CHAPTER

6

Analyse Linear Relations

Vocabulary

x-intercept
parallel lines
perpendicular lines
negative reciprocals
linear system
point of intersection

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Strand:
Analytic Geometry

Overall Expectations

By the end of this course, students will:
AGV.01 determine the relationship between the form of an equation and the shape of its graph with respect to linearity and non-linearity;
AGV.02 determine, through investigation, the properties of the slope and *y*-intercept of a linear relation;
AGV.03 solve problems involving linear relations.

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

By the end of this chapter, students will:
AG1.01 determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation);
AG1.02 identify, through investigation, the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$;
AG1.03 express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$.

Investigating the Properties of Slope

By the end of this chapter, students will:
AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;
AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Using the Properties of Linear Relations to Solve Problems

By the end of this chapter, students will:
AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the *y*-intercept and slope; graph $2x + 3y = 6$ using the *x*- and *y*-intercepts);
AG3.02 determine the equation of a line from information about the line (e.g., the slope and *y*-intercept; the slope and a point; two points);
AG3.03 describe the meaning of the slope and *y*-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);
AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to $50 to $2550);
AG3.05 determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application.

Chapter Problem

The Chapter Problem is introduced in the Chapter Opener. Ask students if they have ever heard of geocaching, or used a Global Positioning System (GPS). Mathcaching is a similar idea. Students use mathematics, in this case coordinate geometry, to find cues to solve a puzzle. There are four Chapter Problem questions (Sections 6.1, 6.2, 6.5, and 6.7), in which students will find clues to a mystery city located somewhere in Ontario. You may wish to have students complete the Chapter Problem questions as part of the regular exercises or assign the Chapter Problem questions all at once as a review or performance task. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 355, in which students are invited to design a mathcaching puzzle of their own.

Chapter 6 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 6 Opener • 15 min	292–293			
Get Ready • 60–80 min	294–295	• BLM G10 Grid Paper • BLM 6.GR.1 Practice: Get Ready	• BLM 6.GR.2 Get Ready Self-Assessment Checklist	**Tools** • grid paper
6.1 The Equation of a Line in Slope *y*-Intercept Form: $y = mx + b$ • 80 min	296–307	• BLM G16 Investigate Graph • BLM G2 Vertical Number Line • BLM G10 Grid Paper • BLM 6.1.1 Practice: The Equation of a Line in Slope *y*-Intercept Form: $y = mx + b$	• BLM A9 Communication General Scoring Rubric	**Tools** • masking tape • metre sticks • stopwatches or watches that measure seconds **Technology Tools** • graphing calculators • CBR™ motion sensor
6.2 The Equation of a Line in Standard Form: $Ax + By + C = 0$ • 80 min	308–314	• BLM G10 Grid Paper • BLM G16 Investigate Graph • BLM G2 Vertical Number Line • BLM 6.2.1 Practice: The Equation of a Line in Standard Form: $Ax + By + C = 0$ • BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator	• BLM A8 Application General Scoring Rubric • BLM 6.2.2 Achievement Check Rubric	**Tools** • grid paper **Technology Tools** • graphing calculators • Computer Algebra System • TI-89 calculators
6.3 Graph a Line Using Intercepts • 80 min	315–322	• BLM G16 Investigate Graph • BLM G10 Grid Paper • BLM 6.3.1 Practice: Graph a Line Using Intercepts • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM A2 Attitudes Assessment Checklist	**Tools** • grid paper **Technology Tools** • *The Geometer's Sketchpad*® • computers • graphing calculators • CBR™ motion sensor
Use Technology: Use *The Geometer's Sketchpad*® to Explore Parallel and Perpendicular Lines • 80 min	323–325	• BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 The *The Geometer's Sketchpad*® 4 • BLM 6.UT.1 Use Technology: Using a Slider • BLM 6.UT.2 Use Technology: Use the TI-83 Plus or TI-84 Graphing Calculator to Explore Parallel and Perpendicular Lines • BLM G4 Protractor	• BLM A9 Communication General Scoring Rubric	**Tools** • protractors **Technology Tools** • *The Geometer's Sketchpad*® • computers • graphing calculators
6.4 Parallel and Perpendicular Lines • 80 min	326–329	• BLM G16 Investigate Graph • BLM G10 Grid Paper • BLM G4 Protractor • BLM 6.4.1 Practice: Parallel and Perpendicular Lines • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM A7 Thinking General Scoring Rubric • BLM 6.4.2 Achievement Check Rubric	**Tools** • grid paper • protractors **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • computers
6.5 Find an Equation for a Line Given the Slope and a Point • 80 min	330–337	• BLM G10 Grid Paper • BLM 6.5.1 Practice: Find an Equation for a Line Given the Slope and a Point • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM A5 Problem Solving Checklist	**Tools** • grid paper **Technology Tools** • *The Geometer's Sketchpad*® • computers • graphing calculators
6.6 Find the Equation for a Line Given Two Points • 80 min	338–343	• BLM G16 Investigate Graph • BLM G10 Grid Paper • BLM 6.6.1 Practice: Find the Equation for a Line Given Two Points • BLM 6.6.2 Student Success: Think Aloud	• BLM A18 My Progress as a Problem Solver	**Tools** • grid paper • rulers **Technology Tools** • CBR™ motion sensor • graphing calculators

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
6.7 Linear Systems • 80 min	344–351	• BLM G16 Investigate Graph • BLM G10 Grid Paper • BLM 6.7.1 Ski Club Plans • BLM 6.7.2 Practice: Linear Systems • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM A20 Learning Skills Checklist • BLM 6.7.3 Achievement Check Rubric	**Tools** • grid paper • rulers **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • computers
Chapter 6 Review • 80 min	352–353	• BLM G10 Grid Paper • BLM 6.CR.1 Chapter 6 Review	• BLM A16 My Progress as a Mathematician	**Tools** • grid paper
Chapter 6 Practice Test • 60–80 min	354–355	• BLM G10 Grid Paper	• BLM 6.PT.1 Chapter 6 Practice Test • BLM 6.CT.1 Chapter 6 Test	**Tools** • grid paper
Chapter 6 Problem Wrap-Up • 40–80 min	355		• BLM 6.CP.1 Chapter 6 Problem Wrap-Up Rubric	**Technology Tools** • Internet • computers
Chapters 4 to 6 Review • 80 min	356–357	• BLM G10 Grid Paper • BLM A14 Self-Assessment Recording Sheet • BLM A15 Self-Assessment Checklist		**Tools** • grid paper
Task: Salary and Commission • 20–40 min	358	• BLM G10 Grid Paper • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM 6.T1.1 Task: Salary and Commission Rubric	**Tools** • grid paper **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • computers
Task: Cod Fish Catches • 20–40 min	359	• BLM G10 Grid Paper • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM 6.T2.1 Task: Cod Fish Catches Rubric	**Tools** • grid paper **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • computers

