CHAPTER

8

Measurement Relationships

Vocabulary

hypotenuse
Pythagorean theorem
surface area
volume
pyramid
lateral faces
cone
sphere

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site, at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Strand:
Number Sense and Algebra

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Overall Expectations

By the end of this course, students will:

NAV.01 demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions;

NAV.02 manipulate numerical and polynomial expressions, and solve first-degree equations.

Specific Expectations

Operating With Exponents

By the end of this chapter, students will:

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]).

Strand:
Measurement and Geometry

Overall Expectations

By the end of this course, students will:

MGV.01 determine, through investigation, the optimal values of various measurements;

MGV.02 solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

MGV.03 verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

By the end of this chapter, students will:

MG2.01 relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$;

MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.03 solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles);

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \frac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \frac{(\text{area of base})(\text{height})}{3}$;

MG2.05 determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Chapter Problem

The Chapter Problem deals with measurement concepts that might be encountered in a landscape design business. Have students explore their understanding of the topic by asking them to suggest measurement skills that a landscape designer might find useful. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions incorporate new skills in the sections where they are introduced, and are designed to help students move toward the Chapter Problem Wrap-Up on page 473.

Alternatively, you may wish to assign only the Chapter Problem Wrap-Up when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment. It is open-ended in nature to allow students originality in their solutions.

Chapter 8 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter Opener • 15 min	412–413			
Get Ready • 80-160 min	414–417	• BLM 8.GR.2 Practice: Get Ready • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM 8.GR.2 Get Ready Self-Assessment Checklist	**Technology Tools** • *The Geometer's Sketchpad*® • computers
8.1 Apply the Pythagorean Theorem • 80 min	418–425	• BLM 8.1.1 Practice: Apply the Pythagorean Theorem • BLM G10 Grid Paper	• BLM A8 Application General Scoring Rubric • BLM A21 Opinion Piece Checklist	**Tools** • grid paper • rulers • an assortment of cardboard boxes **Technology Tools** • *The Geometer's Sketchpad*® • computers
8.2 Perimeter and Area of Composite Figures • 80 min	426–435	• BLM 8.2.1 Practice: Perimeter and Area of Composite Figures • BLM G5 Tangram	• BLM A9 Communication General Scoring Rubric • BLM 8.2.2 Achievement Check Rubric	**Tools** • tangrams, pattern blocks, or geoboards
8.3 Surface Area and Volume of Prisms and Pyramids • 80–160 min	436–443	• BLM 8.3.1 Net for a Pyramid • BLM 8.3.2 Net for a Prism • BLM 8.3.3 Practice: Surface Area and Volume of Prisms and Pyramids	• BLM A5 Problem Solving Checklist • BLM 8.3.4 Achievement Check Rubric	**Tools** • empty 250-mL milk cartons • construction paper • scissors • tape • sand, rice, or another suitable materials • Bristol board • pyramid models • interlocking cubes
8.4 Surface Area of a Cone • 80–160 min	444–450	• BLM 8.4.1 Practice: Surface Area of a Cone • BML T4 *The Geometer's Sketchpad*® 3 • BML T5 *The Geometer's Sketchpad*® 4	• BLM A18 My Progress as a Problem Solver	**Tools** • models of cones • construction paper • scissors • rulers • compasses • tape **Technology Tools** • *The Geometer's Sketchpad*® • computers
8.5 Volume of a Cone • 80 min	451–456	• BLM 8.5.1 Practice: Volume of a Cone • BML T4 *The Geometer's Sketchpad*® 3 • BML T5 *The Geometer's Sketchpad*® 4 • BLM T3 Microsoft® *Excel*	• BLM A11 Group Work Assessment Recording Sheet	**Tools** • empty cylindrical cans • construction paper • scissors • tape • sand, rice or other suitable materials **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • Microsoft® *Excel* • computers
8.6 Surface Area of a Sphere • 80 min	457–461	•BLM G9 Centimetre Grid Paper •BLM 8.6.1 Practice: Surface Area of a Sphere •BML T4 *The Geometer's Sketchpad*® 3 •BML T5 *The Geometer's Sketchpad*® 4	• BLM A1 Assessment Recording Sheet	**Tools** • oranges • string • rulers • centimetre grid paper • plastic balls **Technology Tools** • *The Geometer's Sketchpad*® • computers • graphing calculators

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
8.7 Volume of a Sphere • 80 min	462–469	• BLM 8.7.1 Practice: Volume of a Sphere • BML T4 *The Geometer's Sketchpad®* 3 • BML T5 *The Geometer's Sketchpad®* 4	• BLM A6 Knowledge/ Understanding General Scoring Rubric • BLM 8.7.2 Achievement Check Rubric	**Tools** • cylindrical containers that each just hold three tennis balls • three tennis balls for each group of students • water • containers to catch the overflow water • identical small spheres (e.g., marbles, tennis balls, or table tennis balls) **Technology Tools** • graphing calculators • *The Geometer's Sketchpad®* • computers
Chapter 8 Review • 80 min	470–471	• BLM 8.CR.1 Chapter 8 Review		
Chapter 8 Practice Test • 50–70 min	472–473		• BLM 8.PT.1 Chapter 8 Practice Test • BLM 8.CT.1 Chapter 8 Test	
Chapter 8 Problem Wrap-Up • 30–60 min	473	• BLM G10 Grid Paper • BLM G9 Centimetre Grid Paper	• BLM 8.CP.1 Chapter Problem Wrap-Up Rubric	**Tools** • compasses • grid paper • centimetre grid paper **Technology Tools** • graphing calculators

Chapter 8 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 8.GR.1	Practice: Get Ready	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 8.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
8.1: Apply the Pythagorean Theorem			
	BLM A8	Application General Scoring Rubric	Assessment
	BLM 8.1.1	Practice: Apply the Pythagorean Theorem	Practice
	BLM G10	Grid Paper	Student Support
	BLM A21	Opinion Piece Checklist	Assessment
8.2: Perimeter and Area of Composite Figures			
	BLM 8.2.1	Practice: Perimeter and Area of Composite Figures	Practice
	BLM 8.2.2	Achievement Check Rubric	Assessment
	BLM G5	Tangram	Student Support
	BLM A9	Communication General Scoring Rubric	Assessment
8.3: Surface Area and Volume of Prisms and Pyramids			
	BLM 8.3.1	Net for a Pyramid	Student Support
	BLM 8.3.2	Net for a Prism	Student Support
	BLM A5	Problem Solving Checklist	Assessment
	BLM 8.3.3	Practice: Surface Area and Volume of Prisms and Pyramids	Practice
	BLM 8.3.4	Achievement Check Rubric	Assessment
8.4: Surface Area of a Cone			
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM 8.4.1	Practice: Surface Area of a Cone	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
8.5: Volume of a Cone			
	BLM A11	Group Work Assessment Recording Sheet	Assessment Group Work
	BLM 8.5.1	Practice: Volume of a Cone	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM T3	Microsoft® *Excel*	Technology

	BLM	Title	Purpose
8.6: Surface Area of a Sphere			
	BLM G9	Centimetre Grid Paper	Student Support
	BLM 8.6.1	Practice: Surface Area of a Sphere	Practice
	BLM A1	Assessment Recording Sheet	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
8.7: Volume of a Sphere			
	BLM 8.7.1	Practice: Volume of a Sphere	Practice
	BLM A6	Knowledge/Understanding General Scoring Rubric	Assessment
	BLM 8.7.2	Achievement Check Rubric	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
Chapter 8 Review			
	BLM 8.CR.1	Chapter 8 Review	Practice
Chapter 8 Practice Test			
	BLM 8.PT.1	Chapter 8 Practice Test	Diagnostic Assessment
	BLM 8.CT.1	Chapter 8 Test	Summative Assessment
Chapter 8 Problem Wrap-Up			
	BLM G10	Grid Paper	Student Support
	BLM G9	Centimetre Grid Paper	Student Support
	BLM 8.CP.1	Chapter 8 Problem Wrap-Up Rubric	Summative Assessment

Get Ready

Student Text Pages
414 to 417

Suggested Timing
80–160 min

Technology Tools
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM 8.GR.1 Practice: Get Ready
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 8.GR.2 Get Ready Self-Assessment Checklist

Common Errors

- Some students may not show all their work or use proper form in their solutions.

R_x Have students show the formula that they are using. In the next line, have them show the values they are substituting into the formula, and then show the rest of their solution following the proper order of operations. Encourage students to use equals signs appropriately and show their solution flowing vertically as they move from line to line.

- Some students may need to be reminded to use proper units.

R_x Ensure that students have a clear understanding of the difference between linear units (e.g., m and cm) and square units (e.g., m^2 and cm^2). Use a visual representation to clarify the distinction. Have students discuss possible measurement situations and the units that would be used for each (e.g., installing new baseboard trim in the classroom, putting in floor tiles, painting walls, etc.).

Accommodations

Perceptual—Have students use diagrams and different forms of a formula when calculating dimensions.

Spatial—Encourage students to draw nets of the three-dimensional shapes to help them understand the formulas for surface area and volume.

Teaching Suggestions

- Have students work with a partner or in small groups.
- You may wish to assign the Calculate Perimeter and Circumference and the Apply Area Formulas sections as homework, since most students will recall these skills fairly readily.
- More class time may be necessary for the Calculate Surface Area and Volume section.
- You may wish to use **BLM 8.GR.1 Practice: Get Ready** as remediation or extra practice.
- Computer access is necessary for the Use *The Geometer's Sketchpad®* section. You may wish to use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** to support this activity. If students are already familiar with *The Geometer's Sketchpad®* and have made significant use of it earlier in the course, less time will be necessary here. Alternatively, have students use the Student Edition of *The Geometer's Sketchpad®* and complete some of this section as homework.
- The OSAPAC (Ontario Software Acquisition Program Advisory Committee) has licensed the student edition of *The Geometer's Sketchpad®* for use at home by students. Make students aware of this opportunity.
- It may be useful to use a computer with a projection display to demonstrate the features of *The Geometer's Sketchpad®* used in this chapter that students may not be familiar with, such as the **Measure** menu.

