CHAPTER

2

Relations

Vocabulary

hypothesis
statistics
primary data
secondary data
sample
population
census
random sample
simple random sampling
systematic random sampling
stratified random sampling
non-random sampling
bias
inference
dependent variable
independent variable
outlier
interpolate
extrapolate
linear relation
line of best fit
curve of best fit
distance–time graph

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Strand:
Linear Relations

Overall Expectations

By the end of this course, students will:
LRV.01 apply data-management techniques to investigate relationships between two variables;
LRV.02 demonstrate an understanding of the characteristics of a linear relation;
LRV.03 connect various representations of a linear relation.

Specific Expectations

Using Data Management to Investigate Relationships

By the end of this chapter, students will:
RE1.01 interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant (e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student);
RE1.02 pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;
RE1.03 design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs);
RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding Characteristics of Linear Relations

By the end of this chapter, students will:
RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;
RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);
RE2.03 identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;
RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Connecting Various Representations of Linear Relations

By the end of this chapter, students will:
RE3.01 determine values of a linear relation by using a table of values, by

using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;
RE3.02 describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;
RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model).

Chapter Problem

The Chapter Problem is introduced in the Chapter Opener. Have students discuss their understanding of the topic. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 99.

Alternatively, you may wish to assign only the Chapter Problem when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment.

When discussing factors for a high jumper, look for factors such as height, speed, jumping ability, strength, and conditioning. A key component is for students to explain their thinking. This is a good opportunity to practise reasoning and communication skills.

Chapter 2 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 2 Opener • 15 min	38–39			
Get Ready • 80 min	40–41	• BLM G10 Grid Paper • BLM 2.GR.1 Practice: Get Ready • BLM 2.GR.2 How to Draw a Scatter Plot	• BLM 2.GR.3 Get Ready Self-Assessment Checklist	**Tools** • grid paper
2.1 Hypotheses and Sources of Data • 80 min	42–47	• BLM 2.1.1 Practice: Hypotheses and Sources of Data	• BLM 2.1.2 Achievement Check Rubric • BLM A9 Communication General Scoring Rubric	
2.2 Sampling Principles • 80–160 min	48–55	• BLM 2.2.1 Practice: Sampling Principles • BLM T6 *Fathom*™	• BLM A21 Opinion Piece Checklist • BLM A4 Presentation Checklist	**Technology Tools** • graphing calculators • *Fathom*™ • computers
2.3 Use Scatter Plots to Analyse Data • 80 min	56–67	• BLM T6 *Fathom*™ • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 2.3.1 Practice: Use Scatter Plots to Analyse Data • BLM G10 Grid Paper	• BLM A7 Thinking General Scoring Rubric	**Tools** • grid paper **Technology Tools** • graphing calculators • *Fathom*™ • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
2.4 Trends, Interpolation, and Extrapolation • 80 min	68–76	• BLM G10 Grid Paper • BLM T6 *Fathom*™ • BLM 2.4.1 Practice: Trends, Interpolation, and Extrapolation	• BLM A10 Observation General Scoring Rubric • BLM 2.4.2 Achievement Check Rubric	**Tools** • grid paper **Technology Tools** • graphing calculators • *Fathom*™ • computers
2.5 Linear and Non-Linear Relations • 80–160 min	77–87	• BLM G10 Grid Paper • BLM T6 *Fathom*™ • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 2.5.1 Practice: Linear and Non-Linear Relations	• BLM A5 Problem Solving Checklist • BLM 2.5.2 Achievement Check Rubric	**Tools** • grid paper **Technology Tools** • graphing calculators • *Fathom*™ • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
2.6 Distance-Time Graphs • 80 min	88–94	• BLM 2.6.1 Practice: Distance-Time Graphs • BLM G10 Grid Paper	• BLM A9 Communication General Scoring Rubric	**Tools** • large ball (such as a basketball or volleyball) • grid paper **Technology Tools** • graphing calculators • CBR™ (calculator-based rangefinder) • link cables
Chapter 2 Review • 80 min	95–97	• BLM 2.CR.1 Chapter 2 Review • BLM G10 Grid Paper		**Tools** • grid paper
Chapter 2 Practice Test • 80 min	98–99	• BLM G10 Grid Paper	• BLM 2.PT.1 Chapter 2 Practice Test • BLM 2.CT.1 Chapter 2 Test	**Tools** • grid paper
Chapter 2 Problem Wrap-Up • 80 min	99	• BLM G10 Grid Paper	• BLM A17 Teamwork Self Assessment • BLM 2.CP.1 Chapter 2 Problem Wrap-Up Rubric	**Tools** • grid paper

Chapter 2 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM G10	Grid Paper	Teacher Support
	BLM 2.GR.1	Practice: Get Ready	Practice
	BLM 2.GR.2	How to Draw a Scatter Plot	Student Support
	BLM 2.GR.3	Get Ready Self-Assessment Checklist	Student Self-Assessment
2.1: Hypotheses and Sources of Data			
	BLM 2.1.1	Practice: Hypotheses and Sources of Data	Practice
	BLM 2.1.2	Achievement Check Rubric	Assessment
	BLM A9	Communication General Scoring Rubric	Assessment
2.2: Sampling Principles			
	BLM 2.2.1	Practice: Sampling Principles	Practice
	BLM T6	*Fathom*™	Technology
	BLM A21	Opinion Piece Checklist	Assessment
	BLM A4	Presentation Checklist	Assessment
2.3: Use Scatter Plots to Analyse Data			
	BLM T6	*Fathom*™	Technology
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft® *Excel*	Technology
	BLM 2.3.1	Practice: Use Scatter Plots to Analyse Data	Practice
	BLM A7	Thinking General Scoring Rubric	Assessment
	BLM G10	Grid Paper	Student Support
2.4: Trends, Interpolation, and Extrapolation			
	BLM G10	Grid Paper	Student Support
	BLM T6	*Fathom*™	Technology
	BLM 2.4.1	Practice: Trends, Interpolation, and Extrapolation	Practice
	BLM A10	Observation General Scoring Rubric	Assessment
	BLM 2.4.2	Achievement Check Rubric	Assessment

