
10	10	12	32	120
11	11	10	32	110
12	12	8	32	96
13	13	6	32	78
14	14	4	32	56
15	15	2	32	30

3. a) 8 m × 16 m

b) Answers will vary. Sample solution: The result is consistent with the hypothesis.

c) The hedge will allow Brandon to enclose twice as much space.

4. $\ell = 2w$

5. a) Answers will vary. Sample solution: 10 m × 20 m

b) Answers will vary.

6. Divide the sum by 4 to get the dimension of the width; the length is twice the width.

$$\text{Width} = \frac{\text{sum of the three sides}}{4}$$

$$\text{Length} = 2 \times \text{width}$$

Communicate Your Understanding Responses (page 487)

C1. Create a square field. Use a hedge or a wall to fence one side of the field.

C2. a) When all four edges need to be built/fenced.

b) When three or fewer sides need to be fenced (i.e. one or more sides are already fenced/built).

C3. $\ell = 2w$

Connect and Apply

Question 2 is similar to the Investigate. Point out to students that the amount of light entering the room is dependent on the area of the window.

In question 5, have students focus on the number of pieces of fencing. Use toothpicks to model the pieces. Once the students determine that a square is the best shape, and they should calculate the actual length of each side using the fact that each piece is 2.8 m.

Question 6 is similar to Investigate B since the corral is fenced on only three sides. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3 or BLM T5 *The Geometer's Sketchpad*® 4** to support question 6.

The Chapter Problem (question 7) involves enclosing only two sides of a rectangle.

Question 8 gives students the opportunity to consolidate the results of the various investigations they have carried out (enclosing a rectangle on four, three, and two sides).

Question 9 involves using a table or a spreadsheet to record results. This question is similar to question 6 and Investigate B. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.

The Achievement Check question will be easier for students who have already completed Chapter Problem question 7. You may wish to use **BLM 9.2.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 490)

12. a) The maximum area is 9 m^2. The optimal dimensions are 3 m by 3 m, a square.

b) Again, complete a table from diagrams on paper or using manipulatives:

Rectangle	Width (m)	Length (m)	Fence Used (m)	Area (m^2)
1	1	10	12	10
2	2	8	12	16
3	3	6	12	18
4	4	4	12	16
5	5	2	12	10
6	6	0	12	0

The maximum area is 18 m^2 and this occurs when the dimensions are 3 m by 6 m.

c) The maximum area is 36 m^2 and this occurs when the dimensions are 6 m by 6 m.

Extend

Questions 13 and 14 involve minimizing the perimeter for a fixed area.

The Geometer's Sketchpad® could be used for the investigation in question 15. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Math Contest question 16 requires the application of the Pythagorean theorem. The diagonal of the inscribed square is the diameter of the circle. Solving for the length of the side of the square will require good algebraic skills.

Students who have completed question 7 in Section 9.1 will be more likely to think about a circle as a possibility in question 17. If students have not been assigned this question from the previous section, they may need to be encouraged to think beyond rectangular shapes. Suggest they consider other polygons, for example a hexagon or a dodecagon. This should lead them to realize that a circle is possible.

Exercise Guide

Category	Question Number
Minimum	1, 2, 4, 5, 10
Typical	1–6, 8–11
Extension	13–18

9.3 Minimize the Surface Area of a Square-Based Prism

Strand:
Measurement and Geometry

Student Text Pages
491 to 497

Suggested Timing
80–160 min

Tools
- interlocking cubes
- isometric dot paper

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM 9.3.1 Square-Based Prism Data Recording Table
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 9.3.2 Practice: Minimize the Surface Area of a Square-Based Prism
BLM G8 Isometric Dot Paper
BLM A5 Problem Solving Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

Get Ready questions 3 and 4 review the surface area and volume of prisms that are necessary skills for this section.

Warm-Up

Have students use interlocking cubes to form a prism with a volume of 16 cubic units. Students could share their models with the class to illustrate the different shapes that are possible. Ask students which of these prisms are "square-based." (This section only deals with square-based prisms and students need to get accustomed to this.) Have students predict which shape would have the least surface area. (Surface area calculations are not necessary at this point. These calculations can be left to the Investigate.) This will provide an excellent lead-in to the Investigate, where they determine the shape with the least surface area.

Teaching Suggestions

- Have students work with a partner or in small groups to complete the Investigate. You may wish to use **BLM 9.3.1 Square-Based Prism Data Recording Table** for the Investigate.
- Method 1 uses interlocking cubes and records the results in a table. If interlocking cubes are not available, use cube- or rectangular prism-shaped wrapped candy, such as Jolly Rancher, Starburst, or caramels. Be sure to check for students' allergies before introducing any food into the classroom. (10–15 min)
- Method 2 is similar, but uses a spreadsheet with formulas. If computers are available, this is a good opportunity to have students use a spreadsheet. The technology is advantageous because it allows students to investigate dimensions other than whole numbers. Instruct students to save their spreadsheet for future use in the exercise questions. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity. (20 min)
- Review Examples 1 and 2. (10–15 min) Students may need considerable

Common Errors

- Some students may need assistance in visualizing prisms involved in this section.

R_x Give students many opportunities to work with models for the prisms. Interlocking cubes and empty boxes, such as cereal boxes, juice boxes, tissue boxes, and other cardboard cartons, are very useful for modelling the prisms. Be sure to check for students' allergies before introducing any food containers into the classroom. Students will have more success in understanding the concepts of surface area and volume with these models available. It will also help them understand the practical limitations involved in packaging. (Would a cube-shaped 2-L milk carton be easy to hold?) The models also can be used to examine the extra material required for flaps.

- Some students may have trouble drawing three-dimensional sketches.

R_x Use isometric dot paper, such as **BLM G8 Isometric Dot Paper** to make it easier for students to draw in three dimensions.

Ongoing Assessment

- Chapter Problem question 6 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

help in finding the cube root of a number as required in Example 1. Students who completed some of the more challenging questions from the last chapter (for example, determine the radius of a sphere given its volume) will have better skills in this area.

- Discuss the Communicate Your Understanding questions. Question C2 emphasizes that minimizing surface area is not always practical. (For example, a cube-shaped cereal box would be difficult for a young child to handle.) (5–10 min)
- Assign Practise questions 1 to 4. (10–15 min)
- You may wish to use **BLM 9.3.1 Practice: Minimize the Surface Area of a Square-Based Prism** for remediation or extra practice.

Investigate Answers (page 491)

Method 1

1.