Assessment

Assess student readiness to proceed by informal observation as students work on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; you may wish to give students **BLM 8.GR.2 Get Ready Self-Assessment Checklist**, and have them place a checkmark beside topics in which they feel confident with the necessary skills. Remedial action can be taken in small groups or with a whole class skill review.

8.1 Apply the Pythagorean Theorem

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
418 to 425

Suggested Timing
80 min

Tools
- grid paper
- rulers
- an assortment of cardboard boxes

Technology Tools
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G10 Grid Paper
BLM A8 Application General Scoring Rubric
BLM 8.1.1 Practice: Apply the Pythagorean Theorem
BLM A21 Opinion Piece Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.01 relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$;

Operating With Exponents
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

Discuss with students that a square is a special rectangle, and that its area formula can be expressed as $A = s^2$, where s is the length of the side of the square. Review the area of a rectangle in the Get Ready section Apply Area Formulas.

Warm-Up

1. Calculate the area of a square with sides of length:
a) 5 cm **b)** 20 cm **c)** 1.2 m **d)** 24 m **e)** $\sqrt{3}$ cm **f)** $\sqrt{24}$ m

2. Calculate the following square roots (round to two decimal places where necessary):
a) $\sqrt{36}$ **b)** $\sqrt{121}$ **c)** $\sqrt{20}$ **d)** $\sqrt{1000}$ **e)** $\sqrt{7}$ **f)** $\sqrt{5.2}$

Warm-Up Answers

1. a) 25 cm^2 **b)** 400 cm^2 **c)** 1.44 m^2 **d)** 576 m^2 **e)** 3 cm^2 **f)** 24 m^2

2. a) 6 **b)** 11 **c)** 4.47 **d)** 31.62 **e)** 2.65 **f)** 2.28

Teaching Suggestions

- Students will have used the Pythagorean theorem previously. Discuss the photo and Pythagoras. (5 min)
- Assign the Investigate, Method 1. (5–10 min)
- You may wish to use **BLM G10 Grid Paper** to support this activity. Many students will recall the formula $c^2 = a^2 + b^2$, but fewer will be able to explain and use the formula properly. Some students will take longer than others to complete this activity. Have students share their results. You may wish to have some students share their results with the class using the blackboard or an overhead.
- Alternatively, you may want to do Method 2. Have students work with a partner for this activity. Direct students to the McGraw Hill Ryerson web site for the interactive proof. Go to http://www.mcgrawhill.ca/links/principles9. (10–20 min)
- Another approach would be to begin with pencil and paper and then demonstrate using *The Geometer's Sketchpad®*. You may wish to conduct the demonstration yourself, or ask a gifted student. (5–10 min for the pencil and paper Investigate, 10 min for the demonstration)

Common Errors

- Some students may misuse the Pythagorean formula $c^2 = a^2 + b^2$, letting c be the unknown side, even when it is not the hypotenuse.

R_x Stress to students that the c value is always the length of the hypotenuse. Always have students start with the equation in the form $(\text{hypotenuse})^2 = \ldots$ and rearrange the formula as necessary to solve for the unknown.

- Some students may think that $c^2 = a^2 + b^2$ implies that $c = a + b$.

R_x Use a numerical example, such as $5^2 = 3^2 + 4^2$ and point out that $5 \neq 3 + 4$.

- Some students may rely on measurement formulas without having a clear understanding of what a question is asking. This will be especially true for problems that require multi-step solutions.

R_x Encourage students to draw a diagram to represent a given problem. Most students will more readily understand what is required if they can picture the problem. Encourage students to develop a problem-solving plan when tackling a problem that requires several steps.

Ongoing Assessment

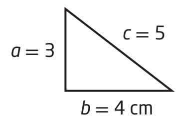
- Question 9, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

- Assign and discuss Examples 1 to 3.
- The OSAPAC (Ontario Software Acquisition Program Advisory Committee) has licensed the student edition of *The Geometer's Sketchpad®* for use at home by students. Make students aware of this opportunity.
- You may wish to use **BLM A8 Application General Scoring Rubric** to assist you in assessing your students.
- You may wish to use **BLM 8.1.1 Practice: Apply the Pythagorean Theorem** for remediation or extra practice.

Investigate Answers (pages 418–420)

Method 1

1. Answers will vary. Sample solution provided.



2. Answers will vary. 5 cm, 4 cm, and 3 cm

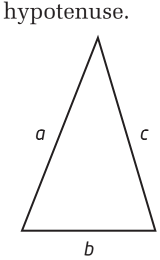
3. Answers will vary based on art from question 1.

a) $c^2 = 25$
b) $a^2 + b^2 = 3^2 + 4^2$
$= 25$
c) $a^2 + b^2 = c^2$

4. Answers will vary based on art from question 1.

a) 5 cm
b) This value is equal to the length of the hypotenuse.

5. No, the Pythagorean relationship does not hold with a non-right triangle.
No, the relationship does not hold true.



6. The Pythagorean theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. In this activity, I constructed a right triangle and measured the lengths of the three sides. Next, I calculated the square of the length of the hypotenuse and the sum of the squares of the lengths of the two shorter sides and found these two values were equal. However, this relation did not hold true for a non-right triangle.

Method 2

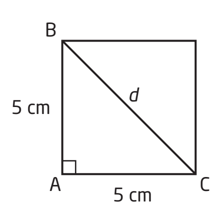
1–5. Answers will vary.

6. No, the Pythagorean relationship does not hold in a non-right triangle. No, the square of the longest side does not equal the sum of the squares of the other two sides.

7. Answers will vary. Sample solution: In this activity, I constructed right and non-right triangles, and saw through examples that the Pythagorean theorem holds in right triangles, but not in non-right triangles.

Communicate Your Understanding Responses (page 422)

C1.



Since BAC is a right triangle, I can apply the Pythagorean theorem to determine the length of the diagonal, or hypotenuse. Use $d^2 = AB^2 + AC^2$ with $AB = AC = 5$ cm to find the value of d.

Accommodations

Gifted and Enrichment—Challenge students to learn more about H.E. Dudeney, the Wheel of Theodorus, and Pythagoras, and to present their findings to the class. As an extra assignment, give students an opportunity to extend the Pythagorean triples (3, 4, 5; 5, 12, 13; 7, 24, 25, etc.) found in the Math Contest question in this section, and to look for as many patterns as they can find.

Visual—Encourage students to construct a triangle with sides 3 units, 4 units, and 5 units using paper and pencil, to draw unit squares on each of the sides of the triangle, and to count the number of unit squares on each side in order to understand the Pythagorean theorem.

Perceptual—Some students may have difficulty visualizing the three-dimensional shapes in the questions. Provide these students with diagrams or have them work together with a classmate to construct the diagrams.

Motor—Let students work with a partner or in small groups when they use technology to complete the Investigate, and allow them to use enlarged grid paper when working with paper and pencil to complete the questions in this section.

Student Success

Have students conduct an Internet search on the history of the Pythagorean theorem. Then, ask them to write a report answering the question, "Should the Pythagorean theorem be named for someone else?" You may wish to use **BLM A21 Opinion Piece Checklist** to assist you in assessing your students.

C2. Count the squares to find the base and the height of the right triangle formed by the dotted lines and line segment AB. Apply the Pythagorean theorem to find the length of AB.

$(AB)^2 = 4^2 + 3^2$

$(AB)^2 = 25$

$AB = 5$ units

C3.

Step 1: Apply the Pythagorean theorem to the triangle.

$5^2 + b^2 = 13^2$

Step 2: Solve the equation to find b.

$b^2 = 13^2 - 5^2$

$b = 12$ cm

Step 3: Substitute 12 cm for the base and 5 cm for the height into the formula for the area of a triangle.

$\text{Area} = \frac{1}{2} \times 12 \times 5$

$\text{Area} = 30\text{ cm}^2$

Practise

Questions 1 to 3 are similar to Examples 1 to 3. Students should be able to model their solutions by reviewing the Examples.

Connect and Apply

For question 7, remind students that it is just the reverse of question 6, where they were given the side length and asked for the diagonal. Here they are given the diagonal and are asked for the side length. Some students might find their algebra skills useful here. Suggest to students that they label the unknown sides of the square with a single variable.

Extend

Some of these questions deal with three dimensions. Boxes may help students visualize three-dimensional problems. For example, in question 10, it is easier for students to demonstrate a space diagonal of a box using a model. Use a shoebox to model the room in question 11. Flatten the shoebox and use the net to show students the actual path that the spider crawls. (Note that the spider and the fly are on opposite end walls of the room. The visual might appear that the spider and fly are on adjacent walls.) Some students may be interested in researching more problems by H.E. Dudeney on the Internet. Direct students to http://thinks.com/puzzles/dudeney/dudeney.htm.