	BLM	Title	Purpose
2.5: Linear and Non-Linear Relations			
	BLM G10	Grid Paper	Student Support
	BLM T6	*Fathom*™	Technology
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft® *Excel*	Technology
	BLM 2.5.1	Practice: Linear and Non-Linear Relations	Practice
	BLM A5	Problem Solving Checklist	Assessment
	BLM 2.5.2	Achievement Check Rubric	Assessment
2.6: Distance-Time Graphs			
	BLM 2.6.1	Practice: Distance-Time Graphs	Practice
	BLM A9	Communication General Scoring Rubric	Assessment
	BLM G10	Grid Paper	Student Support
Chapter 2 Review			
	BLM 2.CR.1	Chapter 2 Review	Practice
	BLM G10	Grid Paper	Student Support
Chapter 2 Practice Test			
	BLM 2.PT.1	Chapter 2 Practice Test	Diagnostic Assessment
	BLM 2.CT.1	Chapter 2 Test	Summative Assessment
	BLM G10	Grid Paper	Student Support
Chapter 2 Problem Wrap-Up			
	BLM A17	Teamwork Self Assessment	Student Self-Assessment
	BLM G10	Grid Paper	Student Support
	BLM 2.CP.1	Chapter 2 Problem Wrap-Up Rubric	Summative Assessment

Get Ready

Student Text Pages
40 to 41

Suggested Timing
80 min

Tools
- grid paper

Related Resources
BLM G10 Grid Paper

BLM 2.GR.1 Practice: Get Ready

BLM 2.GR.2 How to Draw a Scatter Plot

BLM 2.GR.3 Get Ready Self-Assessment Checklist

Common Errors

- Some students may misinterpret question 1c) and not refer to the graph, but rather to their own experiences or opinions.

R_x Remind students that the graph needs to be interpreted appropriately.

- Some students may have forgotten how to draw a scatter plot.

R_x Remind students that the dots represent ordered pairs, for example, (10, 16). The first value, 10, is plotted on the horizontal or *x*-axis, and the second value, 16, is plotted on the vertical or *y*-axis. The dot is placed at the intersection of the two values.

- Some students will not divide appropriately when finding unit rate.

R_x Have students consider which quantity should be the "unit" and to divide by that quantity.

Accommodations

Gifted and Enrichment—Encourage students to create many ways to solve the same problem.

Teaching Suggestions

- Questions 1 and 2 provide a good opportunity for students to work in pairs to discuss their observations about the graphs.
- Refer to the Link to Get Ready in the chapter sections of the Teacher's Resource.
- You may wish to have students use **BLM G10 Grid Paper** for questions 3a) and 4a). You can use **BLM 2.GR.2 How to Draw a Scatter Plot** to provide students with detailed instructions on drawing scatter plots.
- You may wish to use **BLM 2.GR.1 Practice: Get Ready** for remediation or extra practice.
- All BLMs referred to throughout this chapter can be found in the *Principles of Mathematics 9* Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; using **BLM 2.GR.3 Get Ready Self-Assessment Checklist**, students can place a check mark beside topics in the Get Ready in which they feel confident with the necessary skills. Remedial action can be taken in small groups or with a whole class skill review.

2.1 Hypotheses and Sources of Data

Strand:
Linear Relations

Student Text Pages
42 to 47

Suggested Timing
80 min

Related Resources
BLM 2.1.1 Practice: Hypotheses and Sources of Data
BLM 2.1.2 Achievement Check Rubric
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☐ Representing
- ☑ Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.02 pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;

RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations.

Warm-Up

Have students state their opinions on topics, such as:

- Who is the richest person in the world?
- Can you throw a ball farther on Earth or on the moon?
- How are your marks affected by the amount of time you study for a test?

Discuss how their answers to these questions are all hypotheses and that they can be tested and proven to be true or false.

Teaching Suggestions

- Begin with the Warm-Up. (5 min)
- Have students complete the Investigate. (10 min)
- Conduct a class discussion on the results. (5 min)
- As a class, take up Examples 1 and 2, or give similar examples. In Example 2, discuss that the use of data is what makes it primary or secondary. Statistics Canada may have posted original data, making it primary data. If, for example, a student uses the data in a study, it then becomes secondary data. (15 min)
- Assign Practise questions 1 to 4 and take up in class. (15 min)
- Review the key concepts, primary, and secondary data before proceeding to the rest of the exercises.
- You may wish to use **BLM 2.1.1 Practice: Hypotheses and Sources of Data** for remediation or extra practice.

Common Errors

- Some students may write numeric hypotheses with exact values, such as "most students use the telephone for 2 h per day."

R_x Have students consider ranges of values, such as "between 2 h and 4 h," or "at least 1 h."

Ongoing Assessment

Use Achievement Check question 11 to monitor student success. See the Achievement Check Answers and **BLM 2.1.2 Achievement Check Rubric** to assist you in assessing your students.

Communicate Your Understanding questions can be used as quizzes to assess student Communication skills (see **BLM 2.1.2 Achievement Check Rubric** for levels).

You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Accommodations

Gifted and Enrichment—Challenge students to make hypotheses and collect primary or secondary data to prove if the hypotheses are true.

Visual—Let students work with a reading buddy in order to have the information and the questions in this section read to them.

Perceptual—Let students work with an educational assistant, if possible, who can help to explain what is being asked in the questions in this section.

Language—Give students photocopies of the notes for this section and encourage them to highlight the key ideas.

ESL—Allow students extra time in class to understand the meaning of the words in this section. Encourage students to use a translator to ensure that they understand the meaning of the words being used and the context.

Investigate Answers (page 42)

1. Answers will vary.

a) Fifteen percent of students at my school will get their driver's license this year.

b) Television advertising is more effective than newspaper advertising.

c) People consider price more important when buying toothpaste.

d) Girls prefer studying with music playing while boys prefer studying in a quiet room.

e) People rarely use a phone book more than five times a year.

f) Teenagers like tennis the most.

2. Not all hypotheses use numbers. Answers will vary. There are hypotheses that pose a question used to guide research; some hypotheses are just statements presenting a certain viewpoint; while other hypotheses state statistics.

3. Answers will vary.

a) The total number of students in the school and the number that will get their driver's license this year.

b) The statistics for television and newspaper advertising.

c) The statistics from different toothpaste companies to see if price or brand name affected sales more.

d) You will need to find out what different study habits boys and girls prefer.

e) Find the average number of times a person will use a phone book.

f) Find the preferred sport among teenagers.

Communicate Your Understanding Responses (page 44)

C1. Answers will vary.

a) Any multiple of 2 is always an even number.

b) Candy is more popular than gum.

c) Prove the first hypothesis using algebra and conduct a survey to test the second hypothesis.

C2. No. Secondary data may be more reliable.

Practise

In questions 1 and 2, caution students that the opposite hypothesis is not always as simple as adding the word "not." For example, the opposite of "less than" is "greater than or equal to."