Chapter 6 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 6.GR.1	Practice: Get Ready	Practice
	BLM G10	Grid Paper	Student Support
	BLM 6.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
6.1: The Equation of a Line in Slope *y*-Intercept Form: $y = mx + b$			
	BLM G16	Investigate Graph	Student Support
	BLM G2	Vertical Number Line	Student Support
	BLM G10	Grid Paper	Student Support
	BLM 6.1.1	Practice: The Equation of a Line in Slope *y*-Intercept Form: $y = mx + b$	Practice
	BLM A9	Communication General Scoring Rubric	Assessment
6.2: The Equation of a Line in Standard Form: $Ax + By + C = 0$			
	BLM G10	Grid Paper	Student Support
	BLM G16	Investigate Graph	Student Support
	BLM G2	Vertical Number Line	Student Support
	BLM 6.2.1	Practice: The Equation of a Line in Standard Form: $Ax + By + C = 0$	Practice
	BLM A8	Application General Scoring Rubric	Assessment
	BLM 6.2.2	Achievement Check Rubric	Assessment
	BLM T7	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
6.3: Graph a Line Using Intercepts			
	BLM G16	Investigate Graph	Student Support
	BLM A2	Attitudes Assessment Checklist	Assessment
	BLM G10	Grid Paper	Student Support
	BLM 6.3.1	Practice: Graph a Line Using Intercepts	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
Use Technology: Use *The Geometer's Sketchpad*® to Explore Parallel and Perpendicular Lines			
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 6.UT.1	Use Technology: Using a Slider	Student Support Technology
	BLM 6.UT.2	Use Technology: Use the TI-83 Plus or TI-84 Graphing Calculator to Explore Parallel and Perpendicular Lines	Student Support Technology
	BLM A9	Communication General Scoring Rubric	Assessment
	BLM G4	Protractor	Student Support

	BLM	Title	Purpose
6.4: Parallel and Perpendicular Lines			
	BLM G16	Investigate Graphs	Student Support
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM 6.4.1	Practice: Parallel and Perpendicular Lines	Practice
	BLM A7	Thinking General Scoring Rubric	Assessment
	BLM 6.4.2	Achievement Check Rubric	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
6.5: Find an Equation for a Line Given the Slope and a Point			
	BLM G10	Grid Paper	Student Support
	BLM 6.5.1	Practice: Find an Equation for a Line Given the Slope and a Point	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM A5	Problem Solving Checklist	Assessment
6.6: Find the Equation for a Line Given Two Points			
	BLM G16	Investigate Graph	Student Support
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM G10	Grid Paper	Student Support
	BLM 6.6.1	Practice: Find the Equation for a Line Given Two Points	Practice
	BLM 6.6.2	Student Success: Think Aloud	Student Success
6.7: Linear Systems			
	BLM G16	Investigate Graph	Student Support
	BLM G10	Grid Paper	Student Support
	BLM 6.7.1	Ski Club Plans	Student Support
	BLM A20	Learning Skills Checklist	Assessment
	BLM 6.7.2	Practice: Linear Systems	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology

	BLM	Title	Purpose
Chapter 6 Review			
	BLM G10	Grid Paper	Student Support
	BLM A16	My Progress as a Mathematician	Student Self-Assessment
	BLM 6.CR.1	Chapter 6 Review	Practice
Chapter 6 Practice Test			
	BLM PT.1	Chapter 6 Practice Test	Diagnostic Assessment
	BLM 6.CT.1	Chapter 6 Test	Summative Assessment
	BLM G10	Grid Paper	Student Support
Chapter 6 Problem Wrap-Up			
	BLM 6.CP.1	Chapter 6 Problem Wrap-Up Rubric	Summative Assessment
Chapters 4 to 6 Review			
	BLM G10	Grid Paper	Student Support
	BLM A14	Self-Assessment Recording Sheet	Student Self-Assessment
	BLM A15	Self-Assessment Checklist	Student Self-Assessment
Task: Salary and Commission			
	BLM G10	Grid Paper	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 6.T1.1	Task: Salary and Commission Rubric	Summative Assessment
Task: Cod Fish Catches			
	BLM G10	Grid Paper	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 6.T2.1	Task: Cod Fish Catches Rubric	Summative Assessment

Get Ready

Student Text Pages
294 to 295

Suggested Timing
60–80 min

Tools
- grid paper

Related Resources
BLM G10 Grid Paper

BLM 6.GR.1 Practice: Get Ready

BLM 6.GR.2 Get Ready Self-Assessment Checklist

Common Errors

- Some students may assign variables to improper axes.

R_x Review the concepts of dependent and independent variable. By mathematical convention, the independent variable is normally assigned to the (horizontal) *x*-axis.

- Some students may struggle with applying the slope formula properly, either by incorrect substitution or by integer calculation errors.

R_x Have students label the coordinates of the points they are going to use in the formula with x_1, x_2, y_1, and y_2 before substituting. Provide remediation for adding and subtracting integers, as needed. You may wish to use **BLM 6.GR.1 Practice: Get Ready** for remediation or extra practice.

Accommodations

Gifted and Enrichment–Encourage students to investigate geocaching on the Internet.

Visual–For students with visual challenges, consider enlarging graphs and providing grid paper with larger grid squares.

Motor–For students with weak motor skills, consider providing grid paper with larger grid squares.

Memory–For students with memory difficulties, consider allowing them to create and use a formula sheet. This may help them with remembering things, such as the slope formula.

Teaching Suggestions

- Have students work independently on the Get Ready questions.
- You may wish to use **BLM G10 Grid Paper** to support the Get Ready activities.
- You may wish to use **BLM 6.GR.1 Practice: Get Ready** for remediation or extra practice.
- All BLMs referred to throughout this chapter can be found in the *Principles of Mathematics 9* Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; have students place a checkmark beside topics in the Get Ready in which they feel confident, or have students use **BLM 6.GR.2 Get Ready Self-Assessment Checklist**. Remedial action can be taken in small groups or with a whole class skill review.

6.1 The Equation of a Line in Slope *y*–Intercept Form: $y = mx + b$

Strand:
Analytic Geometry

Student Text Pages
296 to 307

Suggested Timing
80 min

Tools
- masking tape
- metre sticks
- stopwatches or watches that measure seconds

Technology Tools
- graphing calculators
- CBR™ motion sensor

Related Resources
BLM G16 Investigate Graph
BLM G2 Vertical Number Line
BLM G10 Grid Paper
BLM 6.1.1 Practice: The Equation of a Line in Slope *y*-Intercept Form: $y = mx + b$
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

AG1.01 determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation);

Investigating the Properties of Slope

AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

Review basic instructions on how to operate the graphing calculators and motion sensors.

Teaching Suggestions

- Students have been introduced to linear relations, including distance versus time using motion sensors, in Chapter 2 Relations, where they studied distinguishing characteristics of linear and non-linear relations. In Chapter 5 Modelling with Graphs, they will have seen linear relations again from the perspective of direct and partial variation.
- The purpose of this chapter is to revisit linear relations from the perspective of coordinate geometry. Make connections between the content of this chapter and related work done earlier. For example:
  - The slope of a line corresponds to the variable part of a direct or partial variation

Common Errors

- Some students may miss negative signs when reading slope or *y*-intercepts from the equation of a line.