Literacy Connections

Word Origins

Have students choose two of the words in this section and research their word origins. Ask them to write a paragraph comparing their origins.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–8
Extension	10–13

8.2 Perimeter and Area of Composite Figures

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
426 to 435

Suggested Timing
80 min

Tools
- tangrams, pattern blocks, or geoboards

Related Resources
BLM 8.2.1 Practice: Perimeter and Area of Composite Figures
BLM G5 Tangram
BLM 8.2.2 Achievement Check Rubric
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- [x] Problem Solving
- [ ] Reasoning and Proving
- [x] Reflecting
- [x] Selecting Tools and Computational Strategies
- [x] Connecting
- [x] Representing
- [x] Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.03 solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles)

Operating With Exponents
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

The Get Ready segments Calculate Perimeter and Circumference and Apply Area Formulas provide the needed skills for this section. Students who experience trouble with composite shapes may benefit from completing Get Ready questions 1 to 5 before starting this section. You may wish to use **BLM 8.2.1 Practice: Perimeter and Area of Composite Figures** as remediation or extra practice.

Warm-Up

Make copies of **BLM G5 Tangram** for each student. Tangrams are seven-piece puzzles of familiar composite shapes that originated in ancient China. The area of the finished tangram is the sum of the areas of the separate pieces. Have students use tangram sets, if available.

Discuss with students how the areas of the pieces are related to each other. For example, ask students, *If the area of the smaller triangle is one square unit, what are the areas of each of the other tangram pieces?* Have students create composite shapes with the tangram pieces, and calculate the areas of these figures. Have students share shapes using overhead tangram pieces, if they are available.

There are many Internet sites with tangram puzzles. You may wish to have students do an Internet search for such sites.

Alternatively, you may wish to use pattern blocks to generate the same type of discussion. One pattern block is assigned an area value, and then the areas of the remaining blocks are determined relative to it. Have students create composite shapes with the pattern blocks and calculate the areas of these figures.

Geoboards can also be used to introduce composite shapes. Construct a shape on an overhead geoboard, and ask students to explain how they would calculate the area of the figure.

Alternatively, you could ask students to create a composite shape with a given area (e.g., 12 square units) on the geoboard. Have students share their solutions with the class.

Common Errors

- Some students may include interior dimensions in their calculations when calculating the perimeter of composite shapes.

R_x Stress to students that the perimeter of a figure consists of outside measurements only. Usually these inner dimensions are shown as dotted lines on diagrams for this reason.

Ongoing Assessment

- Use Achievement Check question 13 to monitor student success. See Achievement Check Answers and **BLM 8.2.2 Achievement Check Rubric**.
- Question 8, the Chapter Problem question, can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Teaching Suggestions

- Begin the Investigate by leading the class through some estimation techniques. Stress that they are not making detailed calculations at this point, but trying only to get a rough estimate. When estimating, students will begin to see the steps involved in completing the rest of the Investigate. Suggest that students pick their estimate of the cost of the patio from a range of values. For example, ask, *Will the cost be under $2000, $2000–$3000, $3000–$4000, $4000–$5000, or over $5000?*
- Assign the Investigate, and have students work with a partner or in small groups. (10–15 min) The main focus here is to apply their skills for determining perimeter and area to calculate the costs involved in building the patio. Follow up with a class discussion. Ask students, *Is it necessary to add 10% for waste?* Students may draw on their own experiences with similar projects at home.
- Assign Examples 1 and 2 and have students work with a partner. (5–10 min)
- Some students may use the trapezoid formula in Example 1, especially if Get Ready question 5b) involving a trapezoid has already been assigned. Encourage students to try both strategies. Either solution to Example 1 is acceptable.
- Discuss the Communicate Your Understanding questions. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students. (5–10 min)
- Assign Practise questions 1 and 2. (5–10 min)
- You may wish to use **BLM 8.2.1 Practice: Perimeter and Area of Composite Figures** for remediation or extra practice.

Investigate Answers (pages 426–427)

1. Answers will vary. $4000

2. a) The perimeter of the semicircle can be calculated using $\frac{1}{2}$ × circumference of a circle. $\frac{1}{2} \times 2\pi r = \pi r$

The length of the two unlabelled sides of the triangles can be found using the Pythagorean theorem.

b) The perimeter of the round part is 9.4 m. The hypotenuse of the smaller triangle is 5.0 m. The hypotenuse of the larger triangle is 6.4 m.

c) The total perimeter is 34.8 m. The perimeter plus 10% is 38.3 m.

3. a) two triangles, one rectangle, and one semicircle

b) The dimensions of each of the simple shapes are already known. Use the area formula for each shape.

c) total area = 54.1 m^2, area plus 10% = 59.5 m^2

4. a) $3744.39

b) $4306.05

c) Answers will vary. $4000 is fairly close to $4306.05.

5. I can use the formulas for simple shapes to easily calculate the missing dimension and area of each shape. Then, these can be added together to find the perimeter and area of the complex figure.

Accommodations

Gifted and Enrichment—Challenge students to think of a business and design a logo for their business using computer design software. Have students research Leonardo of Pisa and Fibonacci numbers on the Internet.

Perceptual—Encourage students to colour-code the different shapes in the composite figures to the formulas that they are using to calculate a specific composite area.

Language—Let students use words instead of formulas when working with the formulas in this section.

Memory—Review with the students the steps required to calculate percents.

Student Success

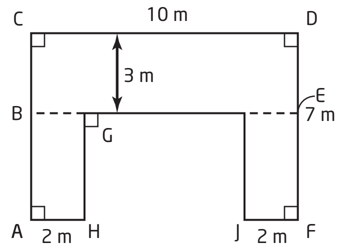
Have students design a room in their dream house, and write a report. Their report should include a fully labelled diagram, and all calculations involving construction and costs. Alternatively, students can construct a three-dimensional model of the room.

Communicate Your Understanding Responses (page 431)

C1. The area of the patio can be determined by adding the area of a trapezoid (formed by the two triangles and the rectangle) and a semicircle.

C2. a) Use known lengths and subtraction to find the lengths of the unknown sides.

b)



As shown in the figure, the yard can be divided into three rectangles. Add the areas of the three rectangles to find the area of the yard.

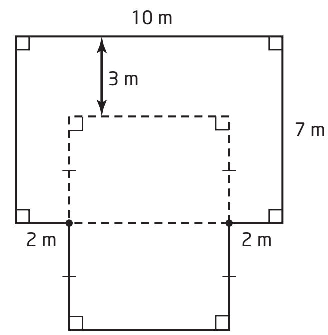
c) Subtract the area of rectangle HGIJ from the area of rectangle ACDF.

C3. a) Some of the sides of the rectangles do not belong to the perimeter of the yard.

b) The perimeter of this yard is greater than the perimeter of a yard that is 10 m by 7 m.

C4. a) Perimeters are the same.

b) Divide the yard into two rectangles and add their areas to determine the area of this yard.



c) The area of this yard is greater than the area of the yard in question C2.

Practise

Use the visuals provided for each of these questions to make it easier to discuss strategies with students and for students to work together to develop a strategy.

Questions 1c), d), and e) require the Pythagorean theorem to determine missing dimensions so that the perimeter can be determined. For example, in question 1c) students need to determine the length of the hypotenuse first and then calculate the perimeter. Encourage students to do this in two distinct steps.

When calculating areas, for example in question 2b), have students draw the two shapes separately (the rectangle and the isosceles triangle) that make up the composite shape. Then, they can calculate the area of each shape showing the appropriate formulas, and then calculate the sum of the areas, as shown in the examples. More students will be successful if they handle the solution in separate steps, so encourage this technique. If students choose to consider the sum of the areas from the start, ensure that they use proper mathematical form throughout their solutions.

Connect and Apply

For question 4, remind students that the arrow is painted on the front side of the sign only.

Question 11 is a revisit of the concepts of section 8.1. Students will need to use the given area to determine the length of the sides of the square.

In question 12, some students will determine the area of the frame by calculating the area of the four trapezoids. Another, more elegant, solution would be to subtract the area of the window from the area of the window and the frame together.

Achievement Check Answers (page 434)

13. a) Each section of the roof must be calculated separately:

$$A = 18 \times 9 + 2\left(\frac{(20 + 18)4.5}{2}\right) + 2\left(\frac{(11 + 9)4.5}{2}\right)$$
$$= 423$$

The area of the roof is 423 m^2.

b) If there were no waste, Susan would need 43 packages.

c) Answers will vary. One approach would be to add 10% (e.g., four packages). Another approach would be to treat each section as a rectangle to accommodate waste and round up to the nearest package for each.

Extend

For question 14, many students will solve the problem by applying the Pythagorean theorem to determine the lengths of the sides of the inner square. Encourage students to realize that the area of the inner square is half the area of the outer square. Drawing the diagonals of the inner square will divide the entire area into eight congruent triangles and help them see this relationship. Students who have completed question 14 will have more insight to help them with question 18.

For question 18, point out to students that the figure on the inside of the rectangle is a rhombus. The area of the rhombus is equal to the area of the shaded area. Students will be able to visualize this if the diagonals of the rhombus are constructed to create four congruent triangles. With this insight, an easy way to calculate the shaded area is to take half of the area of the rectangle. $\frac{1}{2}(80\text{ cm}^2) = 40\text{ cm}^2$

Literacy Connections

Word Origins II

Have students look up the meaning of and history behind the term *composite*. Ask, *Are there any other terms in this section for which you can find a historical reference?* Have students write a paragraph to compare the origins of any such terms.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 11, 12
Typical	1–6, 10–12
Extension	14–18

8.3 Surface Area and Volume of Prisms and Pyramids

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
436 to 443

Suggested Timing
80–160 min

Tools
- empty 250-mL milk cartons
- construction paper
- scissors
- tape
- sand, rice, or other suitable materials
- Bristol board
- pyramid models
- interlocking cubes

Related Resources
BLM 8.3.1 Net for a Pyramid
BLM 8.3.2 Net for a Prism
BLM A5 Problem Solving Checklist
BLM 8.3.3 Practice: Surface Area and Volume of Prisms and Pyramids
BLM 8.3.4 Achievement Check Rubric

Mathematical Process Expectations Emphasis
- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);
MG2.05 determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);
MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating With Exponents
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8^3 by using a calculator]);

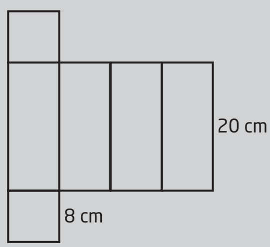
Link to Get Ready
The Get Ready segment Calculate Surface Area and Volume provides the needed skills for this section. Have students complete Get Ready questions 6 to 8 before starting this section.