Connect and Apply

Students may have some difficulties with questions 5b) and 6c). Although they may not understand random sampling, these questions allow students to consider methods of sampling a population. Question 8 refers to the Chapter Problem and can be used as an ongoing assessment tool. You may wish to assign this problem, but not take up the answers in class. Questions 9 and 10 introduce the Internet as a research tool and should be assigned in preparation for further Internet use. Question 11 is a good opportunity to analyse and reflect on hypotheses and makes a good formative assessment piece. Use **BLM 2.1.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 47)

11. a) No. Most participants caught at least one fish (35 out of the 55 entrants), rather than no fish at all as Heather predicted.

b) No. Most people either caught nothing or caught more than two fish (31 out of the 55 entrants).

c) Heather: Most entrants would catch at least one fish.
George: Most entrants would catch nothing or more than two fish.

d) Yes. Two explanations are possible: Both George and Heather's hypotheses are false, so the opposite of these hypotheses must be true. Alternatively, by looking at the data, you can show that 35 out of 55 entrants caught one or more fish, and 31 entrants caught no fish or more than two fish, so both hypotheses in part c) are true.

e) The data from the local newspaper are secondary since they were collected by someone other than George.

f) Several correct answers are possible for each prediction.
Two of the simplest are
- Heather predicts that most entrants will catch fewer than two fish.
- George predicts that most entrants will catch from one to three fish.

Note that the question asks for *new* hypotheses, so, the answers should not be the same as the opposite hypotheses given in part c).

Extend

Questions 12 and 13 are interesting extensions, and students may notice that in question 12, the relationship is linear, but in question 13 it is not. Set aside some class time for a discussion about non-linear relationships.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6
Typical	1–11
Extension	12–14

2.2 Sampling Principles

Strand:
Linear Relations

Student Text Pages
48 to 55

Suggested Timing
80–160 min (160 min if the surveys are done in questions 15 and 16)

Technology Tools
- graphing calculators
- *Fathom*™
- computers

Related Resources
BLM 2.2.1 Practice: Sampling Principles
BLM T6 *Fathom*™
BLM A21 Opinion Piece Checklist
BLM A4 Presentation Checklist

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☐ Representing
- ☑ Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.03 design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs);

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations.

Link to Get Ready

The Get Ready segment Rates provides the needed skills for this section. You may wish to have students complete or review Get Ready questions 5 and 6 before starting this section.

Teaching Suggestions

- Have students read the introductory paragraph about the phone-in radio show. As a class, discuss the inaccuracies of this type of survey.
- Assign the Investigate activity. This activity would work well in groups of three students. (5 min)
- After the students discuss the Investigate questions in their groups, define the terms, **population**, **sample**, and **census**, and have a whole class discussion on how to collect a sample to properly represent a population. Stress that the various types of random sampling are not necessarily appropriate in a given situation. For example, **stratified random sampling** is only necessary when it is important to have various groups represented proportionally, such as when polling students in a school. It may be important to stratify them by grade. **Systematic random sampling** is an easy way of selecting large samples from an organized list of names. (15 min)
- Discuss the term **bias** as it applies to statistics. Suggest that it represents the possibility, not a guarantee, of inaccurate results.
- Review the vocabulary before assigning Communicate Your Understanding. Allot 15 minutes for students to complete C1, C2, and Practise questions 1, 2a), and 2b).
- Discuss the answers to Communicate Your Understanding as a class before assigning additional exercises.
- You may wish to use **BLM 2.2.1 Practice: Sampling Principles** for remediation or extra practice.

Common Errors

- Some students may use the word *random* inappropriately. Part of teenage language is to say, for example, "I am going shopping at some random mall on Saturday."

R$_x$ Explain to the students that this usage means they haven't yet made up their minds and that the mathematical meaning of the word is that each member of the population has an equal chance of being chosen.

Ongoing Assessment

Communicate Your Understanding questions can be used as quizzes to assess student Communication skills.

Accommodations

Gifted and Enrichment—Encourage students to create and conduct surveys to collect data for the class to predict trends.

Visual—Give students opportunities to work with a classmate who will take notes for them.

Perceptual—Let students work with an educational assistant, if possible, or with a student who can help to explain the concepts in this section.

Language—Let students use dictionaries to ensure that they understand the meaning the words in this section.

ESL—Allow students extra time in class to understand the meaning of the words in this section. Encourage students to use a translator to ensure that they understand the meaning of the words being used and the context.

Student Success

Go to www.mcgrawhill.ca/books/principles9 website and choose the link for Census at School. Have your class complete the survey on-line. Then, use the Census at School database for sampling and hypothesizing. Students really enjoy having their own personal data as part of the database.

Investigate Answers (page 48)

1. b) is going to give the best results because the students are randomly selected from across the country.

2. Answers will vary.

Communicate Your Understanding Responses (page 52)

C1. No. The sample size is small and they all live in the same neighbourhood. There is non-random sampling and they don't represent the entire population.

C2. a) Answers may vary. Simple random sample: assign each student a number and randomly select 100 of those numbers using a computer or graphing calculator. Systematic random sample: put all names in alphabetical order. Randomly choose one student as the starting point, then, survey every 5th person before and after the starting point.

b) Answers will vary. Possible answer: Surveying my five best friends.

Practise

Remind students that a population represents the list of objects being studied and need not necessarily be people. They may need some hints to be able to answer question 2. In question 4a) and c), students may think there is bias where there is none.

Connect and Apply

In question 5, students may be uncomfortable with some stratification ideas, such as race or culture. This would be a good opportunity to discuss how governments and school boards may wish to use this type of stratification to see whether all groups are being served appropriately. Question 11 involves the use of a graphing calculator. It is an important tool for generating random numbers. Software, such as *Fathom*™, or a spreadsheet also could be used. You may wish to use **BLM T6 *Fathom*™**. Questions 13 and 14 are important in understanding bias in statistical studies.

Extend

Questions 17 and 19 are a good follow up to questions 13 and 14 in the discussion on bias. Question 18 provides an excellent opportunity for students to analyse a situation.

Literacy Connections

By now, students will probably have developed opinions on the use of statistical information. Assign the following activity:

Write an opinion piece, a series of paragraphs expressing your opinion, on one of the topics below. Develop your main idea with supporting details. Be sure to write your opinion piece with your audience in mind, an adult who is interested in your opinion.

Topics:

a) A census is the best way to collect data.
b) You will never have bias in a survey when you use random sampling.

Review students' opinion pieces and ensure that students have included three paragraphs: an introduction, the body, and a conclusion. Their opinion should be clearly stated and students should provide details with reasons, examples, and facts to support their opinions. You may wish to use **BLM A21 Opinion Piece Checklist** to assess the students. You may also wish

to provide an opportunity for students to present their opinion pieces to the class. Use **BLM A4 Presentation Checklist** to assess students' presentations.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 6–9, 11, 12
Typical	1–9, 11–14, 15 or 16
Extension	17–20

2.3 Use Scatter Plots to Analyse Data

Strand:
Linear Relations

Student Text Pages
56 to 67

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- graphing calculators
- *Fathom*™
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM T6 *Fathom*™
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 2.3.1 Practice: Use Scatter Plots to Analyse Data
BLM A7 Thinking General Scoring Rubric
BLM G10 Grid Paper

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.01 interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant (e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student);

RE1.03 design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs);

RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials).