R_x Have students always look to the left of the number and check for a negative sign.

- Some students may apply the negative sign to both numerator and denominator when graphing a line having a negative slope.

R_x Have students rewrite the equation or slope and move the negative sign to either the numerator or denominator.

- Some students may mix up the forms for horizontal and vertical lines.

R_x Have students use a test point known to be on the line to check that it satisfies the equation.

Ongoing Assessment

- Chapter Problem question 11 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

  - The slope of a line corresponds to the first differences of a linear relation
  - The vertical intercept corresponds to the fixed part of a partial variation
- Students should have some familiarity with distance-time relationships using motion sensors from the work done in Chapter 2 Relations. They have also studied slope, however, not yet as related to the equation of a line. The purpose of this Investigate is to have students discover the geometric significance of the slope (i.e., speed and direction) and vertical intercept (i.e., starting point) of a line produced by a constant motion. A wind-up toy vehicle or battery operated toy vehicle can be used to give students some cleaner data.
- It may not be necessary for students to actually work with these at this point, particularly if they used them in Chapter 2 Relations. You may elect to give a brief demonstration using a student walker, a CBR™ motion sensor, and a viewscreen on an overhead projector.
- Although the point is to introduce the general form of the slope *y*-intercept form of the equation of a line $(y = mx + b)$, students should be made aware fairly early on that x and y can, and perhaps should, be replaced with more logical, contextual variable choices (e.g., $d = mt + b$ for distance versus time, etc.). It is also important for students to identify m in this context as representing the rate of change of distance compared to the rate of change of time, which is called speed.
- Have students work on the Investigate in small groups. (15–20 min) You may wish to have students use **BLM G16 Investigate Graph** to record their work for the Investigate.
- Following the Investigate, conduct a class discussion to consider what happens when speed is not constant. Students should recognize that this produces a non-linear relation (e.g., the *d*–*t* graph is curved), and that the motion cannot be described by an equation in the form $y = mx + b$. Students will study such curves in more depth in grade 10 and beyond (and also in senior physics), and will make connections to the rate of change of speed versus time, which is called acceleration.
- Assign the Examples, and follow up with a discussion. (25–30 min)
- After debriefing the Investigate, work through Example 1. Here, the focus is on identifying slope and *y*-intercept of a given linear graph. Note that when the graph passes through the origin, the *y*-intercept is 0 so b vanishes from the equation (in this case partial variation degenerates to a direct variation). The special cases of horizontal and vertical lines are also presented. Note that in the case of the former, the slope is 0 so the slope term vanishes from the equation. Note also in the case of the latter that the equation of a vertical line cannot be written in the form $y = mx + b$.
- Example 2 illustrates how you can use the slope and *y*-intercept to generate the graph of a line. Indicate that this technique does not require the generation of a table of values. This approach also emphasizes the concept of the slope as a comparison of the rates of change of the two variables in the linear relation. The triangle formed when you "rise" and "run" to move from one point to another is sometimes called a "rate triangle."
- Examples 1 and 2 are deliberately non-contextual, in order to focus on mechanical manipulation skills. In Example 3, attention is returned to contextual applications of linear relations. Note the choice of variables to replace x and y as dictated by the situation. Students should become comfortable with the idea that the choice of variable is purely arbitrary, particularly when applying graphing calculator technology (i.e., they should be able to go back and forth from contextual variables to generic, x and y, variables). The contextual situations provide a good opportunity to draw connections to related concepts of direct and partial variation, as described earlier.
- Assign the exercises as independent work. (balance of period)

Accommodations

Visual—Let students use graphing calculators to check their answers as they work through the Practise questions.

Perceptual—Encourage students to create tables of values for horizontal lines (where each point in the table of values has the same *y*-value) and vertical lines (where each point in the table of values has the same *x*-value).

Spatial—Allow students to write equations in the form $y = mx + 0$ when the *y*-intercept is zero.

Memory—Provide visual cues to remember the formulas in this section.

ESL–Students may need extra time or help with the examples in this section. You may wish to pair ESL students with a strong student to help them succeed in this section.

- You may wish to use **BLM G2 Vertical Number Line** and/or **BLM G10 Grid Paper** to support activities in this section.
- You may wish to use **BLM 6.1.1 Practice: The Equation of a Line in Slope *y*-Intercept Form: $y = mx + b$** for remediation or extra practice.

Investigate Answers (page 296)

4. b) Coordinates will vary. (0,1); This point represents a distance of 1 m from the sensor at a time of 0 s. This value might not be exactly what you expect because you might have started at a distance from the sensor that was slightly less or slightly more than 1 m.

c) Answers will vary. The slope and the speed of the walker have the same value.

7. When someone is moving at a constant speed, an equation of form $y = mx + b$ can be used to describe that person's motion either toward or away from a fixed object. In this case, m represents the number of distance units the person moves per time unit, and b represents the initial distance between the person and the fixed object.

Communicate Your Understanding Responses (page 303)

C1. a) $x = -3$ **b)** $y = 6$ **c)** $y = 2x - 3$ **d)** $y = -x + 4$

C2. First plot the point (0, 2) because it is the *y*-intercept. Now go 5 units to the right and 3 units up, to the point (5, 5). This point must be on the line because the slope of $\frac{3}{5}$ means that the line *rises* 3 units when it *runs* 5 units right. Draw the line through the two points (0, 2) and (5, 5).

C3. a) The person began walking 2 m from the sensor.

b) The person was walking away from the sensor. This is because on the graph, the person's distance from the sensor increases as time goes by.

c) The person was walking at a speed of 1.5 m/s.

d) $d = 1.5t + 2$

C4. a) Yes

b) The part of the graph to the left of the *h*-axis represents the growth of the tomato plant from when it was planted at a plant store to when it was re-planted in the garden.

c) The *h*-intercept is significant because it represents the height of the plant when it was re-planted in its new owner's garden.

Practise

Ensure that students pay attention to the signs associated with the terms in the equation, and include them when identifying the slope and *y*-intercept. When terms are "missing" (e.g., question 1d) and e)), students need to be able to recognize what this implies (e.g., either the slope or *y*-intercept is equal to zero).

When using the slope *y*-intercept method of graphing a line (e.g., question 6b) and c)), some students may get confused when the slope is negative. Encourage them to associate the negative sign with either the numerator or the denominator, but not both.

Connect and Apply

Some students, particularly kinesthetic and visual learners, may benefit from working with CBR™ motion sensors on some of these questions. There are connections to measurement concepts in question 10 part b). A brief review of circumference may be helpful.

Questions 12 and 13 require the ability to integrate mathematical communication into narrative form, and may appeal to students with strong language skills. For students who struggle, suggest performing the mathematical analysis first, before attempting to compose the story. For interest, have a few students share their stories with the class. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Extend

Question 15 is a lead-in to the focus of Section 6.3.