Warm-Up
Construct a net for the following shapes:

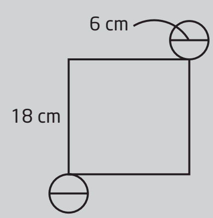
1. a) Square-based prism with dimensions 8 cm by 8 cm by 20 cm
b) Cylinder with a height of 18 cm and a diameter of 6 cm

Warm-Up Answers

1. a) Square-based prism with dimensions 8 cm by 8 cm by 20 cm



b) Cylinder with a height of 18 cm and a diameter of 6 cm



Teaching Suggestions
- Assign the Investigate, and have students work in small groups.
- Have students discuss Part A question 1 in groups, then discuss question 2 as a class. Use **BLM 8.3.1 Net for a Pyramid** for this activity. Its dimensions and height are the same as a 250 mL milk carton, so it can be used in the investigation of the volume of a pyramid in part B.

Common Errors

- Some students may have difficulty distinguishing between the height of a pyramid and the slant height of its lateral sides.

R_x The three-dimensional visuals provided in the student book show the right triangle for most pyramids, and this will help students see the Pythagorean relationship that exists between the height and the slant height. Use three-dimensional models of pyramids wherever possible to help clarify these dimensions for students.

- Some students may use inappropriate units on answers to surface area and volume problems.

R_x Use interlocking cubes to model the volume of prisms. These manipulatives will allow students to see the interlocking cubes as units of volume and the surface area as the sum of the areas of the faces. When students can properly conceptualize surface area and volume, they will be less likely to use inappropriate units.

Ongoing Assessment

- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 8.3.4 Achievement Check Rubric**.
- Question 12, the Chapter Problem question, can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

- As an alternative to Part B, demonstrate for the class, using a milk carton with the top cut off in advance. (Discard the top; it is not used in the rest of the Investigate.) The object is to create a pyramid that has the same base and height as this remaining prism. Discuss with the class what dimensions are known (the length of the square base, and the height) and what dimensions will need to be determined to construct the net for the pyramid. They will need to calculate the slant height, which will be the height of the isosceles triangles in the net for the pyramid. You may want to have students measure the slant height of the pyramid formed from the net in Part A to verify these dimensions.
- You may wish to use **BLM 8.3.2 Net for a Prism** for Part B and have students construct the rectangular prism by using this net rather than an actual milk carton.
- To save time, you may want to construct the pyramid in advance to demonstrate with the class. If you use a heavy grade of paper such as Bristol board, the model will stand up better when filling it with sand or rice.
- Alternatively, three-dimensional plastic models of a prism and pyramid with identical square bases and heights could be used to demonstrate the relationship between their volumes. If such models are available, the investigation will be much faster, but will lack the hands-on approach of the Investigate. Depending on which approach is used, the Investigate could take anywhere from 10 minutes to a full hour.
- Review Examples 1 to 3. (10–20 min)
- You may wish to omit Example 1 if students complete the Investigate, however the pyramid at the Louvre is a wonderful example and may be worth a brief review.
- Spend extra time on Example 2, stressing the difference between the height of the pyramid and its slant height. Some students may have difficulty with this concept. Refer to pyramidal models in the classroom when discussing problems with pyramids.
- You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students.
- You may wish to use **BLM 8.3.3 Practice: Surface Area and Volume of Prisms and Pyramids** for remediation or extra practice.

Investigate Answers (pages 436–437)

Part A

1. a) Square; $(\text{side length})^2$

b) Triangle; $\frac{1}{2} \times \text{base} \times \text{height of triangle}$

c) $\text{Surface Area} = (\text{side length})^2 + 4\left(\frac{1}{2} \times \text{base} \times \text{height of triangle}\right)$
$\text{Surface Area} = s^2 + 2sh$

2. Surface area of a hexagon-based pyramid:
Area of the hexagon + 6 × Area of one lateral surface
Surface area of an octagon-based pyramid:
Area of the octagon + 8 × Area of one lateral surface
To find the surface area of any pyramid:
Step 1: Find the area of the base.
Step 2: Find the area of one lateral surface and multiply it by the number of sides.
Step 3: Add the areas from steps 1 and 2 together to obtain the total surface area of the pyramid.

Part B

1. Answers will vary.

2. Answers will vary. Sample answer: 3:1.

Accommodations

Gifted and Enrichment—Challenge students to prepare a research essay on the history and construction of the pyramids in Egypt.

Perceptual—Encourage students to draw diagrams when completing the questions in this section.

Spatial—Let students change three-dimensional shapes to two-dimensional nets by creating models of the shapes and unfolding them.

Memory—Review with the students the steps for multiplying or dividing by powers of 10. For instance when multiplying by 100, move the decimal two places to the right, and when dividing by 100, move the decimal two places to the left.

ESL—Allow students to use their dictionaries or translators to understand the meanings of the new words in this section.

Student Success

- Have students build an ongoing **journal** of sketches and formulae for three-dimensional shapes. Students can include worked examples of volume and surface area for each shape.
- Ask students to collect a **portfolio** of three-dimensional items and compute their volumes and surface areas (see the photo on the first page of Section 8.3 for a starting point). This portfolio should continue until the end of Section 8.7, and then be presented to the class.

3. a) Answers will vary.

b) approximately 3

c) Volume of the pyramid is about $\frac{1}{3}$ the volume of the prism.

4. Given the same base and height, the volume of the pyramid is $\frac{1}{3}$ the volume of the prism.

Communicate Your Understanding Responses (page 440)

C1. Answers will vary.
Alike: All three shapes have flat bases with sides of equal lengths, and equal heights. All the surfaces are flat.
Different: A and B are prisms but C is a pyramid.

C2. For A and B, multiply the area of the base by the height. For C, multiply the area of the base by the height and divide the result by 3.

C3. A has the greatest volume. The triangular prism's base has only half the area of the rectangular prism, and their heights are the same, so, the triangular prism's volume will be half of the rectangular prism's. The pyramid has the same base area and height as the rectangular prism, so we know the pyramid's volume will be one-third that of the prism.

C4. In each figure, find the area of each surface and add.

C5. For B: You need to find the length of the hypotenuse of the triangular base. Use the Pythagorean theorem to calculate this.
For C: You need to find the slant height of any one lateral surface. The slant height will be the hypotenuse of the right triangle where the other two sides are the perpendicular distance from the vertex of the pyramid to the middle of the base, and half the length of the base.

Practise

If you have covered the Investigate and Examples 1 to 3 in class, students should not have difficulty with questions 1 to 5.

Connect and Apply

Questions 6, 8, and 10 involve determining one of the dimensions of a prism or pyramid given the volume and another dimension. Students may need assistance with these at first. Remind students to rearrange the formulas to solve for the unknown. This will require algebra skills from previous chapters.

Achievement Check Answers (page 443)

14. Answers will vary

a) The box could have dimensions 20 cm by 20 cm by 20 cm (found by trial and error—three numbers whose product is 8000).

For the cylinder, start with a radius of 10 cm (this makes a diameter of 20 cm which is similar to the box).

The area of the base is 314.159…

Dividing the volume by the area of the base gives the height of about 25.46 cm.

b) The surface area of the prism is 6(20)(20) or 2400 cm^2.

The surface area for the cylinder is
$2(314.16) + 25.46(2)(\pi)(10)$ or 227 cm^3.

c) Answers will vary.
The cylinder would seem to be the better choice since it requires less material. Other choices for dimensions could lead to different conclusions.

Another consideration might be aesthetics, such as approximating the Golden ratio for one face's dimensions (i.e., side view). Boxes may be more efficient to pack in shipping containers since there are no gaps between boxes. Consumer research may indicate that one shape is preferred over the other.

Extend

In question 16, to calculate the surface area of the frustum, students may need some guidance. The easiest way to calculate the surface area is to find the areas of the four trapezoidal faces and then add the areas of the square top and bottom. Note that in question 16b), the bottom of the frustum is not painted and so this base should be excluded in the surface area calculation.

Literacy Connections

Word Origins III

Ask students, *Can you think of other situations in which we use the word* lateral*?* Have students find three words that have Latin origins, and three words that have Greek origins. Encourage them to find ways to use these words in their everyday conversations this week.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 7
Typical	1–7, 9–11, 13
Extension	15–18

8.4 Surface Area of a Cone

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
444–450

Suggested Timing
80–160 min

Tools
- models of cones
- construction paper
- scissors
- rulers
- compasses
- tape

Technology Tools
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM A18 My Progress as a Problem Solver
BLM 8.4.1 Practice: Surface Area of a Cone
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis
- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);
MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating With Exponents
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

The Get Ready sections Calculate Perimeter and Circumference and Apply Area Formulas, specifically for circles and surface area of cylinders, will review skills necessary for this section. Assign questions 2, 4b), and 6b) before starting this section, if they have not been completed earlier.

Warm-Up

Present students with models of cones (plastic or wood models, as well as some constructed from construction paper). Discuss what the lateral surface would look like if flattened into a net.

Have students construct different sectors of a circle. For example, using a circle with a radius of 10 cm, have them construct a $\frac{3}{4}$ sector of the circle, a semi-circle, and a $\frac{1}{4}$ sector of the circle. Then, have the students tape the radius edges together to form various cones.