Link to Get Ready

The Get Ready segment Scatter Plots provides the needed skills for this section. You may wish to have the students complete Get Ready questions 3 and 4 before starting this section.

Teaching Suggestions

- Until the end of grade 8, students' experience with scatter plots has been with discrete data. The Investigates and Examples extend their experience to continuous data. These activities engage students in kinesthetic learning activities, along with covering many of the process expectations, most notably, Problem Solving, Reflecting, Connecting, and Communicating.
- Investigate A furthers students' skills in using a graphing calculator as a tool for data analysis and in describing a relationship between handspan and forearm length. Have a class discussion on this relationship. (15 min)

Common Errors

- Some students may misunderstand the concept of a relationship. They may give examples that do not follow the pattern of a relationship.

R_x Explain to students that a relationship is an overall description of how the independent variable generally affects the dependent variable, and that exceptions may occur. These exceptions are called **outliers**.

Ongoing Assessment

Chapter Problem question 7 in Connect and Apply can be used as an assessment tool.

You may wish to use **BLM A7 Thinking General Scoring Rubric** to assist you in assessing your students.

Communicate Your Understanding questions can be used as quizzes to assess student Communication skills.

Accommodations

Visual—Let students work in groups when creating scatter plots using technology.

Spatial—Encourage students to colour code the independent variable and the dependent variable when graphing using pencil and paper.

Motor—Give students extra time to complete the questions in this section and let them to work with a partner when creating graphs using technology.

Memory—Encourage students to review the steps required to use the technology in this section.

Student Success

Link this topic to what is happening in the students' Science class. Have students in groups of four do a brief presentation on where they have used this method in Science. Also, have them identify similarities and differences between the procedures taught in Math class and those taught in Science class.

In a semestered school, not all students will be taking Science simultaneously with Math. Presentations should be confined to students who have completed Science or are taking it concurrently with Math.

- In Investigate B, students learn the scientific process for conducting an experiment. In the follow up discussion, a fun comparison can be made between humans and Martians. (25 min)
- Example 1 is an important overview of identifying independent and dependent variables. If graphing calculators are not available, Example 2 can be used to teach skills involving the use of other technology for making scatter plots. You may wish to use **BLM G10 Grid Paper, BLM T6 *Fathom*™, BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, and/or **BLM T3 Microsoft® *Excel*** to support Example 2. (15 min)
- Discuss the vocabulary, **independent variable**, **dependent variable**, and **outliers**, before assigning the exercise. (2 min)
- Assign Communicate Your Understanding questions C1 and C2. (5 min)
- You may wish to use **BLM 2.3.1 Practice: Use Scatter Plots to Analyse Data** for remediation or extra practice.

Investigate Answers (page 56)

A. 5. The horizontal axis represents the left forearm length and the vertical axis represents the left handspan length.

7. As the forearms get longer, the handspan lengths also get longer.

8. Derf has a long forearm but a short handspan. The ordered pair is below the rest on the graph.

9. Answers will vary. The data could have been recorded incorrectly, Derf could still be growing and hasn't reached his full adult size yet, or it could just be a natural phenomenon.

B. 1. To determine if there is a relationship between a human's forearm length and handspan.

2. Answers will vary. The longer a person's forearm is, the greater their handspan.

4. Answers will vary. The larger a person's handspan is, the longer their forearm is.

5. Answers will vary. A person's handspan and forearm length are related, they are either both larger or both small. This supports my original hypothesis.

6. Answers will vary. A better sample could be used by increasing the size of the sample to the school and randomly selecting 5% of students.

7. Answers will vary. Find the average forearm length and handspan for both the aliens and the humans. You can compare the ratio of forearm to handspan between the two species or you could compare the forearms and the handspans separately.

Communicate Your Understanding Responses (page 63)

C1. If the points follow a line or curve, then the variables are related. If the points are scattered randomly, then they are not related.

C2. B. Outliers might tell you something interesting has occurred.

Practise

Caution students to consider cause and effect when identifying the independent and dependent variables.

Connect and Apply

Compare the relationships in questions 4 (linear) and 5 (non-linear).
For question 8, the following web sites can be used to update dietary information:

- Harvey's—http://www.harveys.ca/eng/site.php (download pdf. file);
- Mr. Sub—http://www.mrsub.ca/consumers/nutritional_subs_wraps.html;

- Pizza Pizza—http://www.pizzapizza.ca/english/pages/menu/sub_nutrition/n_pizza.htm; KFC—http://www.yum.com/nutrition/menu.asp?brandID_Abbr=2_KFC;
- Swiss Chalet—http://www.swisschalet.ca/ourmenu/nutritionals.pdf.

Note that 1 food Calorie is about 4.19 kJ. In SI, kilojoules replace calories as the unit for food energy. Question 7 refers to the Chapter Problem. You may wish to assign this question, but not take it up in class, if it is being used as an ongoing assessment piece. You may wish to use **BLM G10 Grid Paper** for questions 3, 4, 5, 6, and 9.

Extend

Question 9 describes the relationship and asks the students to draw the graph. Question 10d) adds a level of complexity by having students consider the strength of the relationships.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 8
Typical	1–6, 8
Extension	9–11

2.4 Trends, Interpolation, and Extrapolation

Strand:
Linear Relations

Student Text Pages
68 to 76

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- graphing calculators
- *Fathom*™
- computers

Related Resources
BLM G10 Grid Paper
BLM T6 *Fathom*™
BLM 2.4.1 Practice: Trends, Interpolation, and Extrapolation
BLM A10 Observation General Scoring Rubric
BLM 2.4.2 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

Connecting Various Representations of Linear Relations

RE3.01 determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation.

Link to Get Ready

The Get Ready segment Bar Graphs provides the needed skills for this section. You may wish to have the students complete Get Ready questions 1 and 2 before starting this section.

Teaching Suggestions

- In this section students investigate trends by making and reading bar graphs and scatter plots. Students learn when to interpolate and extrapolate. You may wish to use **BLM G10 Grid Paper** for this section.
- As a class, read the introductory paragraph and have a short discussion on the meaning of the word *trend*. Explain to students that a trend is a pattern in data that has occurred over a period of time, or a general tendency of an event. It may or may not be extrapolated into the future. (2 min)
- If students have skills in using graphing calculators or *Fathom*™, using technology allows the students to concentrate on the analysis of the data and observe trends. This also invites a good discussion on when and whether to use interpolation and extrapolation. You may wish to use **BLM T6 *Fathom*™** for this activity. (15 min)
- The Example reinforces the same skills as the Investigate. If you choose to use graphing calculators or *Fathom*™, have the students enter only the end year of each interval. You may experience some difficulties if you enter the range of years (e.g., 2002–2003). Further information on movie attendance can be found at these web sites: http://www.mptac.ca/stats.