Literacy Connections

It Just Makes Sense

Often in mathematics we use terms that fit the situation. For example, the slope *y*-intercept form of the equation of the line is set up so we can read the slope and the *y*-intercept right from the equation. This is true in many places in mathematics. As you work through this chapter, try to find four more places where "it just makes sense."

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 6, 7a), c), 8–10
Typical	1–10, 12, 13
Extension	14–16

6.2 The Equation of a Line in Standard Form: $Ax + By + C = 0$

Strand:
Analytic Geometry

Student Text Pages
308 to 314

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- graphing calculators
- Computer Algebra System
- TI-89 calculators

Related Resources
BLM G10 Grid Paper
BLM G16 Investigate Graph
BLM G2 Vertical Number Line
BLM 6.2.1 Practice: The Equation of a Line in Standard Form: $Ax + By + C = 0$
BLM A8 Application General Scoring Rubric
BLM 6.2.2 Achievement Check Rubric
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

AG1.02 identify, through investigation, the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$;
AG1.03 express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$.

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);
AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);
AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

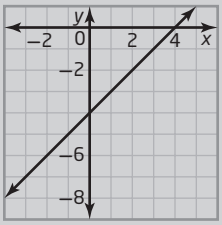
1. Identify the slope and y-intercept of each line.
a) $y = x - 4$ **b)** $y = -x + 1$

2. Use the slope and y-intercept to graph each line in question 1.

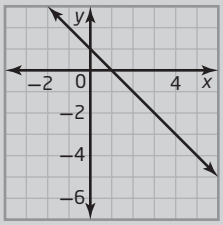
Warm-Up Answers

1. a) The slope is 1, and the y-intercept is –4.
b) The slope is –1, and the y-intercept is 1.

2. a)



b)



Common Errors

- Some students may perform steps for rearranging equations in the wrong order.

R_x Review the processes for rearranging formulas, presented in Section 4.4. Provide remedial exercises if needed. Often thinking of reverse BEDMAS can be helpful. You may wish to use **BLM 6.2.1 Practice: The Equation of a Line in Standard Form: $Ax + By + C = 0$**.

- Some students may mix up signs when rearranging equations.

R_x Review integer operations and provide remedial practice, as needed. You may wish to use **BLM 6.2.1 Practice: The Equation of a Line in Standard Form: $Ax + By + C = 0$**.

Ongoing Assessment

- Use Achievement Check question 10 to monitor student success. See Achievement Check Answers and **BLM 6.2.2 Achievement Check Rubric**.
- Chapter Problem question 6 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Teaching Suggestions

- Assign the Warm-Up as individual work. You may wish to have students use **BLM G10 Grid Paper** for this activity. (5 min)
- To introduce this section, as an alternative to The Beatles reference, or as an additional reference, ask the class to suggest a current popular group or artist. After posing the two questions in the introduction, have students refer to the Literacy Connections box, and ask them to suggest a situation in which each form of $\frac{1}{4}$ would be preferable.
- Assign the Investigate and have students work individually or with a partner. You may wish students to use **BLM G16 Investigate Graph** for this activity. (5–10 min)
- Students will discover that the equation of a line can be written in more than one way. This section emphasizes the ability to rearrange an equation given in standard form into slope *y*-intercept form. The reverse process is less important, and is provided as an Extend exercise at the end of the section.
- Assign the Examples, and follow up with a class discussion. (20–25 min)
- Example 1 focuses on the mechanical steps of rearranging an equation into slope *y*-intercept form. Help students recall the processes for rearranging equations, which they learned in Chapter 5 Modelling With Graphs. Remind students to pay careful attention to integer operations.
- Example 2 uses a contextual situation and provides an opportunity to draw connections between coordinate geometry and partial variation. This example also illustrates the advantage of being able to isolate a term of interest on either side of the equation. In this case, it is much simpler to isolate C on the right side. Encourage students to watch for and take advantage of this option.
- In parts c) and d) of Example 2, two solution methods are presented: graphing and using the equation. Part c) uses a graphing calculator. Refer students to the Technology Appendix for help with this tool.
- Encourage students to weigh the relative advantages and disadvantages of either approach. Take this opportunity to strengthen students' understanding of how the geometry and algebra of the situation are related.
- Assign the exercises and have students work individually or with a partner. (balance of period)
- You may wish to use **BLM G10 Grid Paper** and/or **BLM G2 Vertical Number Line** to support the activities.
- You may wish to use **BLM 6.2.1 Practice: The Equation of a Line in Standard Form: $Ax + By + C = 0$** for remediation or extra practice.

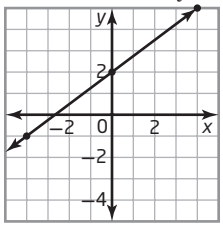
Investigate Answers (page 308)

1. Answers will vary. The graph of this relationship will be linear because it looks like I can rearrange the equation to show the slope and *y*-intercept.

2. (0, 2)

3. Answers will vary (4, 5); (−4, −1)

4. Answers will vary.



5. $\frac{3}{4}$

6. $y = \frac{3}{4}x + 2$

7. Answers will vary. The first equation can be made into the second equation by isolating the y term and then dividing both sides of the equation by its coefficient.

Communicate Your Understanding Responses (page 312)

C1. a) $A = 2; B = 3; C = 6$ **b)** $A = 5; B = -2; C = -1$
c) $A = 1; B = -1; C = 0$

C2. Answers will vary.
Step 2: Isolate the y term by adding $-5x + 20$ to each side
Step 3: Divide each side by -4
Step 4: Simplify the fractions on each side

Accommodations

Gifted and Enrichment—Challenge students to rewrite equations from the form $y = mx + b$ to the form $Ax + By + C = 0$ in several different ways.

Visual—Let students rewrite equations with a negative slope such as $x - y + 3 = 0$ in the form $x + 3 = y$.

Perceptual—Encourage students to show small sequential steps when rewriting equations from the form $y = mx + b$ to the form $Ax + By + C = 0$.

Memory—Encourage students to use highlighting or other visual cues to relate the independent variables and the dependent variables in equations.

Student Success

Provide each student with an equation of a line. Use an **inside/outside circle** to have each student convert the equation to the other form, outlining the steps to his or her partner.

Practise

For question 3, encourage students to rearrange the equations into slope *y*-intercept form first. Although this is implied by the previous two questions, some students may not make this connection independently.

Connect and Apply

Questions 5 and 10 are a lead-in to Section 6.7 Linear Systems. Students should realize that the best option can change, depending on the situation.

Questions 7 to 9 provide a good opportunity to review the concepts of dependent and independent variables. In this case, they can be interchangeable. The concept of inverse functions is alluded to, which students will explore further beyond grade 9. You may wish to use **BLM A8 Application General Scoring Rubric** for question 7 to assist you in assessing your students.