Discuss the slant height of the cone. Point out to students that the radius of their sector has become the slant height of the cone formed. Compare the heights of the various cones and the radii of their bases. No actual measurements are necessary here, just a discussion of their relative sizes.

Use the Warm-Up as a lead-in to the Investigate. The cones created from the sectors of the circle with radius 10 cm are the same ones that will be constructed in the Investigate. This will lessen the time needed for the Investigate, and provide students with the opportunity to have some hands-on experience in forming cones.

Teaching Suggestions

- Assign the Investigate. If the Warm-Up is used, less time may be needed. (15–20 min)
- Follow up on the Investigate with a class discussion. Using the class results from the Investigate, discuss with students that the lateral surface of the cone is proportional to the circumference of the cone. When less lateral surface area was used to form their cone, the circumference of the cone was smaller. For example, if a $\frac{3}{4}$ sector of the circle was used,

Common Errors

- Some students may continue to struggle with the concepts of slant height and height as they did with pyramids.

R_x By doing repeated calculations involving the slant height and height of the cone, students will realize that the slant height is the hypotenuse of the right triangle and the radius and height are the legs of the triangle. Students should begin to pick up on their own errors, recognizing that the slant height should always be longer than the height of the cone.

- Some students may have trouble deciding when the surface area of the cone should include the base and when the base should not be considered.

R_x When working on questions where the cone is presented without a context, students should include the base in the surface area calculation. However there may be cases with real-life scenarios where the base is not part of the cone, for example, a conical drinking cup or a conical pile of sand.

Ongoing Assessment

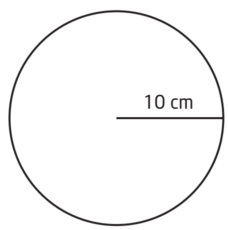
- Question 11, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

the circumference of the cone was $\frac{3}{4}$ the circumference of the original circle. If a semi-circle was used, the circumference of the cone was $\frac{1}{2}$ the circumference of the original circle. When a $\frac{1}{4}$ sector of the circle was used, the circumference of the cone that resulted was $\frac{1}{4}$ the circumference of the original circle.

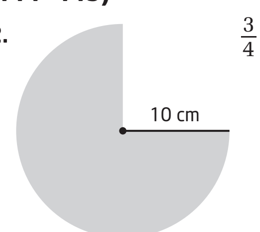
- With this experience with different cones, students are more likely to accept and understand the ratio concepts and the algebra shown in the discussion. Explain to students that they would not be expected to present this proportional reasoning on their own, but they should be able to follow the reasoning as you discuss it with them. The end result of the Investigate is the lateral surface area of a cone, πrs, where r is the radius of the cone and s is the slant height of the cone.
- Assign the Example. (5 min)
- Discuss the Communicate Your Understanding. (5–10 min)
- Assign the Practise questions.
- You may wish to use **BLM A18 My Progress as a Problem Solver** as a student self-assessment.
- You may wish to use **BLM 8.4.1 Practice: Surface Area of a Cone** for remediation or extra practice.

Investigate Answers (pages 444–445)

1.



2.



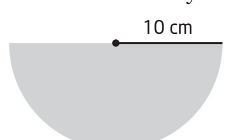
3. Answers will vary. The height should be about 6.61 cm and the radius should be about 7.5 cm.

4. $s = 10$ cm. The length of the third side is equal to the radius of the original circle.

5. 47.1 cm; $\frac{3}{4}$

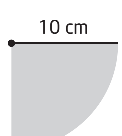
6. $\frac{3}{4}$

7. a) Answers will vary.



b) Step 3: Answers will vary. The height should be about 8.66 cm and the radius should be about 5 cm.
Step 4: The length of the third side, s, is equal to the radius of the original circle.
Step 5: 31.4 cm; $\frac{1}{2}$
Step 6: $\frac{1}{2}$

8. a) Answers will vary.



b) Step 3: Answers will vary. Height should be about 9.68 cm and the radius should be about 2.5 cm.
Step 4: The length of the third side, s, is equal to the radius of the original circle.

Accommodations

Visual—Give students a mathematical model of a cone to help them understand the concept of slant height.

Spatial—Encourage students to use paper to create mathematical models of the shapes in this section.

Language—Allow students to work with a partner when using technology to complete the questions in this section. Have one student be responsible for reading the instructions for using *The Geometer's Sketchpad®* aloud to their partner who will be using the software.

Memory—Let students use cue cards to memorize the formulas in this section.

Step 5: 15.7 cm; $\frac{1}{4}$
Step 6: $\frac{1}{4}$

9. Reflect: The ratio of the areas is the same as the ratio of the circumferences.

$$\frac{\text{Lateral area of cone}}{\text{Area of circle}} = \frac{\text{Circumference of cone}}{\text{Circumference of circle}}$$

Communicate Your Understanding Responses (page 447)

C1. Difference: The cone formed from the larger sector has the greater radius. The cone formed by the smaller sector has the greater height.
Similarity: Both cones have the same slant height.

C2. Difference: The cone formed from the sector of the larger circle has the greater radius, height, slant height, and circumference.
Similarity: The two cones are in proportion.

$$\frac{\text{Radius of cone 1}}{\text{Radius of cone 2}} = \frac{\text{slant height of cone 1}}{\text{slant height of cone 2}}$$

$$\frac{\text{Height of cone 1}}{\text{Height of cone 2}} = \frac{\text{circumference of cone 1}}{\text{circumference of cone 2}}$$

C3. No. When the slant height is doubled, the radius and the height must be increased proportionally by the Pythagorean theorem: $s^2 = r^2 + h^2$. So when s is doubled, $(2s)^2 = (2r)^2 + (2h)^2$. Then,

$$\begin{aligned} SA &= \pi(2r)^2 + \pi(2r)(2s) \\ &= 4\pi r^2 + 4\pi rs \\ &= 4(\pi r^2 + \pi rs) \end{aligned}$$

The surface area is actually quadrupled.

Practise

Students should be comfortable with the Practise questions after working through the Example together.

Connect and Apply

Students will quickly realize that the Pythagorean theorem is used in almost every question. Remind students that the surface area of a cone may or may not include the base of the cone. They must decide in each question if the base should be included, and they must learn to make their decision based on the context of the question.

In question 5, the lateral surface area and the radius are given, and students are required to solve for the slant height. This will require rearrangement of the formula. It may be helpful to do an example with the class to review the necessary algebra skills.

Questions similar to questions 6 and 7 have appeared in earlier sections. Many students will now be comfortable with the algebraic manipulation required when looking at the general case. Weaker students should be encouraged to try a particular cone and determine if the surface area doubles or not.

Questions 8 and 9 involve cones that just fit inside a cube or a cylinder. Explain to students that it will be assumed that the inside dimensions of the cube and cylinder are given. (For example, they may use 10 cm as the diameter and height of the cone. The outside dimensions of the cube would be slightly larger than 10 cm.)

Question 10 involving the frustum of a cone will be easier for students who completed question 16 involving the frustum of a pyramid in Section 8.3. In this case, the surface area calculation can be completed by subtracting the lateral surface area of the small cone that has been removed from the lateral surface area of the original large cone. Note that the surface area should include the top of the frustum (the base of the small cone that has been removed), and the base of the frustum (the base of the large cone).

Question 11, the Chapter Problem question, involves a frustum as well. Part a) lets the student decide whether the base of the frustum should be painted with the glaze. Some may argue that it should be to make the base weatherproof, but others may argue that a special glaze is not necessary on the bottom of the base of a birdbath. Either answer is acceptable.

Extend

Question 13 is an extension to question 8.

Question 14 is similar to question 5, but the re-arrangement of the lateral surface area formula is done in part a) before the known values have been substituted.

In question 15, the surface area of the volcano does not include the base.

If computers are available, students would benefit from completing question 16, which is a *The Geometer's Sketchpad®* activity. Remember that students have access to the Student Edition of *The Geometer's Sketchpad®*, so this activity could be completed at home by some students. You may wish to use **BLM *T4 The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** to support this activity.

Literacy Connections

Word Origins IV

Ask students, *Are there other situations in life in which you use the word* sector*?* Have students write a paragraph to explain one of the situations.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4
Typical	1–9, 12
Extension	13–16

8.5 Volume of a Cone

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
451–456

Suggested Timing
80 min

Tools
- empty cylindrical cans (or plastic models of cylinder and cone)
- construction paper
- scissors
- tape
- sand, rice, or other suitable materials

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad*®
- Microsoft® *Excel*
- computers

Related Resources
BLM A11 Group Work Assessment Recording Sheet
BLM 8.5.1 Practice: Volume of a Cone
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM T3 Microsoft® *Excel*

Mathematical Processes Expectations Emphasis
- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \dfrac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \dfrac{(\text{area of base})(\text{height})}{3}$;

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating with Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

The Get Ready sections Apply Area Formulas and Calculate Surface Area and Volume provide the needed skills for this section. You may wish to have students complete the questions involving circles and cylinders before starting this section. Assign questions 4b), 6b), and 7, if they have not been completed yet.

Warm-Up

1. Determine the volume of each figure:
   a) square-based prism 5 cm by 5 cm by 18 cm
   b) square-based pyramid with a base length of 5 cm and a height of 18 cm

2. How do the volumes of the prism and pyramid in question 1 compare?

3. Determine the volume of the following:
   a) a cylinder with radius 6 cm and height 20 cm
   b) a cylinder with height of 4 m and diameter of 2 m

Warm-Up Answers

1. a) 450 cm^3 **b)** 150 cm^3

2. The volume of the pyramid is one-third the volume of the prism.

3. a) 2261.9 cm^3 **b)** 12.6 m^3

Common Errors

- Some students may experience trouble using the volume formula at first because of the fraction involved.