Common Errors

- Some students may extrapolate trends indefinitely.

R_x Have students consider the Investigate and predict the temperature in June and October, based on the trend. Explain that other factors, or variables, may influence the dependent variable outside the given time span.

Ongoing Assessment

Use Achievement Check question 8 to monitor student success. See the Achievement Check Answers and **BLM 2.4.2 Achievement Check Rubric** to assist you in assessing your students.

Communicate Your Understanding questions can be used as quizzes to assess student Communication skills (see **BLM 2.4.2 Achievement Check Rubric** for levels).

html, http://www.mptac.ca/statistics.html, and http://www.statcan.ca/english/freepub/87F0009XIE/2005001/sumtable.htm. (10 min)

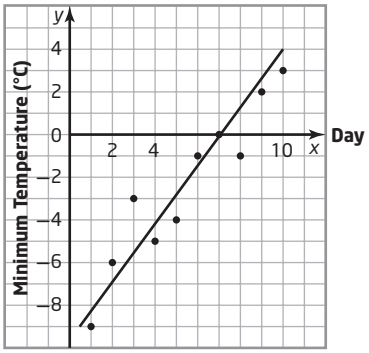
- Review the vocabulary, **interpolate** and **extrapolate**, before assigning the exercises. (2 min)
- Assign Communicate Your Understanding questions C1 and C2, and Practise questions 1 and 2. (15 min)
- Further data for question C2 can be found at this web site: http://www.vs.gov.bc.ca/stats/annual/2001/tab03.html.
- You may wish to use **BLM 2.4.1 Practice: Trends, Interpolation, and Extrapolation** for remediation or extra practice.
- You may wish to use **BLM A10 Observation General Scoring Rubric** at any point during this section to assist you in assessing your students.

Investigate Answers (page 68)

Method 1

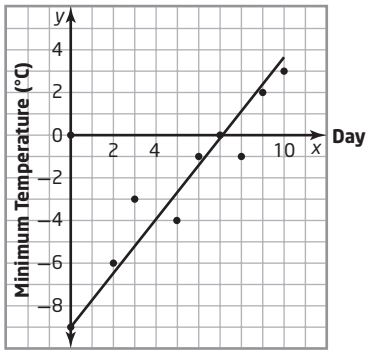
5. The horizontal axis represents the days and the vertical axis represents the minimum temperatures.

6.



7. Answers will vary. The minimum temperatures later in the 10 days were generally higher than the temperatures earlier in the 10 days. The minimum temperatures gradually rose over the 10 days.

8. Answers will vary. To predict temperatures outside of the dates that were surveyed, create a line of best fit (trend line) that gives a good approximation of the pattern. From this trend line, it seems that on March 11 the minimum temperature will be close to 4°C. On March 12, it will be close to 5°C. The minimum temperature on February 26 can be estimated by extending the trend line to the left and looking at its Minimum Temperature value on Day −2.

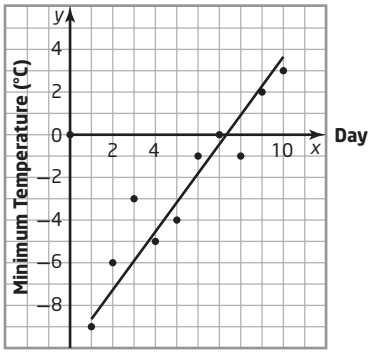


Method 2

4. The horizontal axis represents the days and the vertical axis represents the minimum temperatures.

6. Answers will vary. The minimum temperatures later in the 10 days were generally higher than the temperatures earlier in the 10 days. The minimum temperatures gradually rose over the 10 days.

7. Answers will vary. To predict temperatures outside of the dates that were surveyed, create a line of best fit (trend line) that gives a good approximation of the pattern. From this trend line, it seems that on March 11 the minimum temperature will be close to 4°C. On March 12, it will be close to 5°C. The minimum temperature on February 26 can be estimated by extending the trend line to the left and looking at its Minimum Temperature value on Day −2.



Accommodations

Gifted and Enrichment—Give students opportunities to research data on the Internet for the class to graph.

Motor—Let students use enlarged grid paper when creating graphs using pencil and paper.

Student Success

Have students role play conducting a survey for a market research company. Pose questions, collect data from classmates, analyse and present the results.

Communicate Your Understanding Responses (page 72)

C1. Scatter plot. It is good to show a relationship between the data.

C2. a) Yes. If you follow a trend line it looks like there might be a fertility rate of zero.

b) If you follow a curve pattern, the fertility rate might be 0.81 in 2030.

Practise

For question 1, further information from the City of Guelph's Planning and Building Services can be found at this web site: http://www.guelph.ca/uploads/PBS_Dept/planning/documents/stats_housing.pdf.
In question 2c), the predicted value is not evident. This may provide an opportunity to discuss what factors may contribute to the stabilization of the world's population. Students may suggest saturation, e.g., the limit to the amount of food, water, and land available for increasing numbers of people; people having fewer children; epidemics, etc. Further information on the United Nations' report can be found at these web sites: http://www.chem.brown.edu/chem12/un%20population/unPopulation.html; http://www.un.org/esa/population/publications/longrange2/2004worldpop2300reportfinalc.pdf, and http://www.un.org/esa/population/publications/sixbillion/sixbilpart1.pdf.

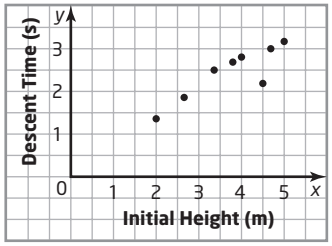
Connect and Apply

Most questions are straightforward in their graphing and interpretation of the data. Answers will vary for the reasons behind trends or relationships. For further information, you may wish to visit these web sites: http://www.givingandvolunteering.ca/factsheets.asp (question 5); and http://www.cria.ca/stats.php (question 7). For question 8, you may wish to use **BLM 2.4.2 Achievement Check Rubric** to assist you in assessing your students. You may also wish to use **BLM G10 Grid Paper** for any or all of the questions in this exercise.

Achievement Check Answers (page 75)

8. a) Independent: initial height; dependent: descent time. The time it takes to get to the bottom of the ramp will depend on where the skateboarder starts.

b) **Descent Times at Various Heights**



c) As the initial height increases, so does the descent time.

d) Point (4.5, 2.2) is an outlier. I would check this data point by performing the experiment again before excluding it.

e) 2.5 m, if (4.5, 2.2) is excluded from the data set.