Achievement Check Answers (page 313)

10. a) Knights: $C = -25n - 1250$; Legions: $C = -30n - 995$

b) Knights: the fixed cost is \$1250 and the variable cost is \$25/person. Legions: the fixed cost is \$995 and the variable cost is \$30/person.

c) Knights: \$2375; Legions: \$2345.

d) Knights offers the better deal (by \$30) if there are 45 people. Since Legions has a lower variable cost, the difference in cost lowers by \$5 for each additional guest. For example, if there are six more guests, the two halls are the same price. If there are more than six additional guests, then Legions offers the better price.

Extend

In question 11, students explore rearranging the equation of a line from slope *y*-intercept form into standard form. Although generally not as useful a form, some students will benefit from the exercise. They may need some assistance in clearing fractions, which was briefly covered in Chapter 4.

For question 12, students who enjoy learning with technology will benefit from exploring the power of a Computer Algebra System to rearrange linear equations. This can become even more useful when students encounter equations of increasing complexity in future studies. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–5, 7–9
Extension	11, 12

6.3 Graph a Line Using Intercepts

Strand:
Analytic Geometry

Student Text Pages
315 to 322

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- graphing calculators
- CBR™ motion sensor

Related Resources
BLM G16 Investigate Graph
BLM A2 Attitudes Assessment Checklist
BLM G10 Grid Paper
BLM 6.3.1 Practice: Graphing a Line Using Intercepts
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

1. Solve for x.
a) $3x + 2(0) = 6$ **b)** $5x - 2(0) = -10$

2. Solve for x.
a) $0 + 2x = 8$ **b)** $5(0) - 3x = -12$

Warm-Up Answers

1. a) 2 **b)** -2
2. a) 4 **b)** 4

Teaching Suggestions

- Assign the Warm-Up as individual work. (5 min)
- Assign the Investigate and have students work individually or with a partner. Use **BLM G16 Investigate Graph** to support this activity. (5–10 min)
- The Investigate provides a scenario in which the intercepts of a linear relation have a significant meaning and can be used to build a linear model. In this case, however, some points on the line do not have contextual meaning. This provides a good opportunity to discuss the relationship between a mathematical model and the situation that it represents. Although the linear model is useful for solving problems, students must have sufficient understanding to be able to identify its limitations.

Common Errors

- Some students may misunderstand the language. [e.g., when asked for a point, they may say −4 when they should say (−4, 0).]

R_x Reinforce the difference between intercept points and intercept.

- Some students may neglect to include brackets around a point (e.g., 4, 0).

R_x Reinforce the appropriate form.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- After debriefing the Investigate, present the third common form of the equation of a line, and discuss its usefulness in finding intercepts. Example 1 presents another method for graphing lines, by identifying the *x*- and *y*-intercepts. As students progress through this chapter, they should be able to judiciously apply an efficient graphing technique (e.g., table of values, slope *y*-intercept, intercepts), appropriate to the form of equation that is given to them.
- Assign the Examples, and follow up with a class discussion. (15–20 min)
- In Example 2, explain and then demonstrate this strategy: to quickly (i.e., mentally) solve for the *x*-intercept, cover the *y*-term with your hand (if at the chalkboard), and mentally solve the resulting equation. Use the same technique for the *y*-intercept.
- Example 3 shows how you can use intercepts to quickly find the slope of a line. Point out the triangle that is formed by both intercepts and the origin. Explain that finding the slope using intercepts is easy because the rise and run is simply the values of the intercepts, and if you use the formula, two of the values are zero. Two lines of reasoning are presented here: application of the graph and the formula. Students should see both methods and appreciate the connection between the geometry and algebra of the slope *y*-intercept, intercepts relationship.
- Assign the exercises as individual work or with partners. (balance of period)
- You may wish to use **BLM A2 Attitudes Assessment Checklist** to assist you in assessing your students.
- You may wish to use **BLM G10 Grid Paper** to support the activities.
- You may wish to use **BLM 6.3.1 Practice: Graph a Line Using Intercepts** for remediation or extra practice.

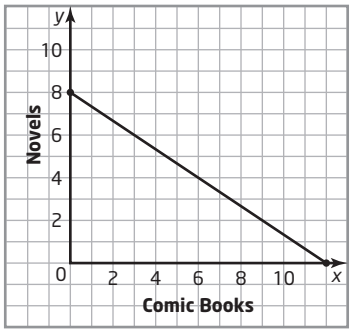
Investigate Answers (page 315)

1. 12 comic books

2. 8 novels

3. (12, 0); (0, 8)

4. **Joanne's Book Store Purchases**



5. a) 2 novels and 9 comic books; 4 novels and 6 comic books; 6 novels and 3 comic books; any other combination of novels and comic books that uses fewer of one or both types of books.

b) Answers will vary. The students could use trial and error. Alternatively, a graph can be used to discover combinations that work by looking at the line and seeing which ordered pairs of whole numbers lie on it.

6. a) Answers will vary. The graph can be used to discover combinations that work by looking at the line and seeing which ordered pairs of whole numbers lie on it. For example, the pair (9, 2) lies on the line, so Joanne can buy 9 comic books and 2 novels as these cost $9 \times 4 + 2 \times 6 = 48$ dollars.

b) Answers will vary. The point $\left(\frac{3}{2}, 7\right)$ lies on the line but $\frac{3}{2}$ in the *x* position represents the number of comic books that Joanne could buy. She can't buy $\frac{3}{2}$ comic books. Only whole numbers in the two positions of the ordered pair (x, y) will make sense as *x* and *y* represent the number of comic books and the number of novels that Joanne could buy.

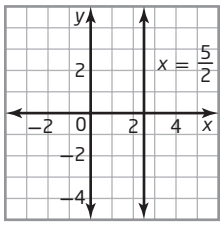
c) Answers will vary. Possible answers: $\left(11, \frac{2}{3}\right)$; $\left(\frac{15}{2}, 3\right)$; These points have no meaning because one coordinate in each point is an improper fraction and these fractions cannot represent numbers of novels or comic books that Joanne can buy.

Communicate Your Understanding Responses (page 319)

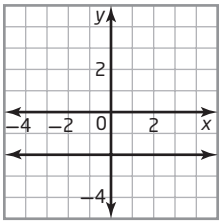
C1. (3, 0); (0, −4)

C2. a) Yes; a vertical line not lying on the *y*-axis has no *y*-intercept

b) Answers will vary. $x = \frac{5}{2}$



C3. This is a horizontal line that intersects the *y*-axis at the point (0, −2);



Accommodations

Gifted and Enrichment—Challenge students to use a graphing calculator to investigate the number of x-intercepts in other simple higher degree equations including quadratic, cubic, and quartic equations.

Perceptual—Encourage students to create a two-point mini table of values to record the x-intercept as the point $(a, 0)$ and the y-intercept as the point $(0, b)$ to show that they represent two different points on the line before graphing the line.

Spatial—Let students use a table of values with five sets of coordinates, where all of the x-values are the same and the y-values are different to graph a vertical line [e.g., $(3, -5)$, $(3, -1)$, $(3, 2)$, $(3, 3)$, $(3, 5)$], and a table of values with five sets of coordinates, where all of the y-values are the same and the x-values are different, to graph a horizontal line [e.g., $(-2, -2)$, $(2, -2)$, $(3, -2)$, $(1, -2)$, $(5, -2)$].