R_x Ensure that students realize that $V = \frac{1}{3}\pi r^2 h$ is the same as $V = \frac{\pi r^2 h}{3}$, is the same as $V = \frac{\pi}{3} r^2 h$. They may need help using their calculators at first. It may be helpful to use an overhead graphing calculator to demonstrate the different ways the values can be entered.

Ongoing Assessment

- Question 10, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

Accommodations

Gifted and Enrichment—Challenge students to create a cone shape and use it to determine the different geometrical shapes, such as a circle or an ellipse, that can be made by cutting the cone in different ways.

Visual—Let students work with a partner when using *The Geometer's Sketchpad®*.

Memory—Review with the students the Pythagorean theorem and the steps required to use a graphing calculator to do a series of calculations.

Teaching Suggestions

- Have students bring in empty cans from home to use for the Investigate. Have students work with a partner or in small groups. (10–15 min)
- You may wish to use **BLM A11 Group Work Assessment Recording Sheet** to assist you in assessing your students.
- Alternatively, you could demonstrate the Investigate for the class or use models of a cylinder and cone with the same base and height. Fill the models with water to demonstrate the volume relationship. (5–10 min)
- Review Examples 1 and 2. (5–10 min)
- Discuss the Communicate Your Understanding questions. (5 min)
- You may wish to use **BLM 8.5.1 Practice: Volume of a Cone** for remediation or extra practice.

Investigate Answers (page 451)

Answers will vary. Sample solution:

1. Radius of can is 10 cm and height is 20 cm.

2. a) $(\text{Slant height})^2 = 20^2 + 10^2$
Slant height = 22.36 cm

3. about 3

4. a) If a cone and a cylinder have the same height and the radius of their bases is also equal, then:
Volume of the cone = $\frac{1}{3}$ × volume of the cylinder

b) $V = \frac{1}{3}\pi r^2 h$

Communicate Your Understanding Responses (page 454)

C1. The volume of the cylinder is three times the volume of the cone. Create a paper cone and a paper cylinder with the same height and radius. Start filling the cone with rice and pouring it into the cylinder. Approximately three cones filled with rice, when poured into the cylinder, should fill the cylinder up.

C2. Doubling the height of a cone will double the volume of the cone.

C3. Doubling the radius of a cone quadruples the volume of the cone.

Practise

Practise questions 1, 2, and 4 are similar to the Examples, so students should have few problems completing these on their own.

Question 3 requires the use of the Pythagorean theorem to determine the height of the funnel given the slant height and radius.

Connect and Apply

In question 5, students should be reminded that when the cone "just fits" inside the cylinder, this means that they can assume the cylinder and cone have congruent bases. This question stresses the relationship between the volumes of a cylinder and a cone. Point out to students that it is not necessary to know the dimensions of the cylinder in a case like this. The volume of the cone will simply be $\frac{1}{3}$(volume of the cylinder) or $\frac{1}{3}(300)$, which is 100 cm^3.

Question 6 allows for some student originality.

Question 8 requires students to solve for the height given the volume and base radius. By now, most students should be comfortable with rearranging the equation to solve for the unknown. Students with weaker algebraic skills may need assistance. Once the equation is set up, encourage students to multiply by 3 to remove the fraction from the equation.

Question 9 will help students realize the different effects that the radius and height have on the volume. In question 9a), some students may predict that the volumes of the cones will be the same. After calculating the volumes in question 9b), students should understand the different roles of r and h in the volume formula.

Question 10, the Chapter Problem, is a revisit of the Chapter Problem from Section 8.4 (question 11). In this question, students are calculating the volume of the concrete in the same fountain that they calculated the surface area for earlier.

In question 11, students may need reminding that 1 L = 1000 cm^3. Question 11a) has students rearrange the formula before substituting, and then in question 11b), solve for the height, as in question 8.

Question 12 is similar to questions 8 and 11, but this time, students will solve for the radius. This requires a higher level of algebraic skills, since students must take the square root in order to determine the radius.

Extend

Question 13 examines the ratio of the volume of a cone as compared to the volume of a cube surrounding it. Stronger students will enjoy estimating this ratio and as an extension could determine this ratio in general.

$$\begin{aligned}\frac{V_{cone}}{V_{cube}} &= \frac{\frac{1}{3}\pi\left(\frac{s}{2}\right)^2 s}{s^3}\\ &= \frac{\frac{1}{12}\ \pi s^3}{s^3}\\ &= \frac{\pi}{12}\end{aligned}$$

Question 14 will extend the students' algebra skills. Have students represent the height with $2r$ in the volume formula and solve for r.

Question 15 has students using a graphing calculator, *The Geometer's Sketchpad*®, or spreadsheet software. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3**, **BLM T5 *The Geometer's Sketchpad*® 4**, or **BLM T3 Microsoft® *Excel*** to support this activity.

In question 16, when students substitute the height of 20 cm, the volume formula becomes $V = \frac{20}{3}\pi r^2$. You may wish to have students use a graphing calculator, consider the radius to be the variable x, and graph $y = \left(\frac{20}{3}\right)\pi x^2$. Discuss the fact that the quadratic relation that results is an example of a non-linear relation.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4
Typical	1–9, 11, 12
Extension	13–17

8.6 Surface Area of a Sphere

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
457–461

Suggested Timing
80 min

Tools
- oranges
- string
- rulers
- centimetre grid paper
- plastic balls

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G9 Centimetre Grid Paper
BLM 8.6.1 Practice: Surface Area of a Sphere
BLM A1 Assessment Recording Sheet
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Operating with Exponents
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8^3 by using a calculator]);

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \frac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \frac{(\text{area of base})(\text{height})}{3}$;
MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Link to Get Ready

The Get Ready sections Calculate the Perimeter and Circumference and Calculate the Surface Area and Volume review skills needed for this section. In particular, the questions involving circles and cylinders, such as questions 2 and 6b) should be completed before starting this section.

Warm-Up

1. Find the area of the following:
  a) a circle with radius 6 cm
  b) a circle with diameter 3 m

2. Find the surface area of each figure:
  a) a cylinder with radius 6 cm and height 8 cm
  b) a cone with radius 6 cm and height 8 cm

3. Find the radius of the following, correct to two decimal places:
  a) a circle with circumference 31.4 cm
  b) a circle with circumference 1 m

Warm-Up Answers

1. a) 113.1 cm^2 **b)** 7.1 m^2
2. a) 527.8 cm^2 **b)** 301.6 cm^2
3. a) 5.00 cm **b)** 0.16 m

Common Errors

- Some students may continue to struggle with the proper units for surface area. Since they are dealing with a three-dimensional object, some will have trouble visualizing the surface area of the sphere.

R_x The Investigate will help reinforce the concepts of surface area as the students flatten the orange peel to cover the centimetre grid paper. A plastic ball cut and flattened out will help students visualize the surface area of the sphere as well.

Ongoing Assessment

- Question 7, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Perceptual—Provide students with three-dimensional shapes to help them understand the concept of a shape such as a cube inscribed in a sphere.

Spatial—Encourage students to use technology to solve the questions in this section.

Teaching Suggestions

- Assign the Investigate. Have students work in small groups. You may wish to use **BLM G9 Centimetre Grid Paper** to support this activity. (15–25 min)
- An orange is needed for each group. Oranges that peel easily, such as Clementine oranges, will be easier and less messy to work with. Ensure that you check student allergies before introducing any food to the classroom. While the students work with the oranges, you could cut apart an inexpensive plastic ball (the type where the plastic does not stretch when inflated), about 30 cm in diameter. Such balls are available from a dollar store. Have groups share their results with the class. Class discussion should lead the students to the formula $SA = 4\pi r^2$.
- Discuss Examples 1 and 2. Example 2 requires finding the radius of a sphere given its surface area. (10 min)
- Discuss the Communicate Your Understanding questions.
- Assign the Practise questions.
- You may wish to use **BLM 8.6.1 Practice: Surface Area of a Sphere** for remediation or extra practice.

Investigate Answers (page 457)

Answers will vary. Sample answers are provided below.

1. An estimate of the surface area of an orange might be between 140 and 170 cm.

2. a) Circumference of the orange is 22 cm.
b) Radius of the orange is 3.5 cm.

3. almost 155 cm^2

4. a) 38.5 cm^2
b) approximately 4:1
c) A possible formula for the surface area of a sphere is four times the area of a circle with the same radius.

5. $SA = 4\pi r^2$

Communicate Your Understanding Responses (page 459)

C1. Step 1: Use a string to go around the surface of the softball.
Step 2: Use the length of this string and the formula for the circumference of a circle to find the radius of the softball.
Step 3: To determine the surface area, substitute the value of the radius into the formula for the surface area of a sphere, $(SA = 4\pi r^2)$.

C2. No, doubling the radius will quadruple the surface area of a sphere. The radius is squared in the formula for the surface area of a sphere.

$SA_{\text{old}} = 4\pi r^2$

$SA_{\text{new}} = 4\pi(2r)^2$

$SA_{\text{new}} = 4(4\pi r^2)$

$SA_{\text{new}} = 4 \times SA_{\text{old}}$

Practise

Practise questions 1 and 2 are similar to Example 1.

Question 3 is similar to Example 2.

Connect and Apply

Questions 4 to 7 are typical application questions.

In question 5, students should recognize that they are assuming that the surface of Earth is smooth and that it is spherical.

In question 8, students predict how the surface area increases when the radius is increased.

Question 9 has students examine the quadratic nature of the surface area function and use the features of the graphing calculator to determine the surface area given the radius and vice versa. You may wish to use **BLM A1 Assessment Recording Sheet** to assist you in assessing students' use of technology.