Extend

Question 9 involves multiple relationships from the same chart. For further information, you may wish to visit this web site: http://www.givingandvolunteering.ca/factsheets.asp.
Question 10 involves research in the library or on the Internet to critically

analyse a prediction regarding world growth. Consider having a class discussion on Internet web sites and appropriate sources. Remind students that anyone can create a web site on any topic on the Internet. Ask students to raise their hands if they have a personal web site or keep an Internet journal (a *blog*). Explain that web sites like these contain personal opinions and information contained on them should be looked at critically. You may wish to review the vocabulary from Section 2.1 on primary and secondary data. This also may provide an opportunity to remind students that personal information should never be revealed over e-mail, in an on-line journal, or a chat-room and that anything that makes them uncomfortable should be reported immediately to their parent or guardian.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 6
Typical	1–7
Extension	9–12

2.5 Linear and Non-Linear Relations

Strand:
Linear Relations

Student Text Pages
77 to 87

Suggested Timing
80–160 min (160 min if more experiments are done)

Tools
- grid paper

Technology Tools
- graphing calculators
- *Fathom*™
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM G10 Grid Paper
BLM T6 *Fathom*™
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 2.5.1 Practice: Linear and Non-Linear Relations
BLM A5 Problem Solving Checklist
BLM 2.5.2 Achievement Check Rubric
BLM 2.5.3 Student Success: Crossword

Mathematical Process Expectations Emphasis

- [x] Problem Solving
- [ ] Reasoning and Proving
- [x] Reflecting
- [x] Selecting Tools and Computational Strategies
- [ ] Connecting
- [x] Representing
- [x] Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.01 interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant (e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student);

RE1.02 pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;

RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

RE2.03 identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;

RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Connecting Various Representations of Linear Relations

RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model).

Common Errors

- Some students may draw the line of best fit so that it goes through the greatest number of points possible.

R_x Give students an example where six points line up in a perfectly straight line, and an additional three points lie scattered above that line. Show that the line going through those six points cannot be the line of best fit because there is not a balance between points above and below the line.

Ongoing Assessment

Use Achievement Check question 10 to monitor student success. See the Achievement Check Answers and **BLM 2.5.2 Achievement Check Rubric** to assist you in assessing your students.

Communicate Your Understanding questions can be used as quizzes to assess student Communication skills (see **BLM 2.5.2 Achievement Check Rubric**).

Teaching Suggestions

- Introduce the Investigate by reading the opening paragraph as a class. Have the students answer the questions individually before having a class discussion to summarize the concept of a linear relation. This will flow into the concept of a line of best fit that can be drawn for a data relationship that appears linear, or a curve of best fit for a data relationship that appears non-linear. (15 min)
- As a class, work through Example 1, or an example of a scatter plot that provides a linear relationship. Although there are formal methods of finding the line or curve of best fit, for now, finding the line or curve that comes closest to as many points as possible is best. You may wish to use **BLM G10 Grid Paper**, **BLM T6 *Fathom*™**, **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, and/or **BLM T3 Microsoft® *Excel*** for this section. (15 min)
- Example 2 explains when it is not appropriate to find a line of best fit. This is an initial stage of critical analysis and should be stressed. (5 min)
- Review the vocabulary, **linear relation**, **line of best fit**, and **curve of best fit**, before assigning the activities. (2 min)
- Assign activities C1, C2, and Practise questions 1–4. (20 min)
- You may wish to use **BLM 2.5.1 Practice: Linear and Non-Linear Relations** for remediation or extra practice.

Investigate Answers (page 77)

1. Answers will vary. Both boys and girls grow the fastest within the first 3 months, and then, their growth rate slows down a little.

2. Answers will vary. Young girls grow a little bit slower and are a few centimetres shorter than boys.

3. Although neither growth rate is linear, they both start to get very close to being linear after 18 months.

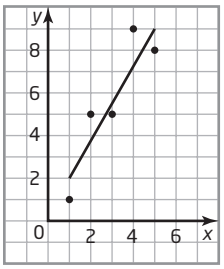
4. From 0 to 18 months, the growth rate of boys and girls appears to be non-linear.

5. Changes in linear graphs are constant while those in non-linear graphs occur a lot faster or slower.

Communicate Your Understanding Responses (page 83)

C1. A. The points lie close to a straight line.

C2. Yes. The points can be scattered evenly above and below the line of best fit. Graphs will vary. Example:



Accommodations

Visual—Give students opportunities to work together with a reading buddy who will read the technology instructions read to them.

Spatial—Encourage students to colour code the *x*- and *y*-axes on a graph when plotting points, linear relations, and non-linear relations.

Language—Let students work with a student note taker or reading buddy who will take notes and read the questions in the exercises for them.

Student Success

Have students collect examples of surveys from newspapers and other print sources. Instruct students to create a collage of the results, highlighting the information and critiquing how the information is presented. Once students have completed Section 2.5, have them complete the crossword puzzle of data management terminology on **BLM 2.5.3 Student Success: Crossword**.

Practise

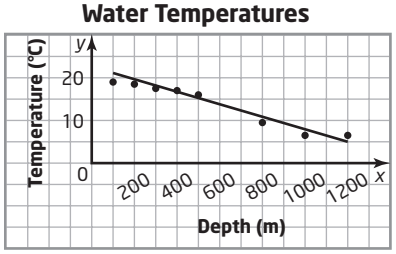
Students should have few difficulties with these questions. Make sure they use appropriate scales when graphing the scatter plots.

Connect and Apply

Question 6d) invites students to consider other factors that affect both the independent and dependent variables. Question 7 is an excellent example of the dangers of extrapolation under changing conditions. You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students. It is important that students carry out a few experiments in this unit. You may wish to assign question 8 to half the class and question 9 to the other half, if you are pressed for time. Question 10 is an excellent formative assessment tool. You may wish to use **BLM 2.5.2 Achievement Check Rubric** to assist you in assessing your students. You may wish to use **BLM G10 Grid Paper** to help with questions 4 to 12.

Achievement Check Answers (page 86)

10. a) and b)



c) As the depth increases, the temperature decreases.

d) At a depth of 700 m, the water temperature would be 12°C.

e) At a depth of 1600 m, the water temperature would be −0.7°C.

f) Interpolating is more accurate than extrapolating because the answer can be checked by the data points around it, therefore, the estimate for d) is likely to be more accurate.