Length	Width	Height	Volume	Surface Area
1	1	16	16	66
2	2	4	16	40
4	4	1	16	48

2. Volume of the prism is 16 cubic units. If the prism has a base with three cubes on each side, the area of the base will be 9 square units. In this case, for the volume to be 16 cubic units, the height of this prism will have to 1.78 units (a fraction). Since, interlocking cubes are whole and cannot be split into fractions, it is not possible to build a prism with this volume that has a square base with three cubes on each side. Below is a set of equation which explains the problem:

Volume = length × width × height

$16 = 3 \times 3 \times \text{height}$

$\frac{16}{9} = \text{height}$

$1.78 = \text{height}$ (which is not possible)

3. The figure with dimensions $2 \times 2 \times 4$ has the minimum surface area. Its shape is the closest to a cube.

4. a) Answers will vary. 4 cubes × 4 cubes × 4 cubes

b)

Length	Width	Height	Volume	Surface Area
1	1	64	64	258
2	2	16	64	136
4	4	4	64	96
8	8	1	64	160

5. 27 cubes are used: 3 cubes × 3 cubes × 3 cubes, 125 cubes are used: 5 cubes × 5 cubes × 5 cubes

6. For a given volume, the square based prism with the minimum surface area is formed when length = width = height.

Method 2

2. Area of square base = $(\text{side length of square base})^2$

3. Consider the following steps:

Volume = length × width × height

length = width (because the base is square)

Volume = $(\text{length})^2 \times \text{height}$

Also,

$$(\text{length})^2 = (\text{area of the square base})$$
$$\text{Volume} = (\text{area of the square base}) \times \text{height}$$
$$64 = (\text{area of the square base}) \times \text{height}$$
$$\frac{64}{(\text{area of the square base})} = \text{height}$$

4. Surface Area of any rectangular prism =
$2 \times (\text{length} \times \text{width} + \text{width} \times \text{height} + \text{height} \times \text{length})$

$SA = 2 \times [(\ell \times w) + (w \times h) + (h \times \ell)]$
$\ell = w$ (because the base is square)
$SA = 2 \times [(\ell^2) + (\ell \times h) + (h \times \ell)]$
$SA = 2 \times (\ell^2) + (4 \times h \times \ell)$
$SA = (2 \times \text{area of the base}) + (4 \times \text{height} \times \text{side length})$
So, $SA = 2*(\text{column B}) + 4*(\text{column A})*(\text{column C})$

5.

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	.064	64	.0258
2	4	.016	64	.0136
3	9	7.111	64	103.332
4	16	.04	64	.096
5	25	2.56	64	101.2
6	36	1.778	64	114.67
7	49	1.306	64	134.57
8	64	.01	64	.016

6. The 4th prism; its dimensions are $4 \times 4 \times 4$. The shape is a cube.

7. a) 5 cm × 5 cm × 5 cm

b)

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	.0125	125	.0502
2	4	31.25	125	.0258
3	9	13.889	125	184.667
4	16	7.813	125	.0157
5	25	.05	125	.0150
6	36	3.472	125	155.333
7	49	2.551	125	169.429
8	64	1.953	125	190.5
9	81	1.543	125	217.556
10	100	1.25	125	.0250

8. a) Answers will vary. Possible answer: 6.7 cm

b)

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	.0300	300	.01202
2	4	.075	300	.0608
3	9	33.333	300	.0418
4	16	18.75	300	.0332
5	25	.012	300	.0290
6	36	8.333	300	.0272
7	49	6.122	300	269.429
8	64	4.689	300	.0278
9	81	3.704	300	295.333
10	100	.03	300	.0320

7 cm × 7 cm × 6.122 cm

c) 6.7 cm × 6.7 cm × 6.7 cm. The shape is a cube.

9. To get the minimum surface area of a square-based prism with a given volume, equate the length, width, and height.

Accommodations

Gifted and Enrichment—Challenge students to calculate third roots, fourth roots, and fifth roots of different numbers using a calculator.

Perceptual—Allow students to use manipulatives or a spreadsheet when working through the questions in this section.

Language—Provide verbal clues for the students as they work through the questions in this section so that they can visualize and record the shapes for the questions in this section.

Memory—Encourage students to use a calculator to determine cube roots.

Student Success

Have students use technology extensively in Sections 9.3 through 9.6.

Communicate Your Understanding Responses (page 495)

C1. Answers will vary. The surface area must be minimized when designing packages and containers to save on heat loss and minimize packaging costs.

C2. Answers will vary. The products may not be cube shaped, they may be long rectangles, and a cube box would be inefficient. Sometimes, a cube-shaped container does not look attractive. Hence, not all boxes are cube-shaped. For example, when creating laptop packaging boxes.

Practise

If the Investigate and Examples 1 and 2 have been covered in class, students should not have difficulty with Practise questions 1–4.

Question 4 involves minimizing heat loss similar to Example 2.

Connect and Apply

In both questions 5 and 9, explain to students that a cube may not always be a practical shape for a container, because it is more difficult for us to handle (as a laundry detergent container and as a juice box). Questions 5 to 7 involve finding the dimensions of a cube with a given volume. In questions 6 and 7, students must determine the surface area of their optimal shape.

Question 8 is an extension of question 7, where the box has no lid.

Question 9 c) provides an opportunity for a Literacy Connection.

Question 10 allows for some student originality. It could be used as an assessment that is somewhat open-ended. You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students.

Extend

Question 12 requires students to design a carton to hold 24 tissue boxes. Students may need help realizing that the tissue boxes should be packed in a “cube-like” configuration. A cube-like configuration is possible here if the boxes are stacked in a 48 cm cube.

Question 13 is similar to the case of the lidless box in question 8.

Question 14 provides a good reminder that extra cardboard is required for the flaps of a box. Students should realize that in most of the previous questions, flaps have been ignored.

Question 15 looks at the flaps of the box in further detail.

Literacy Connections

The Cube

The maximum volume occurs when we have a cube. Have students write a dialogue between two students who are working on this section together, one who thinks this is an obvious concept, and one of who thinks it is very challenging.

Exercise Guide

Category	Question Number
Minimum	1–5
Typical	1–5, 7, 9, 10
Extension	8, 11–15

9.4 Maximize the Volume of a Square-Based Prism

Strand:
Measurement and Geometry

Student Text Pages
498 to 503

Suggested Timing
80–160 min

Tools
- grid paper

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- *The Geometer's Sketchpad*®
- computers

Related Resources
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 9.4.1 Investigate: *The Geometer's Sketchpad*® Method
BLM 9.4.2 Practice: Maximize the Volume of a Square-Based Prism
BLM G10 Grid Paper
BLM 9.4.3 Achievement Check Rubric
BLM A21 Opinion Piece Checklist
BLM A4 Presentation Checklist

Mathematical Process Expectations Emphasis

- ✓ Problem Solving
- ✓ Reasoning and Proving
- ✓ Reflecting
- ✓ Selecting Tools and Computational Strategies
- ✓ Connecting
- ✓ Representing
- ✓ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements
MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);
MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

Assign Get Ready questions 3 and 5 before starting this section, if they have not been completed earlier.

Warm-Up

1. Calculate the surface area and volume of a prism with each set of dimensions.
a) 2 cm by 2 cm by 11 cm **b)** 3 cm by 3 cm by 6.5 cm

2. Compare your answers in question 1. Which shape would be the better choice for a package? Explain your reasoning. Do you think there is a shape that would be even better than the two shapes described? If so, describe the shape.

Warm-Up Answers

1. a) SA = 96 cm^2; V = 44 cm^3 **b)** SA = 96 cm^2; V = 58.5 cm^3

2. Package b) is the better choice as it has greater volume for the same surface area.

Teaching Suggestions

- Assign the Warm-Up questions.
- Review the Warm-Up Answers. Both of these prisms have a surface area of 96 cm^2. Students should determine from their calculations that the 3 cm by 3 cm by 6.5 cm prism has greater volume. Some students may realize that a cube with the same surface area would have even greater volume, but it is not necessary for students to realize this at this point.
- Assign the Investigate. (10–15 min) If the Warm-Up was assigned, less time may be needed.
- Method 2 uses a spreadsheet, if computers are available. (10–15 min) Remind students to save their spreadsheets for future use. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- You may wish to have students use *The Geometer's Sketchpad*® for this activity. If so, use **BLM 9.4.1 Investigate: *The Geometer's Sketchpad*® Method** to support this activity.
- Do Example 1 as a class. (5 min)

Common Errors

- Some students using algebraic methods may not realize that in some questions, the material to be used to form a package comes in a particular shape and so the optimal shape for the package may not be possible.