ESL—Have ESL students consider the words in their language that relate to the ones in the margin items, and see if there are similarities or differences in the sense of the words. Have students work with a partner to discuss the words they find. Non-ESL students may wish to learn some of the terms, such as perpendicular, parallel, slope, and point, in a second language.

Student Success

Have students **use technology** (graphing calculators or software) to construct graphs and equations of lines from their intercepts. Students can then investigate how the equations change as the intercepts are varied.

Practise

For question 2, caution students that the list does not present ordered pairs, but rather provides two points (i.e., the intercepts) in each case.

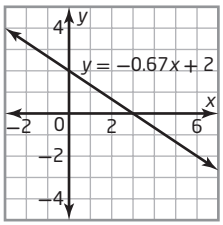
Connect and Apply

Question 6 provides another opportunity to attach physical meaning to the intercepts of a linear graph. A CBR™ demonstration at the front of the classroom may be helpful.

For questions 8 to 11, students may struggle with some of these problems. Consider having them work with a partner to solve these.

You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support question 10. Question 10 can be done using Cabri Jr. Alternatively, if you have access to TI-Navigator then you can have this image pre-constructed and send it to all student calculators. The students can then begin processing the question.

Sample image:



Extend

Question 13 presents a good opportunity for students to explore non-linear relations, which they will do more of beyond grade 9. Have graphing calculators or graphing software available for this exercise. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Question 15 is a good exercise in algebraic reasoning, a skill that some students find challenging. Have students work with numerical values first, and then try to generalize to construct the formulas. Note it is not critical that students be able to memorize and apply these formulas.

Literacy Connections

It Just Makes Sense II

To intercept something is to cross it or touch it. Is this another situation where "it just makes sense"? Did you have this picked out already as one of the places in this chapter where things "just make sense"?

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 7
Typical	1–9, 11
Extension	12–15

Use Technology

Use *The Geometer's Sketchpad*® to Explore Parallel and Perpendicular Lines

Strand:
Analytic Geometry

Student Text Pages
323 to 325

Suggested Timing
80 min

Tools
- protractors

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- graphing calculators

Related Resources
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 6.UT.1 Use Technology: Using a Slider
BLM 6.UT.2 Use Technology: Use the TI-83 Plus or TI-84 to Explore Parallel and Perpendicular Lines
BLM A9 Communication General Scoring Rubric
BLM G4 Protractor

Common Errors

- Some students may find that the desired options do not appear to be available in various pull-down menus.

R_x Have students check that they do not have either too many or too few things selected before searching the menu. Remind students to deselect before selecting new objects to work with.

Accommodations

Gifted and Enrichment—Challenge students to extend their knowledge of parallel and perpendicular lines by creating different families of parallel lines with positive, negative, and zero slopes.

Visual—Provide verbal instructions for students to complete the Investigate.

Specific Expectations

Investigating the Properties of Slope

AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

Review the basic commands of *The Geometer's Sketchpad*®.

Teaching Suggestions

- Point out to students that *The Geometer's Sketchpad*® displays perpendicular lines accurately, that is, they really do meet at 90°, unlike some graphing calculator displays.
- You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Although function terminology and function notation are not expectations at the grade 9 level, *The Geometer's Sketchpad*® uses them. You may wish to give a basic introduction to these concepts. Students will learn these concepts in more depth in later grades, and should not be assessed on their understanding at this level.
- Another way to dynamically change the slope of a line is to use a slider. You may wish to use **BLM 6.UT.1 Use Technology: Using a Slider** for instructions.
- This Investigate can also be performed using graphing calculators or graphing software, however these may not have the same dynamic capabilities (e.g., motion controllers, etc.). You may wish to use **BLM 6.UT.2 Use Technology: Use the TI-83 Plus or TI-84 to Explore Parallel and Perpendicular Lines** to support this activity.
- For question 5, you may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.
- For question 8, you may wish to use **BLM G4 Protractor** to support this activity.

6.4 Parallel and Perpendicular Lines

Strand:
Analytic Geometry

Student Text Pages
326 to 329

Suggested Timing
80 min

Tools
- grid paper
- protractors

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G16 Investigate Graph
BLM G10 Grid Paper
BLM G4 Protractor
BLM 6.4.1 Practice: Parallel and Perpendicular Lines
BLM A7 Thinking General Scoring Rubric
BLM 6.4.2 Achievement Check Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☐ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☐ Communicating

Specific Expectations

Investigating the Properties of Slope

AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

Review the processes and keystrokes for graphing lines on a graphing calculator, including the Zoom and Window options. You may wish to refer students to the Technology Appendix.

Teaching Suggestions

- Do the Warm-Up. (5 min)
- For the Investigate, have students work individually or with a partner. Use **BLM G16 Investigate Graph** to support this activity. (15–20 min)
- The purpose of this Investigate is to consolidate and confirm the relationships between the slopes of parallel and perpendicular lines.
- The Investigate activity can be done either on grid paper, or using graphing calculators. Use **BLM G10 Grid Paper** or refer students to the Technology Appendix, depending on which method you choose.
- If you choose to have students use a graphing calculator, setting the Zoom to square will ensure that perpendicular lines actually appear to cross at 90°. If students are graphing lines on grid paper by hand, encourage them to be as accurate as possible.
- Assign the Example, and follow up with a class discussion. (5–10 min)
- The Example illustrates that parallel lines have equal slopes and that perpendicular lines have slopes that are negative reciprocals. After

Common Errors

- Some students may reverse the relationships between parallel lines and perpendicular lines.

R_x Have students picture parallel lines and remember that they have the same direction, and hence the same slope.

- Some students may forget either the negative part or the reciprocal part of negative reciprocals.

R_x Have students check that the product of the negative reciprocals equals -1.

Ongoing Assessment

- Use Achievement Check question 11 to monitor student success. See Achievement Check Answers and **BLM 6.4.2 Achievement Check Rubric.**
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to determine the reason that perpendicular lines do not always appear to be perpendicular when graphing using a graphing calculator.

Perceptual—Encourage students to create tables of values to interpret distance-time graphs.

Spatial—Let students use graphing calculators to graph the linear relations in this section.

Memory—Allow students to work with a partner or in small groups to review the sequential steps to rewrite equations from standard form ($Ax + By + C = 0$) to slope y-intercept form ($y = mx + b$).

Student Success

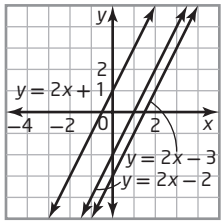
Use **concept attainment** to have students identify the relationships between parallel and perpendicular lines and their slopes.

working through part b), take some time to consolidate the concept of negative reciprocal. A negative reciprocal is easiest to identify if the number is expressed as a proper or improper fraction, so, some numeric manipulation is sometimes required.