Extend

Question 10 requires the algebra skills necessary to rearrange the surface area formula. Students may need assistance with the rearranging. Example 2 and question 3 have already involved rearranging with substituted values, so this is the same process for the general case. The graphing calculator is used to graph this square root function.

Question 11 is an extension of question 8. It looks at the factor by which the surface area increases when the diameter is tripled.

Question 13 is a *The Geometer's Sketchpad*® activity that looks at the ratio of the surface areas of a cube and a sphere that just fits inside the cube. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity. An extension of this activity would be to consider the algebraic model for this problem:

$$\begin{aligned}\frac{SA\text{ of cube}}{SA\text{ of sphere}} &= \frac{6(2r)^2}{4\pi r^2}\\ &= \frac{24r^2}{4\pi r^2}\\ &= \frac{6}{\pi}\end{aligned}$$

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–6, 8, 9
Extension	10–13

8.7 Volume of a Sphere

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
462–469

Suggested Timing
80 min

Tools
- cylindrical containers that each just hold three tennis balls
- three tennis balls for each group of students
- water
- containers to catch the overflow water
- identical small spheres (e.g., marbles, tennis balls, or table tennis balls)

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM 8.7.1 Practice: Volume of a Sphere
BLM A6 Knowledge/Understanding General Scoring Rubric
BLM 8.7.2 Achievement Check Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \frac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \frac{(\text{area of base})(\text{height})}{3}$;

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating With Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8[3] by using a calculator]);

Link to Get Ready

The Get Ready segment Calculate Surface Area and Volume provides the needed skills for this section. You may wish to have students complete question 7 before starting this section.

Warm-Up

1. Determine the volume of the following figures:
a) a cylinder with radius 8 cm and height 15 cm
b) a cone with radius 8 cm and height 15 cm
c) a cylinder with diameter 4 m and height 4 m
d) a cone with diameter 4 m and height 4 m

Warm-Up Answers

1. a) 3015.9 cm^3 **b)** 1005.3 cm^3 **c)** 50.3 m^3 **d)** 16.8 m^3

Teaching Suggestions

- Discuss the hot air balloon photo. The discussion should include the empty space in the large balloon when it is filled with the ordinary soccer balls. Ask students, *Is the volume of the hot air balloon the same as the volume of the soccer balls inside it? Why or why not?* (5 min)
- Assign the Investigate. (10–20 min) Since it involves the displacement of water, you will need a large container for the overflow of water. Consider performing this Investigate in a science lab where sinks are available. Have students work in small groups for the Investigate, or demonstrate it for the class. The Investigate should lead to the conclusion that the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder.
- Work through Example 1 with the class to develop the formula for the volume of a sphere, $v = \frac{4}{3}\pi r^3$.
- Assign Example 2. (10–15 min)

Common Errors

- Some students may have trouble with the formula because of the fraction involved.

R_x Ensure that students realize that $V = \frac{4}{3}\pi r^3$ is the same as $V = \frac{4\pi r^3}{3}$, is the same as $V = \frac{4\pi}{3}r^3$. They may need help using their calculators at first. Using an overhead graphing calculator to demonstrate the different ways the values can be entered may be helpful.

Ongoing Assessment

- Use Achievement Check question 11 to monitor student success. See Achievement Check Answers and **BLM 8.7.2 Achievement Check Rubric**.
- Question 6, the Chapter Problem question, can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

Accommodations

Gifted and Enrichment—Challenge students to investigate the aerodynamics related to hot air balloons.

Motor—Encourage students to work with a partner when working through the Investigate in this section.

Memory—Allow students to use formulas when completing quizzes and tests.

- Assign and discuss the Communicate Your Understanding questions. (10 min)
- Assign Practise questions 1 to 4.
- You may wish to use **BLM 8.7.1 Practice: Volume of a Sphere** for remediation or extra practice.

Investigate Answers (pages 462–463)

1. Answers will vary. Students will likely estimate the diameter to be about 6.5 cm. Not knowing any formula for a sphere, they might consider the volume of a cube with sides 6.5 cm and estimate that the sphere would fill about $\frac{2}{3}$ of that space. So, $\frac{2}{3} \times 6.5^3$ is about 183 cm^3. Or, they might compare the volume of the sphere to the volume of the cylinder and estimate that it would fill a fraction of that space. Anything in the ballpark from 125 to 225 cm^3 would be reasonable. (The actual volume is about 143.79 cm^3.)

2. a) The diameter of the cylinder is about 7 cm.
b) The height of the cylinder is about 21 cm.

5. The height of the displaced water is about 14 cm.

6. $\frac{2}{3}$; Volume of the displaced water is equal to the volume of the three tennis balls.

7. $\frac{2}{3}$

8. Reflect: The volume of a sphere is two-thirds the volume of a cylinder with the same radius and a height equal to the diameter of the sphere.

Volume of the sphere $= \frac{2}{3} \times$ volume of the cylinder

To find the volume of one tennis ball, divide the volume of the displaced water by 3.

9. Answers will vary.

Communicate Your Understanding Responses (page 465)

C1. Step 1: Use the formula for the surface area of a sphere to find the radius.
Step 2: Substitute the value of the radius into the formula for the volume of a sphere.

C2. If you doubled the volume of a sphere, its volume would be eight times the original volume.

Practise

Practise questions 1 to 4 are similar to Examples 1 and 2. Students should not have difficulty if the Examples are covered with the class before assigning these questions.

Connect and Apply

Question 5 requires that students use proportional reasoning to calculate the mass of the lollipop. This assumes that both lollipops have the same density and that the largest one was also spherical.

Question 6, the Chapter Problem question, is a sphere just fitting inside a cylinder. The students must assume that the dimensions given are of the interior of the cylinder. You might discuss with the class that this cylinder might be made of packing foam to protect the glass gazing ball that it holds. Most students will have experienced packages like this.

Question 7 is similar, but here the spheres are in a rectangular prism package. Having a package of golf balls in the classroom may help students visualize the problem. This question does not require a volume calculation, but an extension would be to calculate the empty space in the box by calculating the volume of the golf balls and the volume of the box.

Question 8 involves a silo. This question also requires use of percent skills and calculating the number of truckloads required to fill the silo to 80% capacity. Students should be reminded to round the number of truckloads up (never down) to the nearest whole number in this kind of question. You may wish to use **BLM A6 Knowledge/Understanding General Scoring Rubric** to assist you in assessing your students.

For question 9, help students realize that the two hemispherical ends of the tank truck go together to make a complete sphere, so the volume can be calculated by finding the sum of the volume of the cylinder and the sphere.

Question 10, a Fermi problem, will lead to some great discussion about basketballs filling the room and how much space would be empty. Students could start by imagining layers of basketballs arranged in the room. Lead the class discussion to realize that more basketballs could fit into the room if the spheres fit more closely together. It would be helpful to have several spheres on hand (marbles, tennis balls or table tennis balls) to illustrate this packing technique. The idea here is to choose a reasonable estimation technique. A range of answers is appropriate.

In question 11, the Achievement Check, lead students to recognize different factors that they should take into consideration in part c). For example, amount of wasted space, amount of material required to make the package, ease of stacking and shipping, etc. Either answer is appropriate with proper justification. You may wish to use **BLM 8.7.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 467)

11. a) The square-based prism has sides 8.5 cm by 8.5 cm by 17 cm.

$$\begin{aligned}\text{Volume of prism} &= 8.5 \times 8.5 \times 17 \\ &= 1228.25\ \text{cm}^3\end{aligned}$$

The cylinder has a diameter of 8.5 cm and a height of 17 cm.

$$\begin{aligned}\text{Volume of cylinder} &= \pi \times (4.25)^2 \times 17 \\ &= 964.7\ \text{cm}^3\end{aligned}$$

b) The amount of empty space in each package is the volume of the package minus the volume of the two tennis balls.

$$\begin{aligned}\text{Volume of 2 tennis balls} &= 2 \times 4 \div 3 \times \pi \times (4.25)^3 \\ &= 643.1\ \text{cm}^3\end{aligned}$$

$$\begin{aligned}\text{Empty space in the square-based prism} &= 1228.25 - 643.1 \\ &= 585.15\ \text{cm}^3\end{aligned}$$

$$\begin{aligned}\text{Empty space in the cylinder} &= 964.7 - 643.1 \\ &= 321.6\ \text{cm}^3\end{aligned}$$

There would be almost twice as much empty space in the square-based prism package.

c) Answers may vary. The cylinder will be a better choice if the point is to leave less empty space in the package. The cylinder is also the better choice if it is important to use less packaging material.

$$\begin{aligned}SA_{\text{cylinder}} &= 2 \times \pi \times (4.25)^2 + 2 \times \pi \times 4.25 \times 17 \\ &= 567.5\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}SA_{\text{Square-based prism}} &= 2 \times (8.5)^2 + 4 \times 8.5 \times 17 \\ &= 722.5\ \text{cm}^2\end{aligned}$$

The package design may also vary depending on the style of print and colour that is used to make it catch the eye. (e.g., triangular prism, colour panels).

Extend

Question 12 involves rearranging the volume formula. Some of your students will be quite adept at this rearranging by now, but to solve for the radius it is necessary to take the cube root, which has not been done previously.

For question 13, enter $y = \frac{4}{3}\pi x^3$ into the graphing calculator. Have students choose an appropriate window for this function. This question requires the use of the Trace feature on the graphing calculator. Students will find this question easier if question 9 from Section 8.6 Surface Area of a Sphere was assigned previously. Have students graph both the surface area and the volume functions simultaneously to compare them as an extension activity. Have a discussion about their relative rates of change. You may wish to use the terms *quadratic* and *cubic* to describe these functions, but simply treating the graphs as examples of non-linear relations would suffice. You may wish to tell students that they will study functions like these in more detail in future courses.