R_x Give students a sheet of 8.5" by 11" paper (about 21.5 cm by 28 cm) or grid paper and have them cut and tape the paper to form the largest square-based prism possible. You may wish to use **BLM G10 Grid Paper**. This hands-on approach will illustrate that a cube is not always possible if the faces of the prism are made from a single piece of paper. This activity could be also be used for a summative assessment.

Ongoing Assessment

Use Achievement Check question 9 to monitor student success. See the Achievement Check Answers and **BLM 9.4.3 Achievement Check Rubric**.

Chapter Problem question 8 can also be used as an assessment tool.

Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Assign the Communicate Your Understanding questions to students as a think-pair-share activity. (5 min)
- Assign the Practise questions.
- Use **BLM 9.4.2 Practice: Maximize the Volume of a Square-Based Prism** for extra practice or remediation.

Investigate Answers (page 498)

Method 1

1.

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	24	5.5	5.5
2	2	4	24	2.0	8.0
3	3	9	24	0.5	4.5

2. Prism 2 has the maximum volume. Its shape is a cube.

3. a) Answers will vary. Possible answer: Prism 3. Dimensions: 3 cm × 3 cm × 3 cm

b)

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	54	13.0	13
2	2	4	54	5.75	23
3	3	9	54	3.0	27
4	4	16	54	1.375	22
5	5	25	54	0.2	5

4.

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	2	4	96	11.0	44.0
2	3	9	96	6.5	58.5
3	4	16	96	4.0	64.0
4	5	25	96	2.3	57.5
5	6	36	96	1.0	36.0

5. The maximum volume of a square based prism with a given surface area is achieved when a cube is formed.
length = width = height

Method 2

2. The formula for height is found by rearranging the formula for surface area.

$$2 \times (\text{area of base}) + (4 \times s \times h) = 24$$

$$4 \times s \times h = 24 - [2 \times (\text{area of base})]$$

$$h = \frac{24 - [2(\text{area of base})]}{4(\text{side length of base})}$$

3. 2 cm × 2 cm × 2 cm. This cube has the greatest volume at 8 cm^3.

4. a) Answers will vary. Possible answer: Cube with 3 cm sides.

b)

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	54	13.0	13
2	2	4	54	5.75	23
3	3	9	54	3.0	27
4	4	16	54	1.375	22
5	5	25	54	0.2	5

Accommodations

Gifted and Enrichment—Challenge students to determine why some companies do not use rectangular prism shapes for packaging that are not the most economical.

Visual—Work with groups of students to help them to solve equations sequentially using steps.

Spatial—Encourage students to draw scale diagrams to represent the shapes in this question.

Motor—Encourage students to work in groups when completing the questions in this section.

Memory—Encourage students to create more “formula cue-cards” to record the new formulas that they have learned in this Chapter.

5. a) Answers will vary. Possible answer: Cube with 4 cm sides.

b)

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	96	23.5	23.5
2	2	4	96	11.0	44.0
3	3	9	96	6.5	58.5
4	4	16	96	4.0	64.0
5	5	25	96	2.3	57.5
6	6	36	96	1.0	36.0

6. The maximum volume of a square-based prism with a given surface area is achieved when a cube is formed.
length = width = height

Communicate Your Understanding Responses (page 501)

C1. Answers will vary. Possible answer: Packaging companies need to minimize the materials used on packaging products by maximizing the volume.

C2. We know that the maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube. Box C is a cube and hence has the greatest volume.

Practise

Question 1 is similar to Communicate Your Understanding question C2.
Questions 2 and 3 are similar to the Example.

Connect and Apply

Have students use their spreadsheets from Investigate: Method 2 to complete Connect and Apply questions 4 to 9. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity. These questions also could be done using algebraic techniques like Example 1. Note to students that extra material required for seams and flaps in the containers will be ignored for all calculations in all the questions.

The Chapter Problem question 8 involves shipping an item in the largest box that can be made from 2500 cm^2 of cardboard. The box will be a cube, and students are required to calculate the empty space in the box.

The Achievement Check question 9 involves making a box with and without a lid. This will require students to alter the algebra in their solutions, dividing the given surface area by 6 for the box with a lid, but by 5 for the box without a lid. The answers in this case will require rounding.

Achievement Check Answers (page 503)

9. a) 1500 cm^2

b) Using the fact that the maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube, the area of each square face is $\frac{1500}{6}$ cm^2 or 250 cm^2.
Therefore, the dimensions of each face are 15.8 cm by 15.8 cm.
$(\sqrt{250} \doteq 15.811\ldots)$

c) Set up a table to explore this relationship:

Box	Width (cm)	Length (cm)	Height (cm)	Area (cm^2)	Volume (cm^3)
1	.010	.010	.035	1500	.03500
2	.020	.020	13.8	1500	.05520
3	.030	.030	.05	1500	.04500
4	.025	.025	8.8	1500	.05500
5	.021	.021	12.6	1500	.05557
6	.022	.022	11.5	1500	.05588
7	.023	.023	10.6	1500	.05583
8	22.5	22.5	11.0	1500	.05590
9	22.6	22.6	10.9	1500	5589.2
10	**22.4**	**22.4**	**11.1**	**1500**	**5590.14**
11	22.3	22.3	11.2	1500	5590.10

The maximum volume is reached by making the base 22.4 cm on a side.

d) Answers may vary. For example: ignore material for seams or overlap flaps.

Extend

In Question 10 there is one sheet of plywood to be used. It is not possible to use all the plywood and make the box a cube with a lid with each face made from a single piece of wood. If each face is 60 cm by 60 cm, there will be 60 cm by 120 cm, or 7200 cm^2, of plywood wasted. Some students may decide to piece the faces together to form a bigger box. Discuss how the pieces of wood might be joined. This question is designed to be open-ended in this way. Ensure that students identify any assumptions they are making in their solutions.

Question 11 is similar to question 10. If the box is made with a lid, it can be formed by using six 10 cm by 10 cm faces and all the stained glass will be used. If the box has no lid, the only way to create a bigger box than a 10 cm cube is to piece the faces together. In this case, the box is being made out of stained glass, so students may argue that it is reasonable to assume that pieces of glass can be joined together to form a face for the box. It is possible to get a bigger box without a lid if this is done. Any method is acceptable as long as students realize the assumptions that are being made.

You may wish to use **BLM G10 Grid Paper** for questions 10 and 11.

Literacy Connections

Opinion Piece

Assign the following activity. Write an opinion piece, a series of paragraphs expressing your opinion, on the topic below. Develop your main idea with supporting details. Be sure to write your opinion piece with your audience in mind, an adult who is interested in your opinion.
Topic: The cylinder is the best shape for a beverage container.

Review students' opinion pieces, and ensure that students have included three paragraphs: an introduction, the body, and a conclusion. Their opinion should be clearly stated and students should provide details with reasons, examples, and facts to support their opinions. You may wish to use **BLM A21 Opinion Piece Checklist** to assess the students. You may also wish to provide an opportunity for students to present their opinion pieces to the class. Use **BLM A4 Presentation Checklist** to assess students' presentations.

Exercise Guide

Category	Question Number
Minimum	1, 2, 3, 5
Typical	1–7
Extension	10, 11

9.5 Maximize the Volume of a Cylinder

Strand:
Measurement and Geometry

Student Text Pages
504 to 509

Suggested Timing
80 min

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM 9.5.1 Cylinder Data Recording Table
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM A18 My Progress as a Problem Solver
BLM 9.5.2 Practice: Maximize the Volume of a Cylinder

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

The Get Ready questions involving cylinders provide the needed skills for this section. Have students complete Get Ready questions 4 and 6 before starting this section.