Extend

Students who are able to develop the concept of first differences on their own will be successful with question 11. Although question 12 should be a relatively straightforward question, the presence of fractions makes it more challenging. It should be accessible to most level 3 or 4 students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 8
Typical	1–7 and 8 or 9
Extension	11–14

2.6 Distance–Time Graphs

Strand:
Linear Relations

Student Text Pages
88 to 94

Suggested Timing
80 min

Tools
- large ball (such as a basketball or volleyball)
- grid paper

Technology Tools
- graphing calculators
- CBR™ (calculator-based rangefinder)
- link cables

Related Resources
BLM 2.6.1 Practice: Distance-Time Graphs
BLM A9 Communication General Scoring Rubric
BLM G10 Grid Paper

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.01 interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant (e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student);
RE1.02 pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;
RE1.03 design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs);
RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;
RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

Connecting Various Representations of Linear Relations

RE3.02 describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;
RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model).

Teaching Suggestions

- Read the opening paragraph as a class. This is a good introduction to the CBR™ (calculator-based rangefinder) for graphing calculators. The CBR™ is to be used as a data collection tool so that analysis of the data can follow. (2 min)
- Investigates A and B are excellent exercises in understanding distance-time graphs. Ideal group sizes are two or three students. Allow students to experiment with the CBRs™ until they have a good sample. (40 min)

Common Errors

- Some students may have difficulty describing motion or matching graphs.

R_x This type of concept takes time to understand. Have students investigate what happens to the distance–time graph when they stand still, walk at a slow steady pace, walk at a faster steady pace, speed up, slow down, and walk backwards. Have them copy their graphs onto a piece of paper or **BLM G10 Grid Paper** and write a full description of their actions.

Ongoing Assessment

Communicate Your Understanding questions can be used as quizzes to assess student Communication skills.

- At the end of the activities, have a class discussion on the importance of distance-time graphs and the speed relationships they show. This will help students' understanding later in the course when they learn about rates of change and slope. Stress that the steepness of the graph represents the speed of the object. Also, stress the importance of using time as the independent variable. (10 min)

Tips on using the CBR™:

a) The CBR™ works best at distances of 1 m to 3 m. Measurements of distances less than 1 m or more than 3 m can be inaccurate. CBR2s™ have a better range, approximately 15 cm to 6 m.

b) Soft clothing absorbs the ultrasonic pulses from the ranger. The target student can improve the reflection of the pulses by carrying a piece of stiff cardboard.

c) Hard surfaces, such as desks and chalkboards, can reflect the ultrasonic pulses and cause erroneous readings. A wide hallway is a good place to conduct these investigations.

- If you do not have access to CBRs™, discuss the graphs in question 7 of Investigate A. Create a few more simple graphs so the students can act out and describe the motion implied by the graph. Have the students act out some simple motions toward a wall, while their partner(s) draw the appropriate graphs.
- For Investigate B, have students roll a ball slowly down a ramp with a metre stick or tape measure fastened to the side. Have one student tap off the seconds from a stopwatch, while a second student calls out the distances every 2 s. A third student can record them on a chart. Steps 5, 6, and 8 can be answered for this activity as well.
- Assign and discuss questions C1 and C2. (10 min)
- You may wish to use **BLM 2.6.1 Practice: Distance-Time Graphs** for remediation or extra practice.

Investigate Answers (page 88)

A. 5. The horizontal axis represents time and the vertical axis represents distance. Time is the independent variable because distance is being measured relative to time.

6. Answers will vary.

a) The graph shows the distance decreasing in a straight line for the first 2 s when approaching the wall.

b) The graph shows the distance increasing at an increasing pace (non-linear).

c) The graph shows a horizontal line when I was standing still.

d) The parts of the graph that represent the motion towards the wall and standing still are straight lines while the motion away from the wall is a curved line. The straight part represents motion at a constant speed and the curved part represent motion at an increasing speed (non-constant).

7. a) The graph represents motion toward a wall at a constant speed and then, standing still.

b) The graph represents motion toward a wall at an increasing speed and then, slowing down to a complete stop and standing still for a while.

8. Motion at a constant speed is represented by straight lines on a distance-time graph and motion at an increasing or decreasing speed is represented by a curved line.

B. 5. The horizontal axis represents time and the vertical axis represents distance. Time is the independent variable because distance is being measured relative to time.

6. The point on the graph where the distance stops increasing (where the graph peaks) is the point where the ball hit the floor and bounced back.

7. The coordinates of this point represent how far the ball was from the rangefinder when the ball hit the floor.

8. Answers will vary. The relationship between distance and time is not linear because the graph shows their relationship as a curve not a straight line. We can conclude that the speed of the ball is increasing.

Communicate Your Understanding Responses (page 91)

C1. AB: slowest movement, BC: fastest movement, CD: no movement, DE: backward movement

C2. a) Person 1 walks towards the CBR™ at a steady pace and then, stops in front of it.

b) Person 2 walks at a faster pace than Person 1.

c) Person 3 has a running start towards the CBR™ but then gradually slows down.

Accommodations

Gifted and Enrichment—Give students opportunities to create their own distance-time graphs.

Visual—Provide students with opportunities to orally describe the graphs in this section.

Motor—Encourage students to work in groups when using technology.

Memory—Allow students to create and use "sequential cue card" instructions to remember the steps required to use technology in this section.

Practise

These questions should not pose difficulties for the students.

Connect and Apply

Most questions support the basic concepts taught in the Investigate activities. Question 5 is a key question to consolidate the students' understanding of distance–time graphs. Question 9 is an excellent activity for making students think about appropriate motion for a given graph and to communicate their ideas. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students. You may wish to use **BLM G10 Grid Paper** for questions 6 to 9.

Extend

In question 12, the Ball Bounce program shows the distance of the ball from the floor, rather than from the CBR™. This way, it provides what looks like a ball bouncing along a floor. Remind students that the independent axis represents time.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 5
Typical	1–9
Extension	11–14

Chapter 2 Review

Student Text Pages
95 to 97

Suggested Timing
80 min

Tools
- grid paper

Related Resources
BLM 2.CR.1 Chapter 2 Review
BLM G10 Grid Paper

Ongoing Assessment

- Students can create a summary sheet of the skills in the chapter (to be holistically checked by teacher before the Chapter Test).
- Upon completing the Chapter Review, students can also answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related section in the text for Examples or Key Concepts? For which questions was this necessary?*

Using the Chapter Review

Each question reviews different skills and concepts. Have copies of **BLM G10 Grid Paper** available for students to use. The students might work independently to complete the Chapter Review, then, in pairs to compare solutions. Alternatively, the Chapter Review could be assigned for reinforcing skills and concepts in preparation for **Chapter Practice Test**. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult. Use **BLM 2.CR.1 Chater 2 Review** for extra review.

After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty and include the related sections and teaching examples. They can use this to focus their studying for **Chapter Test** on the chapter's content.

The Chapter Review contains a considerable number of communication questions and may take the students longer than expected. You may wish to spend a few minutes a day for a few days taking these up.
For further data for question 8, go to the web site:
http://www.statcan.ca/english/Pgdb/demo03.htm.