Question 14 requires significant algebra skills, but is appropriate for your stronger students. As an extension, have students look at the general case when surface area is doubled. If r represents the original radius and if R represents the radius of the larger sphere,

$$\begin{aligned} 2(4\pi r^2) &= 4\pi R^2 \\ 2r^2 &= R^2 \\ \sqrt{2r^2} &= R \\ \sqrt{2}r &= R \end{aligned}$$

Now, if the volume of the large sphere is considered,

$$\begin{aligned} V &= \frac{4}{3}\pi\left(\sqrt{2}r\right)^3 \\ &= \frac{4}{3}\pi\left(2\sqrt{2}r^3\right) \\ &= 2\sqrt{2}\left(\frac{4}{3}\pi r^3\right) \\ &= 2\sqrt{2}\text{ (original volume)} \end{aligned}$$

Students would use the decimal approximation for $2\sqrt{2}$, but you could show them that this is the exact value. This would only be appropriate for your best students.

Question 15 is suitable for most students, although the estimating in part a) will be more difficult for weaker students. Again, your better students could be encouraged to look at the general case for this question, which appears in question 16.

Question 17 is a *The Geometer's Sketchpad®* activity that students could try at home with the Student Edition of *The Geometer's Sketchpad®* or in class, if time allows and computers are available. You may wish to use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** to support this activity.

Question 18, a Math Contest question, is a good question to summarize the volume concepts from this chapter. Most students can handle this question by doing the volume calculations. Encourage your better students to try to answer the question without doing the actual calculations and by taking a more algebraic approach.

Question 19, a Math Contest question, is an extension of question 6, the Chapter Problem question, and would be a great extension to the Investigate where a can of tennis balls was used.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4
Typical	1–5, 7–10
Extension	12–19

Student Text Pages
470–471

Suggested Timing
80 min

Related Resources
BLM 8.CR.1 Chapter 8 Review

Ongoing Assessment

- Upon Completing the Chapter Review, students can also answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *Which questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related example in the text to help you with the questions? For which questions did you have to do this?*

Using the Chapter Review

Each question reviews different skills and concepts. The students might work independently to complete the Chapter Review, then with a partner to compare solutions. Alternatively, the Chapter Review could be assigned to reinforce skills and concepts in preparation for the Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with concepts they find difficult. Use **BLM 8.CR.1 Chapter 8 Review** for the Chapter Review.

After students complete the Chapter Review, encourage them to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content.

Chapter 8 Practice Test

Student Text Pages
472–473

Suggested Timing
50–70 min

Related Resources
BLM 8.PT.1 Chapter 8 Practice Test

BLM 8.CT.1 Chapter 8 Test

Summative Assessment

After students complete **BLM 8.PT.1 Chapter 8 Practice Test**, you may wish to use **BLM 8.CT.1 Chapter 8 Test** as a summative assessment.

Accommodations

Perceptual—Assign specific questions to groups of students and have them present their answers to the class.

Motor—Allow students to do fewer questions when completing the Chapter Review and Chapter Test.

Memory—Provide formula sheets for the students to use when completing the questions in this section.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	8.7	Example 1 (page 463)
2	8.2	Example 2 (page 429)
3	8.3	Example 1 (page 437)
4	8.4	Example (page 446)
5	8.1	Example 2 (page 420)
6	8.3	Example 1, 2 (pages 437, 438)
7	8.3	Example 1 (page 437)
8	8.7	Example 1 (page 463)
9	8.4	Example (page 446)
10	8.5	Example 1 (page 452)
11	8.7	Example 2 (page 464)
12	8.7	Example 2 (page 464)

Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?
- Apply the Pythagorean theorem where appropriate to determine unknown dimensions in measurement problems
- Calculate the perimeter and area of composite figures (combinations of rectangles, triangles, parallelograms, trapezoids, and circles)
- Calculate the surface area and volume of rectangular prisms, triangular prisms, cylinders, and pyramids
- Draw the net for a rectangular prism, triangular prism, cylinder, and pyramid
- Recognize from their investigative experiences that the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height
- Calculate the surface area of a cone given its radius and slant height
- Calculate the surface area of a cone given its height and either the radius or the slant height
- Calculate the volume of a cone given its radius or diameter and its height
- Recognize from their investigative experiences that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height
- Calculate the height of a cone given its radius and slant height in order to determine the volume of the cone
- Calculate the surface area of a sphere
- Calculate the volume of a sphere
- Calculate the surface area and volume of composite figures (combinations of prisms, pyramids, cones, cylinders, and spheres or hemispheres)
- Solve problems that involve the surface area and volume of prisms, pyramids, cylinders, cones, spheres, and composite figures

Student Text Pages
473

Suggested Timing
30–60 min

Tools
- compasses
- grid paper
- centimetre grid paper

Technology
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM G9 Centimetre Grid Paper
BLM 8.CP.1 Chapter 8 Problem Wrap-Up Rubric

Summative Assessment
- Use **BLM 8.CP.1 Chapter 8 Problem Wrap-Up Rubric** to assess student achievement.

Using the Chapter Problem

Students can complete the Chapter Problem individually or with a partner.

The Chapter Problem Wrap-Up can be used as an assessment piece, either completed in class or at home.

The Chapter Problem is designed to allow for student creativity. The diameter of the cylindrical base is given, but otherwise the students will determine the dimensions of the fountain. You may wish to set up a spreadsheet with the relevant formulas to check the dimensions chosen by each individual student.

The cone can be set on top of the cylindrical base, inside the cylinder, or inverted on the cylinder. Students will need to fully describe the fountain and its construction on their sketch.

Ask students to submit their sketch showing all dimensions in advance to allow you to identify any problems that students may encounter with their design. Conference with particular students who may need guidance with their design to complete the next steps in calculating the volume and surface area of their fountains.

Although students have had lots of experience with surface area and volume of three-dimensional shapes in this chapter, many of them will benefit from having models of these shapes available in the classroom. You may wish to modify the first part of the problem to require students to make a scale model of their fountain as well as a sketch. Another useful aid would be to have posters visible in the classroom showing the relevant surface area and volume formulas. Ensure that graphing calculators are available for student use.

If the Chapter Problem questions have been assigned throughout the chapter, review each of them, emphasizing the skills and formulas that were used. This could be done as a class or in small groups, perhaps using a jigsaw technique.

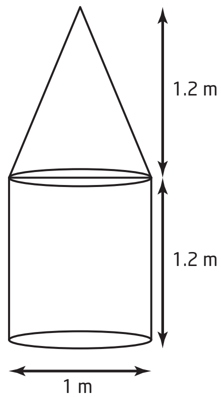
Due to the complexity of this problem, students would benefit from an opportunity to brainstorm approaches to the problem, with a partner or in small groups, before they begin. One strategy is to allow students to discuss the problem but not to write anything down until they begin individual work. Another strategy is to introduce the problem one day, but not assign it to be completed until another day.

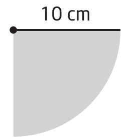
When assigning this problem, it is important to allow enough time for students to think, plan, and complete the work. Thirty minutes will be required for a strong class and more for an average class.

Students will find it useful to have compasses in order to create their sketches. They may also find it helpful to do the sketches on grid paper (regular or centimetre). You may wish to use **BLM G9 Grid Paper** or **BLM G10 Centimetre Grid Paper** to support this activity.

Level 3 Sample Response

a) This is my sketch of the fountain.





b) I need to find the volume of the two pieces.

For the cylinder, $V = \pi r^2 h$
$= \pi(0.5)^2(1.2)$
$= 0.94\ \text{m}^3$

For the cone, $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}(0.94)$
$= 0.31\ \text{m}^3$

The total volume of concrete I need to make the fountain is $1.25\ \text{m}^3$.

c) The surface area to be painted is made up of two surfaces: the slant side of the cone and the vertical side of the cylinder.

For the cone, the slant height is found using the Pythagorean theorem.

$s^2 = (1.2)^2 + (0.5)^2 \longrightarrow s^2 = 1.69 \longrightarrow s = 1.3\ \text{m}$

The surface area of the cone is $A = \pi rs$
$= \pi(0.5)(1.3)$
$= 2.04\ \text{m}^2$

The area of the side of the cylinder is a rectangle.
$A = \pi(1)(1.2)$ or $3.77\ \text{m}^2$.

The total surface area to be painted is $5.81\ \text{m}^2$.

d) The cost of the fountain is

C = concrete + paint
$= (1.25)(100) + 2(17.50)$
$= \$160.00$

I needed two cans of paint because my area of $5.81\ \text{m}^2$ was just over the coverage of one can.

Level 3 Notes

At this level, look for the following:

- A rough sketch labelled with all necessary dimensions
- Volume and area are calculated with only minor errors
- Relevant formulas are evident
- Surface area calculations may include unnecessary areas such as the bottom of the cone and the top of the cylinder
- Some justification of reasoning for steps taken

What Distinguishes Level 2

At this level, look for the following:

- A sketch with some necessary dimensions missing
- Sketch may be two-dimensional rather then three-dimensional
- Volume and areas are partially calculated or may contain major errors
- Calculations may be completed without reference to formulas
- Surface area calculations may only involve one part of the fountain
- Partial, unclear, or incomplete justification of steps taken

What Distinguishes Level 4

At this level, look for the following:

- A precise sketch, drawn to scale, with all necessary dimensions shown
- Volume and area are calculated with no errors
- Formulas and calculations are integrated into the solutions
- Solution may explain why surface area calculation includes the chosen areas
- Complete justification for steps in the solution
- May include comments about the proportions chosen for the fountain