Warm-Up

1. Determine the volume and surface area of a cylinder with radius 6 cm and height of 20 cm.

2. Determine the height of a cylinder with a surface area of 375 cm^2 and a radius of

a) 7 cm **b)** 5 cm

Warm-Up Answers

1. $V = 2261.9$ cm^2; $SA = 980.18$ cm^2

2. a) 1.53 cm **b)** 6.94 cm

Teaching Suggestions

- The Investigate Method 1 requires algebraic skills to isolate the height in the formula for surface area of a cylinder. Work through the first step of the Investigate as a class. (Rearranging formulas was done previously in Section 4.4.)
- Review the necessary algebra skills. (5 min)
- Assign the rest of the Investigate. (15–20 min)
- Students can use copies of **BLM 9.5.1 Cylinder Data Recording Table** to record their results.
- Alternatively, once students understand the rearranging of the formula, have them use a spreadsheet and complete Method 2. If students are familiar with spreadsheets, this method will be advantageous. It will take less time and allow students to complete the Investigate for different surface areas. Remind students to save their spreadsheets for future use. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- Follow up the Investigate (Method 1 or 2) with a class discussion to ensure that students realize that a cylinder with a height equal to its diameter has the largest volume for a given surface area.

Common Errors

- Some students may struggle with the algebraic skills required in this section.

R_x Once you have led your students through the algebraic skills necessary for the Investigate, you may want to keep the focus on the results. It is important that students understand that the optimal shape for the cylinder with a given surface area is one with a height equal to its radius. Again, once you have led your students through the algebraic technique in the Example, ensure that they have the key concept $SA = 6\pi r^2$. Many of your students will have no trouble understanding that this becomes $SA = 5\pi r^2$ when the cylinder has no lid since the one circular lid (with area πr^2) has been removed.

Ongoing Assessment

- Chapter Problem question 5 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Review Example 1 as a class. (5–10 min)
- Discuss the Communicate Your Understanding questions as a class. (5 min)
- You may wish to use **BLM A18 My Progress as a Problem Solver** as a self-assessment for students as they work through the Investigate.
- Use **BLM 9.5.2 Practice: Maximize the Volume of a Cylinder** for extra practice or remediation.

Investigate Answers (page 504)

Method 1

1.

Step	Explanation
$SA = 2\pi r^2 + 2\pi rh$	
$375 = 2\pi r^2 + 2\pi rh$	Substitute $SA = 375$
$375 - 2\pi r^2 = 2\pi r^2 + 2\pi rh - 2\pi r^2$	Subtract $2\pi r^2$ from both sides.
$375 - 2\pi r^2 = 2\pi rh$	Simplify.
$\frac{375 - 2\pi r^2}{2\pi r} = \frac{\cancel{2\pi r}h}{\cancel{2\pi r}}$	Divide both sides by $2\pi r$.
$h = \frac{375 - 2\pi r^2}{2\pi r}$	Simplify to isolate h.

2. a) $h = \frac{375 - 2\pi}{2\pi} = \frac{375}{2\pi} - 1 \approx 58.68$ cm

b) $V = 184.36$ cm^3

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	58.68	184.36	375

3.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
2	27.84	349.87	375
3	16.89	477.68	375
4	10.92	548.94	375
5	6.94	544.80	375
6	3.95	446.42	375
7	1.53	234.93	375

4. Maximum volume = 548.94 cm^3, height = 10.92 cm, radius = 4 cm

5. Not necessarily. Extend the investigation to use decimal values for the radius.

Method 2

2.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	58.68	184.36	375
2	27.84	349.87	375
3	16.89	477.68	375
4	10.92	548.94	375
5	6.94	544.80	375
6	3.95	446.42	375
7	1.53	234.93	375

When the radius is 8 cm, the surface area of the top and bottom is 402.12 cm^3, which is greater than the entire surface area of the cylinder. Therefore, to compensate, the height becomes negative.

3. Maximum volume at 4 cm radius. Yes. Using decimals to the nearest tenths, the greatest volume is 557.47 cm^3 when the radius is 4.5 cm.

4. Volume = 557.539 cm^3, radius = 4.46 cm

5. a) radius = 4.46 cm, height = 8.92 cm

b) $r = \frac{h}{2}$ **c)** diameter = height

6. Volume = 858.36 cm^3, radius = 5.15 cm, height = 10.30 cm. The radius is half the height.

7. Volume = 2427.88 cm^3, radius = 7.28 cm, height = 14.58 cm. The radius is half the height.

8. a) $r = \frac{h}{2}$

b) In a square based prism, the maximum volume for a fixed surface area occurs when all the dimensions of are equal (length = width = height). However, in a cylinder, the maximum volume for a fixed surface area occurs when the radius of the base equals half the height or the diameter of the base equals the height.

Communicate Your Understanding Responses (page 508)

C1. Answers will vary. Sample solution: It will be necessary to maximize the volume of a cylinder, given its surface area when trying to find the dimensions for oxygen cylinders to be kept in rockets going into the space.

C2. Cylinder B has the greatest volume because it is the one whose height is the closest to its diameter.

C3. Answers will vary. Glasses with the maximum volume are not very practical because they are hard to hold. Also, an important consideration is how the glasses appear to the customer. The customer would want the glasses to look elegant and appear to hold more liquid.

Accommodations

Perceptual—Encourage students to colour-code "like terms" when simplifying the formula for surface area.

Motor—Let students work together and use a spreadsheet to complete the questions requiring technology in this section.

Language—Allow students to use a metric converter to change the units from cm to mm, etc.

Memory—Review with the students the steps required to find a square root on a calculator.

Practise

Practise questions 1 and 2 are similar to the Example, so students should have few problems completing these on their own.

Connect and Apply

If Method 2 was used for the Investigate, have students use their spreadsheets to complete some of the questions. Ensure that they can also solve the questions algebraically.

The Chapter Problem question 5 requires students to determine the best height for the cylinder, and then determine how many CDs can be stacked in a cylinder of this height. Students might describe assumptions about the amount of empty space allowed in the cylinder.

Question 6 is an extension to the previous questions with an open-topped cylinder. Advanced students will be able to handle the implications this makes on the algebra involved, but other students may require assistance with this.

Extend

Question 7 compares the volume of a square-based prism and a cylinder made with the same amount of sheet metal. Again, students who have created a spreadsheet for Investigate could use these spreadsheets to determine the volume of each container. Ensure that students hypothesize their answer before performing the calculations.

Question 8 extends the possibilities to include a sphere. This is a great extension question, especially for advanced students. Remind students about the formulas for the surface area and volume of a sphere (Making Connections, page 509).

Question 9 is an appropriate extension for those students who have created the spreadsheet for a cylinder already. In this question, students would alter the formulas in their spreadsheet for a cylinder with no lid.

Exercise Guide

Category	Question Number
Minimum	1–3
Typical	1–4, 6
Extension	7–10

9.6 Minimize the Surface Area of a Cylinder

Strand:
Measurement and Geometry

Student Text Pages
510 to 515

Suggested Timing
80 min

Tools
- construction paper
- rulers
- scissors
- tape

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 9.6.1 Practice: Minimize the Surface Area of a Cylinder
BLM 9.6.2 Achievement Check Rubric
BLM A23 News Report Checklist
BLM A4 Presentation Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements
MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];
MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);
MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

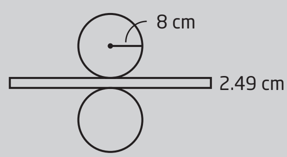
The Get Ready questions involving cylinders provide the needed skills for this section. Have students complete Get Ready questions 4 and 6 before starting this section.