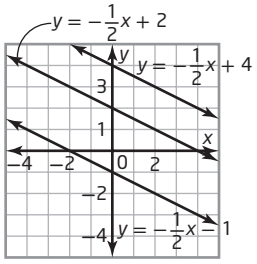
- Assign the questions as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM G4 Protractor** to support the activities.
- You may wish to use **BLM 6.4.1 Practice: Parallel and Perpendicular Lines** for remediation or extra practice.

Investigate Answers (page 326)

1. a)

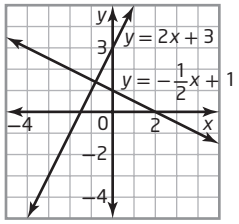


b)

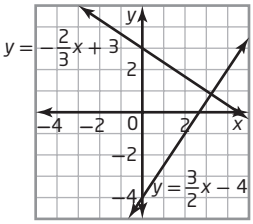


2. Answers will vary. The lines are parallel. The equations differ by a constant but have the same slope.

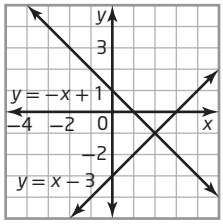
3. a)



b)



c)



4. Answers will vary. The lines are perpendicular. The slope constants of the two equations in each set are negative reciprocals of each other.

5. a) The lines' slopes are the same.
b) The lines' slopes are negative reciprocals.

Communicate Your Understanding Responses (page 327)

C1. Lines **A** and **C**; They are parallel because their slopes are the same.

C2. Lines **A** and **B** and lines **C** and **D**; They are perpendicular because their slopes are negative reciprocals.

Practise

For questions 1 and 2, remind students that it is not always obvious when two lines have equal slopes, or slopes that are negative reciprocals. Encourage students to apply their number sense when comparing numbers in various forms, for example, proper and improper fractions, integers, mixed numbers, and decimals. Some students may find it easier to convert all slopes to proper or improper fractions.

Connect and Apply

Question 3 is similar to questions 1 and 2 from Practise.

Question 6 provides an opportunity to draw connections between parallel lines and direct and partial variation. Remind students that when you change the fixed part (or vertical intercept), you produce a line parallel to the original.

For questions 8 to 11, remind students that right triangles have two sides that meet at 90°, and this infers that the slopes of the segments forming the

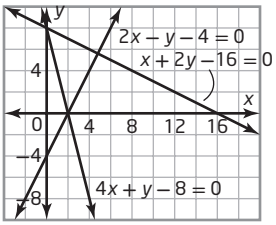
right angle must be negative reciprocals.

For question 10, there are two possible solutions in the Cartesian plane. For an interesting extension to question 10, explain to students that there are infinite solutions in three-dimensional space. Have students consider the segment KL as the axis of rotation about which they can find an infinite number of points M that satisfy the condition. This type of reasoning and visualization will be of benefit to any students who may study Geometry and Algebra in grade 12. You may wish to use **BLM A7 Thinking General Scoring Rubric** for questions 8 and/or 10 to assist you in assessing your students.

Question 11 is an Achievement Check. You may wish to use **BLM 6.4.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 329)

11. a)



b) Yes, the triangle appears to be a right triangle.

c) The slopes of the three lines are -4, 2, and -0.5, respectively. The last two slopes are negative reciprocals (and have a product of -1). Therefore, these two lines are perpendicular and the triangle is a right triangle.

d) Answers will vary. $y = 3x - 5$, $y = -\frac{1}{3}x + 4$, $y = x$. Two equations must have slopes that are negative reciprocals and the third line must have a different slope from the other two lines.

Extend

For questions 12 and 13, consider providing access to graphing software or a graphing calculator. These are good exercises to help students continue to develop perceptual connections between the algebraic representation of a line (i.e., its equation) and its geometric representation (i.e., its graph). You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6, 8
Typical	1–10
Extension	12–14

6.5 Find an Equation for a Line Given the Slope and a Point

Strand:
Analytic Geometry

Student Text Pages
330 to 337

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM 6.5.1 Practice: Find an Equation for a Line Given the Slope and a Point
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM A5 Problem Solving Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

1. Write the equation of a line whose slope and y-intercept are given.
   a) $m = \frac{3}{4}$, $b = -5$
   b) $m = -1$, $b = 6$
2. What is the slope of a line perpendicular to $y = 3x - 7$?
3. What is the slope of a line parallel to $2x - 3y + 8 = 0$?

Warm-Up Answers

1. a) $y = \frac{3}{4}x - 5$ b) $y = -x + 6$
2. $\frac{1}{3}$
3. $\frac{2}{3}$

Teaching Suggestions

- Assign the Warm-Up as individual work. (5 min)
- The focus of this section and Section 6.6 is to build a linear equation based on various pieces of given information. An equation in the form $y = mx + b$ can be produced, or can be used to determine the slope and y-intercept of a line. Various techniques for performing this are illustrated in the examples. The emphasis in the academic course is to be able to apply algebraic reasoning in finding the equation.
- Assign the Examples, and follow up with a class discussion. (20–25 min)
- In Example 1, the slope and a point known to be on a line are given. This information is used to solve for the y-intercept and subsequently to write the equation of the line. The given information is also used to produce the

Common Errors

- Some students may struggle with operations involving fractions and integers when they arise, for example, 1d) and f).

R_x Review basics and provide remediation as needed. Students in the academic program must become proficient at mechanical manipulations of this sort. You may wish to use **BLM 6.5.1 Practice: Find an Equation for a Line Given the Slope and a Point** for remediation or extra practice.

- Some students may substitute the coordinates of a known point into $y = mx + b$ again, after finding m and b, students, when trying to generate the equation of a line.

R_x Remind students that only m and b are required to generate the equation, x and y are left as variables because this will produce an equation that can produce an infinite number of points on the line.

Ongoing Assessment

- Chapter Problem question 7 can be used as an assessment tool. You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to research and study the slopes that are used in everyday life, such as stairs, roofs, and highways.

graph of the line. Ensure that students resist the temptation to substitute the known point again, once the y-intercept has been found. Make the distinction in processes between the first step, in which *particular* information (e.g., a single point) is used to solve for the y-intercept, and the second step, in which the slope and y-intercept are used to solve for the *general* case of an equation that can be used to provide infinitely many points are on the line. This distinction requires a relatively high level of abstract reasoning, and may require ongoing reinforcement.

- Example 2 requires application of the same skill as Example 1 to a contextual situation. There is an obvious connection to partial variation, which students have studied previously. The given information is similar to the previous example, if less obvious: the slope (which corresponds to the variable part) is 2 ($2/km), and the point known to be on the line is (10, 25). From this information, students can find the C-intercept, develop the equation, and produce the graph. Graphic and algebraic solutions are presented and compared in the last part of the example. Students should see both methods and appreciate the connections between the graph and the equation.
- Example 3 shows how other knowledge of coordinate geometry can be synthesized and applied, as needed. In this case, students use the properties of parallel and perpendicular lines to determine the slope of the desired line. Review these properties, as needed.
- Assign the exercises as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM G10 Grid Paper** to support these activities.
- You may wish to use **BLM 6.5.1 Practice: Find an Equation for a Line Given the Slope and a Point** for remediation or extra practice.