Chapter 2 Practice Test

Student Text Pages
98 to 99

Suggested Timing
80 min

Tools
- grid paper

Related Resources
BLM 2.PT.1 Chapter 2 Practice Test
BLM 2.CT.1 Chapter 2 Test
BLM G10 Grid Paper

Summative Assessment

- After students complete the **BLM 2.PT.1 Chapter 2 Practice Test,** you may wish to use **BLM 2.CT.1 Chapter 2 Test** as a summative assessment.

Accommodations

Perceptual—Encourage students work together in study groups.

Motor—Let students dictate their answers to the review questions and practice test, if possible, to an educational assistant who, as a scribe, will record the responses and answers.

Language—If possible, allow students time to complete the review questions and practice test in this section in their school's Language Lab in which the questions are scanned by the computer and read to the student.

Memory—Give students opportunities to give oral responses to the Chapter Review questions and Practice Test. Provide students with extra visual and verbal cues and prompts.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	2.1	Example 2 (page 43)
2	2.2	Example 2 (page 50)
3	2.4	Example 1 (page 70)
4	2.3	Investigate B (page 58)
5	2.1	Example 1 (page 43)
6	2.2	Examples 1, 2, 3 (page 49)
7	2.5	Investigate, Example 1 (pages 77, 78)
8	2.6	Investigate A, B (pages 88, 89)
9	2.3/2.5	Investigate A/Investigate (pages 56, 77)

Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. You may wish to use **BLM G10 Grid Paper** for Chapter Review questions 6a), 7b), 8a), 9a), 10a) and b), 11a), and 13a) and b). If the Practice Test is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?
- Identify population, random sampling techniques, bias, outliers
- State the opposite of a hypothesis
- Make a scatter plot and describe a relationship
- Make a line of best fit from a scatter plot
- Discuss a trend from a scatter plot
- Interpolate and extrapolate as appropriate
- Conduct an experiment involving data collection

Chapter 2 Problem Wrap-Up

Student Text Page
99

Suggested Timing
80 min

Related Resources
BLM A17 Teamwork Self Assessment
BLM 2.CP.1 Chapter 2 Problem Wrap-Up Rubric
BLM G10 Grid Paper

Tools
• grid paper

Summative Assessment

• Use **BLM 2.CP.1 Chapter 2 Problem Wrap-Up Rubric** to assess student achievement.

Using the Chapter Problem

- The Chapter Problem may be an ongoing assessment piece or given as a culminating activity at the end of the unit.
- This problem is well suited to group work. If you choose to have students work in groups, you may wish to use **BLM A17 Teamwork Self Assessment** as a self-assessment tool.
- Part a) will be the most time consuming and part d) will be the most challenging to demonstrate level 3 or 4 work.
- Explain the importance of fully documented steps, a complete set of data, good graphs, and clear explanations and that level 4 work involves providing more depth to their thinking, explanations, and reflection. You may wish to use **BLM G10 Grid Paper** to assist the students.
- Contact a physical education teacher or track and field coach as a guest speaker for the class and to help with the logistics of collecting data.
- If data collection must be restricted to the classroom, the high jump could be modified to measure how high a student can jump off the ground from a stationary start. A practical way to measure jump heights is to have students touch as high as possible up a wall, and then subtract their combined arm and body height. A measuring scale can be taped to the wall beforehand. This will give the height of their feet off the ground. Multiple students could view each jump and their observations could be averaged to provide more accurate data.
- Discuss the problem in a whole-class setting or divide the class into cooperative groups. The objective of such discussions is to clarify the problem, brainstorm possible hypotheses, plan a strategy for collecting and recording data, and examine assessment criteria for the completed task.
- Questions to ask to generate discussion include:
  - How many measurements do you need?
  - How will you record your data?
  - How will you choose the sample of students to collect data from?

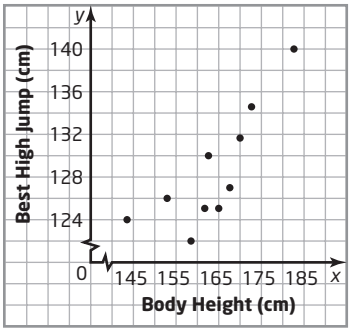
Level 3 Sample Response

a) I examined the relationship between body height and performance in the high jump. My original hypothesis had been that high jump performance increases with a student's body height. The following chart gives the data that was collected.

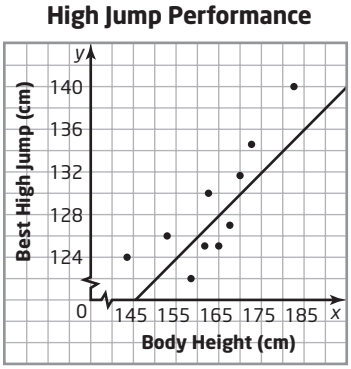
Student	Body Height (cm)	Best High Jump (cm)
1	153	126
2	167	128
3	182	140
4	144	124
5	161	125
6	158	122
7	173	135
8	162	130
9	169	132
10	165	125

b) My scatter plot of this data follows. It suggests a linear relationship is possible but it is not a strong linear relationship. Six of the data points seem to form a good straight line but the other four points are below.

High Jump Performance



c) The line of best fit (least squares line) is added below. The line fits the data fairly well, but there are many points that seem to be away from this line.



d) The hypothesis has been (weakly) supported by the data.

Level 3 Notes

Look for the following:

- Clear statement of hypothesis
- Simple, but complete, description of how data was collected
- Complete data table, scatter plot
- Line or curve of best fit present, may be hand drawn but will smoothly fit the data
- Clear report
- Mostly correct units of measure

What Distinguishes Level 2

At this level, look for the following:

- Statement of hypothesis is present but may be incomplete or ambiguous
- Limited description of how data was collected
- Data table provided but it may have only a few data points or contain a few points with obvious errors
- Line of best fit may be absent or drawn but inaccurately
- Reasonably clear report, but with some lack of clarity
- Some correct units of measure, some may be absent

What Distinguishes Level 4

At this level, look for the following:

- Clear, detailed statement of hypothesis
- Detailed description of how data was collected
- Report may include more than one hypothesis related to the data
- Explanation of how hypothesis was chosen
- Data table and scatter plot are based on a large sample of data
- May include a discussion of the choice of a line of best fit vs. a curve of best fit
- Possible analysis of errors arising from measurement tools and procedures
- Suggestions for how to improve data collection accuracy
- Possible comments about extrapolation of data to taller and shorter athletes
- Creative, sophisticated, or very organized way of providing reports, which may include diagrams, photos, and Internet references
- Correct use of relevant units of measure