Warm-Up

1. Determine the height of a cylinder with a volume of 500 cm^3 and a radius of 8 cm.
2. Draw a net for the cylinder in question 1.

Warm-Up Answers

1. 2.49 cm

2. 

Teaching Suggestions

- Have students work in small groups for the Investigate. (20–25 min)
- Alternatively, have students work with a partner within groups of six students, using a different radius to form one cylinder with a volume of 500 cm^3. Then, have the three partner groups record their results in a table as a group. (10 min)
- Students who have used spreadsheets several times through this chapter will be comfortable with Method 2. You may want to go through the algebraic manipulation necessary to develop the formulas with them. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- Discuss the Example. (10 min) Students should realize from the Investigate that a cylinder with a height equal to its diameter is the optimal shape for a given volume.

Common Errors

- Some students may continue to struggle with the algebraic skills necessary for this section.

R_x Encourage students to use spreadsheets to investigate the optimal shape for the cylinder. It is important that they recognize this optimal shape, even if they have trouble performing the algebraic skills.

Ongoing Assessment

- Use Achievement Check question 10 to monitor student success. See Achievement Check Answers and **BLM 9.6.2 Achievement Check Rubric**.
- Chapter Problem question 8 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Assign and discuss the Communicate Your Understanding questions. (5 min)
- Assign Practise questions 1 to 3.
- You may wish to assign **BLM 9.6.2 Practice: Minimize the Surface Area of a Cylinder** for remediation or extra practice.

Investigate Answers (page 510)

Method 1

1. Answers will vary. Possible answer: radius = 4.3 cm, to the nearest tenth, area = 58.09 cm^2

2. Answers will vary. Possible answer: height = 8.6 cm, to the nearest tenth

3. Answers will vary. Possible answer: circumference = 27.02 cm, to the nearest hundredth

5. a) Answers will vary. Possible answer: area of rectangle = 232.42 cm^2, to the nearest hundredth

b) Answers will vary. Possible answer: 348.67 cm^2, to the nearest hundredth

6.

Cylinder	Radius (cm)	Base Area (cm^2)	Height (cm)	Surface Area (cm^2)
1	4.3	58.088	8.608	348.734

7.

Cylinder	Radius (cm)	Base Area (cm^2)	Height (cm)	Surface Area (cm^2)
2	2.0	12.566	39.789	525.133
3	2.5	19.635	25.465	439.270
4	3.0	28.274	17.684	389.882
5	3.5	38.485	12.992	362.683
6	4.0	50.265	9.947	350.531
7	4.5	63.617	7.860	349.457
8	5.0	78.540	6.366	357.080

8. Cylinder 1 has the least surface area. Height = diameter.

9. For a given volume, the cylinder whose height is equal to the diameter has the least surface area.

Method 2

1.

Cylinder	Radius (cm)	Base Area (cm^2)	Volume (cm^3)	Height (cm)	Surface Area (cm^2)
1	1	3.142	500	159.155	1006.283
2	2	12.566	500	39.789	525.133
3	3	28.274	500	17.684	389.882
4	4	50.265	500	9.947	350.531
5	5	78.540	500	6.366	357.080
6	6	113.097	500	4.421	392.861
7	7	153.938	500	3.248	450.733

Accommodations

Gifted and Enrichment—Challenge students to investigate the food products contained in cylinders and rectangular prisms.

Perceptual—Allow students to work in pairs when completing the questions in this section.

Memory—Remind students to use colour-coding to add "like terms" such as $SA = 2\pi r^2 + 4\pi r^2$
$= 6\pi r^2$
and to use small sequential steps when solving equations.

ESL—Encourage students to work with a partner when working through the questions in this section.

Student Success

Assign the concepts of Sections 9.3 through 9.6 to groups. Each group investigates real-life applications, then shares with the other groups using a **carousel** strategy.

2. radius = 4 cm (whole-numbered radius).

Cylinder	Radius (cm)	Base Area (cm^2)	Volume (cm^3)	Height(cm)	Surface Area(cm^2)
1	3.5	38.485	500	12.992	362.683
2	3.6	40.715	500	12.280	359.208
3	3.7	43.008	500	11.626	356.287
4	3.8	45.365	500	11.022	353.887
5	3.9	47.784	500	10.464	351.978
6	4.0	50.265	500	9.947	350.531
7	4.1	52.810	500	9.468	349.523
8	4.2	55.418	500	9.022	348.931
9	4.3	58.088	500	8.608	348.734
10	4.4	60.821	500	8.221	348.915

3. radius = 4.3 cm and the height = 8.6 cm, or 2 × radius

4. radius = 5.3 cm and the height = 10.65 cm ≈ 2 × radius

5. radius = 6 cm and the height = 12.02 cm ≈ 2 × radius

6. The cylinder whose height is equal to the twice the radius has the minimum surface area.

Communicate Your Understanding Responses (page 513)

C1. Answers will vary. Sample answer: constructing a cylindrical container for a quantity of liquid.

C2. Cylinder B has the least surface area because its height is equal to its diameter.

Practise

Practise questions 1 to 3 are similar to the Example. Students should not have many problems completing these questions.

Connect and Apply

For questions 4 and 5, remind students that 1 L = 1000 cm^3. For question 6, have students use their spreadsheets from the previous sections to justify their answers.

Conduct a class discussion for question 7.

The Achievement Check question 10 is a variation of question 4 from Section 9.5. Use **BLM 9.6.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 515)

10. a) Note that 600 L = 600 000 cm^3.
If the radius is 20 cm, the area of the end is
$\pi(20)^2 \doteq 1256.6$
Then, the height of the cylinder is $\frac{600\ 000}{1256.6}$ cm, or about 477.5 cm.
If the radius is 30 cm, the area of the end is
$\pi(30)^2 \doteq 2827.4$
Then, the height of the cylinder is $\frac{600\ 000}{2827.4}$ cm, or about 212.2 cm.

b) For an optimal design, the height must equal the diameter of the cylinder.

$600\ 000 = 2\pi r^3$

$r = \sqrt[3]{\dfrac{600\ 000}{2\pi}}$

$r \doteq 45.7$

The radius of the cylinder should be 45.7 cm and the height 91.4 cm.

c) The optimal rectangular shape is a cube. The side length would have to be $\sqrt[3]{600\ 000}$ cm, or about 84.3 cm. Both shapes have the same capacity. The surface area of the cube is 46 638 cm^2. The surface area of the cylinder is 26 244 cm^2. The cylinder shape requires less material because its surface area is less. The cylinder may also be preferred since a curved shape is more aerodynamic.

Extend

Question 11 involves a cylinder without a lid. Students will find this question easier if they have completed the lidless cylinder questions 6 and 9 from Section 9.5.

Question 12 will be easier for students who have already completed question 8 from Section 9.5.

The Math Contest questions 13 to 16 extend and consolidate the concepts of this section and Section 9.5.

Literacy Connections

News Report

Assign the following activity. Look at the picture at the beginning of Section 9.6. Imagine that the headline reads: "Acme Company Cans Food in All Sizes!" Write a news report about the photo and the given headline. Remember to have a link between the photo and the headline. Try to answer the questions who, what, where, when, why, and how in your news report. Also, remember to write in the third person.

Review students' news reports, and ensure that students have answered the questions who, what, where, when, why, and how, and written in the third person. You may wish to use **BLM A23 News Report Checklist** to assess your students. You may also wish to provide an opportunity for students to present their news reports to the class. Use **BLM A4 Presentation Checklist** to assess students' presentations.

Exercise Guide

Category	Question Number
Minimum	1, 2, 3, 4
Typical	1–9
Extension	11–16

Chapter 9 Review

Student Text Pages
516 to 517

Suggested Timing
80 min

Related Resources
BLM 9.CR.1 Chapter 9 Review

Ongoing Assessment

- Have students create a summary sheet of the skills in the chapter. Check this list holistically before assigning the Chapter Test.
- Upon completing the Chapter Review, have students answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related section in the text for Examples or Key Concepts? For which questions was this necessary?*

Using the Chapter Review

Each question on **BLM 9.CR.1 Chapter 9 Review** reviews different skills and concepts. Have students work independently to complete the Chapter Review, then with a partner to compare solutions. Alternatively, assign the Chapter Review for reinforcing skills and concepts in preparation for the Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult.

After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content.

Chapter 9 Practice Test

Student Text Pages
518 to 519

Suggested Timing
60 min

Related Resources
BLM 9.PT.1 Chapter 9 Practice Test
BLM 9.CT.1 Chapter 9 Test

Summative Assessment

- After students complete **BLM 9.PT.1 Chapter 9 Practice Test**, you may wish to use **BLM 9.CT.1 Chapter 9 Test** as a summative assessment.

Accommodations

Gifted and Enrichment—Challenge students to prepare an extra Chapter Test for their classmates.

Motor—Allow students to do fewer questions in this section.

Memory—Let students use a formula sheet when working through the questions in the Chapter Test.

ESL—Encourage students to use their translators when completing the questions in this section.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	9.2	Investigate B (page 486)
2	9.3	Example 1 (page 493)
3	9.6	Example (page 512)
4	9.4	Example 1 (page 500)
5	9.2	Investigate A (page 484)
6	9.3, 9.6	Example 1, Example (pages 493, 512)
7	9.3	Example 1 (page 493)
8	9.6	Example (page 512)
9	9.3, 9.6	Example 1, Example (pages 493, 512)

Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?

- Model the areas of rectangles with the same perimeter using geoboards, grid paper, *The Geometer's Sketchpad*®
- Model the perimeter of rectangles with the same area using geoboards, grid paper, toothpicks, *The Geometer's Sketchpad*®
- Conduct an investigation to determine the dimensions of the largest rectangles that can be enclosed (on four, three, or two sides) by a given perimeter by using appropriate techniques (manipulatives, spreadsheet, graphing calculator, or *The Geometer's Sketchpad*®)
- Conduct an investigation to determine the dimensions of the rectangle with a given area that can enclosed (on four, three, or two sides) by the least amount of fencing by using appropriate techniques (manipulatives, spreadsheet, graphing calculator, or *The Geometer's Sketchpad*®)
- Solve problems that involve maximizing the area of a rectangle for a given perimeter
- Solve problems that involve minimizing the fencing to enclose a fixed area (on four, three, and two sides)
- Model the surface area of square-based prisms with a fixed volume using interlocking cubes
- Conduct an investigation to determine the dimensions of the square-based prism of a given volume that has minimal surface area by using appropriate techniques (pencil and paper, spreadsheet)
- Conduct an investigation to determine the dimensions of the square-based prism with the largest volume for a given surface area by using appropriate techniques (pencil and paper, spreadsheet)
- Solve problems that involve maximizing the volume or minimizing the surface area of square-based prisms
- Conduct an investigation to determine the dimensions of the cylinder with the largest volume for a given surface area by using appropriate techniques (pencil and paper, spreadsheet)

- Conduct an investigation to determine the dimensions of the cylinder of a given volume that has minimal surface area by using appropriate techniques (pencil and paper, spreadsheet)
- Solve problems that involve maximizing the volume or minimizing the surface area of cylinders
- Minimize the amount of material in packaging problems that involve square-based prisms and/or cylinders

Chapter 9 Problem Wrap-Up

Student Text Pages
519

Suggested Timing
30–60 min

Related Resources
BLM 9.CP.1 Chapter 9 Problem Wrap-Up Rubric

Summative Assessment

Use **BLM 9.CP.1 Chapter 9 Problem Wrap-Up Rubric** to assess student achievement.

Using the Chapter Problem

The Chapter Problem is designed to incorporate the skills with both cylinders and prisms. Students design a cylindrical container to hold CDs, and then a square-based carton that will hold a number of these cylinders for shipping. This problem can be done individually or with a partner. It can also be used as an assessment piece, either completed in class or at home.

The size of the carton is left up to the student, but the customer needs at least 750 CDs. Students must design two different cartons. Finally, they are asked to recommend the number of CDs that could be packaged in the carton to minimize the packaging.

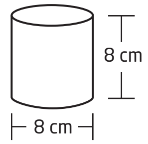
Students will find the design of the cylinder easier to handle if they have previously been assigned question 5 from Section 9.5. Designing the carton may be easier if students have completed question 12 from Section 9.3. (This question involved packing tissue boxes inside a carton using the least amount of material.)

Teaching Suggestions

- The Chapter Problem will have a variety of solutions depending on the student's choice of carton in c).
- Some students may have difficulty providing diagrams in part d). Provide a brief review of perspective drawing or isometric views (top, side, front views).
- Provide concrete materials from which to build models of the situation (e.g., boxes, identical cans, thread spools, cardboard, scissors, tape, etc.).
- If the lead-up problems have been assigned throughout the chapter, review each of them, emphasizing the skills and formulas that were used. Conduct the review as a class or in small groups, perhaps using a jigsaw technique. (30–40 min)
- Provide sufficient time for students to think about this problem. This will lead to clearer, more complete solutions from students. If some students finish quickly, encourage them to consider other packaging options and to compare quantities of cardboard used.
- Due to the complexity of this problem, provide an opportunity to brainstorm approaches to the problem, with partners or in groups, before they begin. One strategy is to allow students to discuss the problem but not to write anything down until they begin individual work. Another strategy is to introduce the problem one day, but not assign it to be completed until another day.

Level 3 Sample Response

a) We learned that the minimum surface area cylinder is created when the height equals the diameter. Therefore, she should make a container 8 cm in diameter and 8 cm in height.

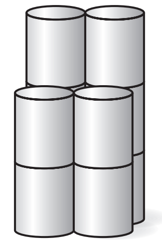


b) Each package will contain $\frac{8}{0.1} = 80$ CDs.

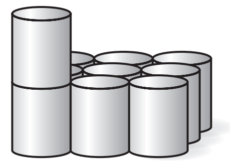
c) Talia will need 10 cylindrical packages to complete the order of 750 CDs, with the last package containing only 30 CDs.

d) Since the carton has a square base, there will be either 4, or 9, cylinders on the bottom layer. One option is a box with two layers of 4 cylinders and a third layer that has only 2 cylinders of CDs.

The dimensions of this cardboard carton are
$2 \times 2 \times 3$ cylinders,
which is 16 cm × 16 cm × 24 cm.



The other option has 9 cylinders on one layer and 1 on the second layer. This is the least practical option.



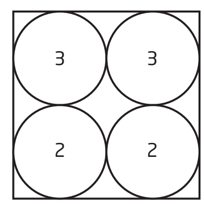
The dimensions of this cardboard carton are
$3 \times 3 \times 2$ cylinders, which is
24 cm × 24 cm × 16 cm.

e) This order had some empty space in the carton. It would be better to order so that a cube box is used. That means 1, 8, 27, etc. packages of CDs. In CDs, this is 80, 640, 2160, etc.

Level 3 Notes

Look for the following:

- A complete solution to most parts of the problem
- Minor computational errors
- Diagrams contain enough detail to demonstrate intended packaging
- Most steps in the solution have justification
- Carton size (correct) may be chosen without reference to other options



What Distinguishes Level 2

At this level, look for the following:

- A few parts of the problem have complete solutions
- Major computational errors
- Diagrams are present but unclear or unrelated to the dimensions of the problem
- Little or no justification for the steps in the solution
- Carton size chosen may not have a square base

What Distinguishes Level 4

At this level, look for the following:

- A complete solution to all parts of the problem
- No computational errors
- Diagrams clearly show how the packages will be placed in the carton, perhaps with multiple views
- Diagrams will be drawn to scale
- May include comments about the limits to the size of the carton due to weight restrictions of the shipping company
- A clear and concise justification is provided for all steps in the solution
- Choice of carton size will be fully justified with reference to alternate choices
- Solution may include extra space in packaging to allow materials to move in and out easily (thus complicating calculations)

Chapters 7 to 9 Review

Student Text Pages
520 to 521

Suggested Timing
80 min

Related Resources
BLM A14 Self-Assessment Recording Sheet
BLM A15 Self-Assessment Checklist

Ongoing Assessment

- This is an opportunity for students to assess themselves by completing selected questions and checking their answers against the answers in the back of the student text. They can then revisit any questions with which they had difficulty.
- Upon completing the Chapters 7 to 9 Review, students can also answer questions such as the following:
  - Did you work by yourself or with others?
  - What questions did you find easy? Difficult? Why?
  - How often did you have to check the related Example in the student text to help you with the questions? For which questions was this necessary?

Using the Chapters 7 to 9 Review

Each question reviews different skills and concepts. Have students work independently to complete the Chapters 7 to 9 Review, then with a partner to compare solutions. Alternatively, assign the Chapters 7 to 9 Review to reinforce skills and concepts in preparation for the specific chapter Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult.

After they complete the Chapters 7 to 9 Review, encourage students to discuss any questions, consider alternative strategies, and ask about questions with features they find difficult. You may wish to use **BLM A14 Self-Assessment Recording Sheet** or **BLM A15 Self-Assessment Checklist** to help students assess their understanding.

Task The Horse Barn

Strand:
Measurement and Geometry

Student Text Pages
522

Suggested Timing
20 min

Related Resources
BLM 9.T1.1 Task: The Horse Barn Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Values of Measurements

MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Teaching Suggestions

- Most students will expect the pen to be a square, however the conditions here do not match the examples studied in the chapter. Caution students to complete all parts of the question to draw an appropriate conclusion.
- This question is equivalent to problems studied in the chapter. To illustrate this, invent an imaginary extra 20 m of fencing to cover the opening of the barn. Now the problem becomes to find the maximum area rectangle for a given fixed perimeter of 220 m. This is a square with side 220 ÷ 4 or 55 m.
- As a class, or in small groups, review similar questions that have been covered in chapter 8 and 9, emphasizing the skills and formulas that were used. (20 min)

Ongoing Assessment

- Use **BLM 9.T1.1 Task: The Horse Barn Rubric** to assess student achievement.

Level 3 Sample Response

a) $A = 40 \times 70$, so the area is 2800 m^2

b) Width $= 40 + x$

c) Length $= \dfrac{200 - (60 + 2x)}{2}$

d)

Pen Number	x (m)	Pen Width (m)	Pen Length (m)	Area (m^2)
1	0	40	70	2800
2	5	45	65	2925
3	10	50	60	3000
4	15	55	55	3025
5	20	60	50	3000
6	25	65	45	2925

e) The optimal pen is 55 m by 55m, which has an area of 3025 m^2.

f) This is the pen with the greatest area. When x is either larger or smaller than 15 m, the area of the pen is less than 3025 m^2.

Level 3 Notes

Look for the following:

- Clear understanding of area and perimeter concepts
- Clear understanding of how to represent lengths with polynomials
- Evidence of knowledge of all possible rectangular configurations
- Mostly accurate calculations for perimeter and area in chart
- Mostly accurate substitution of values for x
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some understanding of area and perimeter concepts
- Some understanding of how to represent lengths with polynomials
- Evidence of knowledge of some of the possible rectangular configurations
- Somewhat accurate calculations for perimeter and area in chart
- Somewhat accurate substitution of values for x
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Very clear understanding of area and perimeter concepts
- Very clear understanding of how to how to represent lengths with polynomials
- Evidence of knowledge of all possible rectangular configurations
- Accurate calculations for perimeter and area in chart
- Accurate substitution of values for x, and observation that areas peak at $x = 15$
- Use of very good form and correct mathematical notation
- May include explanation that this is really similar to previously solved problems

Task — The Ice Rink

Strand:
Measurement and Geometry

Student Text Pages
523

Suggested Timing
20 min

Tools
- grid paper

Related Resources
BLM G10 Grid Paper
BLM 9.T2.1 Task: The Ice Rink Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.03 solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles);

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \frac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \frac{(\text{area of base})(\text{height})}{3}$;

Teaching Suggestions

Review, as a class or in small groups, some of the area and volume formulas and the results for similar questions that were covered in chapters 8 and 9.

Make available concrete materials such as toothpicks and graph paper. You may wish to use **BLM G10 Grid Paper**. Graphing calculators and/or computers would also be helpful. Scientific calculators are a necessity.

Clarify that the barrier sections available to the rink builders are of fixed length and cannot be cut or shortened. This may be a new concept for some students as most of the previous problems have involved lengths that were functions of a continuous variable, not a discrete variable.

Encourage students to refine their solutions, providing more complete explanations for their reasoning.

Level 3 Sample Response

a) Possible solutions include 3 m by 42 m, 6 m by 39 m, 9 m by 36 m, 12 m by 33 m, etc.

b) Using algebra, if x represents the width then $2x$ represents the length. Then, $x + 2x + x + 2x = 30$, so $x = 5$. The dimensions are 15 m by 30 m. This result can also be obtained from examining a complete table of possibilities (see below).

c) No. This can be found by using a chart of all possible shapes or else by using algebra. ($4x = 30$, then $x = 7.5$), which is impossible since the barrier sections are indivisible 3-m lengths.

d) The rink with the greatest area measures 21 m by 24 m. This can be found by creating a chart (see below) or else reasoning that a square has the greatest area for a fixed perimeter and this shape is the closest you can get to a square for this question.

Length (m)	Width (m)	Area (m^2)
3	42	126
6	39	234
9	36	324
12	33	396
15	30	450
18	27	486
21	24	504
24	21	504
27	18	486
…	…	…

e) Volume = volume of rink + volume of paddling pool

$$V = \ell wh + \frac{1}{3}\pi r^2 h$$
$$= (2400)(2100)(3) + \frac{1}{3}\pi \times 500^2 \times 30$$
$$\doteq 38\ 681\ 944.9$$

The volume of water needed is 38 681 944.8 cm^3, or 38 682 L to the nearest litre.

Ongoing Assessment

- Use **BLM 9.T2.1 Task: The Ice Rink Rubric** to assess student achievement.

Level 3 Notes

Look for the following:

- Correct answers to parts a), b), and c)
- A complete chart of possible rink sizes
- An appropriate plan for parts d) and e) with partially complete solutions
- Minor computational errors
- Most steps in the solution have justification
- Mostly correct selection of volume formulas
- Mostly correct units of measure

What Distinguishes Level 2

At this level, look for the following:

- Some answers to a), b), or c) are missing
- An incomplete chart of possible rink sizes
- A minimally appropriate plan for parts d) and e) with an incomplete attempt at solutions
- Major computational errors
- Few steps in the solution have justification
- Incorrect selection of volume formulas
- Few, if any, correct units of measure
- May not consider changing units before computing volume
- May be completed as though each barrier were 1 m long

What Distinguishes Level 4

At this level, look for the following:

- Correct answers to parts a), b), and c). For part c), there may be an explanation of how to create a square rink using less than all of the barriers
- A complete chart of possible rink sizes
- An appropriate plan for parts d) and e) with complete solutions
- Very few computational errors
- All steps in the solution have justification
- Correct selection of volume formulas
- Correct units of measure, including a choice of an appropriate unit of volume for part e)
- May comment on the fact that less water will actually be needed because water expands when it freezes

Task Packing Compressed Air

Strand:
Measurement and Geometry

Student Text Pages
523

Suggested Timing
20 min

Related Resources
BLM 9.T3.1 Task: Packing Compressed Air Rubric

Mathematical Process Expectations Emphasis

- ✓ Problem Solving
- ✓ Reasoning and Proving
- ✓ Reflecting
- ✓ Selecting Tools and Computational Strategies
- ✓ Connecting
- ✓ Representing
- ✓ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Teaching Suggestions

This problem can be addressed at two levels. First, it can be used to model an EQAO problem, using parts a) and b) with a 20-minute time allotment. Second, it can be used as a performance assessment task. Have students work with a partner to develop and carry out a plan for approaching the task.

- Before assigning either a) and b), OR the complete task, review some related material (surface area and volume of a cylinder, calculator skills, making sketches with or without technology, Pythagorean Theorem).
- Make available concrete materials from which to build models of the situation (e.g., boxes, identical cans, thread spools, cardboard, scissors, tape, etc.).
- Some students may have difficulty providing diagrams in part c), and you may wish to provide these as an aid.
- As a class or in small groups, review similar questions that have been covered in chapter 8 and 9, emphasizing the skills and formulas that were used, perhaps using a jigsaw technique. (30 min)
- Provide sufficient time for students to think about this problem. This will lead to clearer, more complete solutions from students. If some students finish quickly, encourage them to consider other packaging options and to compare volumes and surface areas.
- Due to the complexity of part c), allow students to brainstorm approaches, with a partner or in groups, before they begin. One strategy is to allow students to discuss the problem but not to write anything down until they begin individual work. Another strategy is to introduce the problem one day, but not assign it to be completed until another day.

Level 3 Sample Response

a) Since the dimensions are in centimetres, first express the volume in cubic centimetres.

$$0.015\text{ m}^3 = 0.015 \times 100 \times 100 \times 100\text{ cm}^3$$
$$= 15\ 000\text{ cm}^3$$
$$V = \pi r^2 h$$
$$15\ 000 = \pi \times r^2 \times 75$$
$$r^2 = \frac{15\ 000}{\pi \times 75}$$
$$r = \sqrt{\frac{15\ 000}{\pi \times 75}}$$
$$r \doteq 7.98$$

The radius is approximately 7.98 cm. So, the diameter of the cylinder is 16 cm, to the nearest centimetre.

b) In Chapter 8, students learned that the surface area of a cylinder is greatest when the height is equal to the diameter.

In $15\ 000 = \pi r^2 h$, substitute $h = 2r$

$$15\ 000 = \pi r^2 \times 2r$$
$$15\ 000 = 2\pi r^3$$
$$\text{So, } r = \sqrt[3]{\frac{15\ 000}{2\pi}}$$
$$r \doteq 13.365$$

Ongoing Assessment

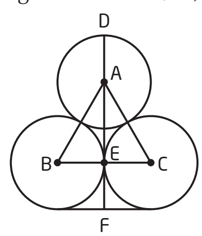
- Use **BLM 9.T3.1 Task: Packing Compressed Air Rubric** to assess student achievement.

Students may use a calculator or trial and error to find the value of r.
Then, for the cylinder of maximum surface area, the height is double the value of r, or 26.73 cm.
The new cylinder has a radius of 13.4 cm and a height of 26.7 cm.
Next, use the formula to calculate the surface area.

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(13.365)^2 + 2\pi(13.365)(26.73) \\ &\doteq 3366.97 \end{aligned}$$

The new cylinder has a surface area of approximately 3367 cm^2.

c) In the case when the cylinders arranged in 4 rows of 5, the dimensions of the box are:
Length: 5 times the diameter of the cylinder is 5×16 cm, or 80 cm
Width: 4 times the diameter of the cylinder is 4×16 cm, or 64 cm
Height: 75 cm
Therefore, the volume of the box is $80 \times 64 \times 75 = 384\ 000$ cm^3, or 0.384 m^3.
In the case when the cylinders are staggered, we need to find the length and width, while the height remains 75 cm. By examining the staggered case drawing, we can see that the length is 5 times the diameter plus one radius to allow for the staggered configuration So, length $= 5 \times 16 + 8$, or 88 cm



In the drawing above, using the Pythagorean theorem in $\triangle AEC$:

$$\begin{aligned} m(DF) &= m(DA + AE + EF) \\ &= 8 + \sqrt{16^2 - 8^2} + 8 \end{aligned}$$

So $m(DF) = 30$ cm, to the nearest centimetre.
The width of the box for the staggered arrangement is twice the length of DF or 60 cm. Hence the volume of the "staggered" box is $88 \times 60 \times 75 = 396\ 000$ cm^3 or 0.392 m^3

So it follows that the arrangement with identical rows has least volume (and also least surface area, 31 840 cm^2, since both boxes have the same height).

Some students may consider a configuration of two identical rows of 10 or one row of 20. Both of these have the same volume, 384 000 cm^3 as in the 4×5 case, but greater surface areas (39 040 cm^2 and 60 640 cm^2)
The case of a staggered 10×2 configuration gives a box with volume 378 000 cm^3 or 0.378 m^3. Below is a table showing the data.

Possible Shipping Boxes		
Arrangement	Volume (cm^3)	Surface Area (cm^2)
Identical Rows: 4×5	384 000	31 840
Identical Rows: 2×10	384 000	39 040
Identical Rows: 1×20	384 000	60 640
Staggered Rows: 4×5	396 000	32 760

Level 3 Notes

Look for the following:
- A complete solution to parts a) and b) of the problem
- An appropriate plan for part c) with a partially complete solution
- Minor computational errors

- Diagrams consider some intended forms of the packaging boxes
- Most steps in the solution have justification
- Correct form of packaging and box size is found but not all cases are considered

What Distinguishes Level 2

Look for the following:

- A partially complete solution to parts a) and b) of the problem
- A minimally appropriate plan for part c) with an incomplete attempt at a solution
- Major computational errors
- Diagrams are included but are unclear and/or unrelated
- Little or no justification for the steps in the solution
- Only a few cases are considered in part c)

What Distinguishes Level 4

At this level, look for the following:

- A complete solution to parts a) and b) of the problem
- A very appropriate plan for part c) with a complete solution
- Very few computational errors
- Diagrams consider all possible forms of the packaging boxes
- Most steps in the solution have justification
- Correct form of packaging and box size is found and cases are analysed