Communicate Your Understanding Responses (page 335)

C1. Answers will vary.

Step 2: Substitute $y = 1$, $m = 3$, and $x = 2$ into the slope y-intercept equation
Step 3: Simplify the equation by multiplying 3 and 2
Step 4: Isolate the b term on the right hand side
Step 5: Simplify the equation by subtracting 6 from 1

C2. a) $-\frac{5}{3}$ **b)** 4 **c)** $-\frac{1}{5}$ **d)** $\frac{2}{7}$

Practise

For question 2, some students may not realize immediately that the information provided regarding parallel and perpendicular lines can be used to identify the slope of the desired line. In the case of vertical and horizontal lines, e.g., question 2e), the concept of negative reciprocals is not useful; instead students need to recall the forms of such lines and apply geometric reasoning.

Connect and Apply

Questions 3 and 4 provide the opportunity for students to make connections between the equation of a line, its graph, partial variation, and first differences. It is important for students to see how these various components of the chapter are interrelated.

Question 7, the Chapter Problem question, is a multi-step problem. After finding the equation of the line, students must then find its intercepts in order to obtain the clues.

For question 8, students should be familiar with problems of this type, using $d = mt + b$, from their earlier work with motion sensors. This problem

takes the concepts to a more meaningful context. The numbers are larger, but the concepts are the same. Encourage students who enjoy problems of this type to study senior physics.

For questions 9 and 10, students will need some familiarity using *The Geometer's Sketchpad®*. Some review of basic commands, most of which are introduced in the Use Technology section, may be required. You may wish to use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** to support this activity. These questions can also be done using a graphing calculator: adjust the WINDOW setting, plot a point from the data in row 1 of L1 and L2, and enter the equation $y = 2x + \#$, where # can be changed until the line passes through the point.

Extend

Question 11 produces a piecewise linear relationship. Suggest breaking the problem up into two parts, before and after Aki changes his speed.

Literacy Connections

It Just Makes Sense III

Look at what you have done in this section: you have used the number value for the slope m and the x and y values from a point and filled the numbers in in the appropriate spot in the slope point form of the equation of the line in order to find out the y-intercept b. Did you have this picked out already as one of the places in this chapter where things "just make sense"?

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1a), c), e), 2, 3, 4
Typical	1–6, 8
Extension	11

6.6

Find an Equation for a Line Given Two Points

Strand:
Analytic Geometry

Student Text Pages
338 to 343

Suggested Timing
80 min

Tools
- grid paper
- rulers

Technology Tools
- CBR™ motion sensors
- graphing calculators

Related Resources
BLM G16 Investigate Graph
BLM A18 My Progress as a Problem Solver
BLM G10 Grid Paper
BLM 6.6.1 Practice: Find an Equation for a Line Given Two Points
BLM 6.6.2 Student Success: Think Aloud

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

1. Find the equation of a line having a slope of -2, and passing through the point $(3, -1)$.
2. Find the slope of a line that passes through $(2, 5)$ and $(-2, 3)$.

Warm-Up Answers

1. $y = -2x + 5$
2. $y = \frac{1}{2}x + 4$

Teaching Suggestions

- Assign the Warm-Up as individual work. (5 min)
- For the Investigate, have students work individually or with a partner. You may wish to use **BLM G16 Investigate Graph** for this activity. (10–15 min)
- You may wish to have students use **BLM A18 My Progress as a Problem Solver** as a self-assessment.
- The Investigate problem is an example of a partial variation in which neither the variable cost nor the fixed cost is known. This is the same as knowing neither the slope nor y-intercept of the equation of the line. What is known are two points that satisfy the linear relationship.
- The purpose of the Investigate is to have students discover that you can still generate the equation of a line with this information. The first step is to use the two known points to calculate the slope of the line, and then proceed as in the previous section to find the y-intercept, and then the equation.

Common Errors

- Some students may struggle with operations involving fractions and integers when they arise, for example, in Practise question 1d).

R_x Review basics and provide remediation as needed. Students in the academic program must become proficient at mechanical manipulations of this sort. You may wish to use **BLM 6.6.1 Practice: Find an Equation for a Line Given Two Points**.

- Some students may become confused by the fact that they use both known points to find the slope, then one of the known points to find the *y*-intercept, and then neither of the points to find the equation of the line.

R_x Ensure that students understand the purpose of each step in the process, and are not just memorizing mechanical steps to an algorithm. Consider applying a literacy strategy to consolidate this understanding. For example, pair students and assign them roles A and B. Then, have them take turns explaining to each other each step in the process for finding the equation of a line, given two points. For example:

A tells B: "First we substitute both points into the slope formula in order to calculate the slope of the line."

B tells A: "Next we select one of the points and..."

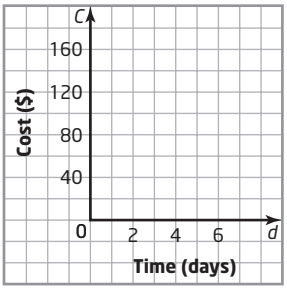
Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- After debriefing the Investigate, summarize the three steps to finding the equation of a line when given two points. Then, use the Example to illustrate and consolidate understanding of the process. Remind students to pay attention to detail when performing mechanical manipulations, particularly those involving integers and fractions.
- Assign the Example, and follow up with a class discussion. (10–15 min)
- Communicate Your Understanding question C5 can be useful as a study aid. As an extension, have students create examples to support their explanations.
- Assign the exercises as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM G10 Grid Paper** to support these activities.
- You may wish to use **BLM 6.6.1 Practice: Find an Equation for a Line Given Two Points** for remediation or extra practice.

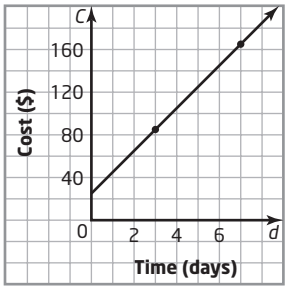
Investigate Answers (page 338)

1. a), b) **Renting a Snowboard**



2. a) The point (3, 85) represents Josh's 3-day snowboard rental at a cost of $85 and the point (7, 165) represents Kylie's 7-day snowboard rental at a cost of $165.

b) **Renting a Snowboard**



3. a) 20. It means that it costs $20 dollars a day to rent the board.

b) 25; The *d*-intercept represents the flat insurance cost of the snowboard rental.

4. $C = 20d + 25$

5. a) By using the graph we find that it costs $125 to rent a board for 5 days.

b) $C = md + b$
$C = 20(5) + 25$
$C = 125$

c) Yes. Both methods resulted in an answer of $125.

6. Answers will vary. Yes. The slope may be determined by subtracting the *C*-values and *d*-values of the two points, and then, dividing the subtracted *C*-values by the subtracted *d*-values. Then, the *C*-intercept of the equation may be determined by substituting the slope and *d*- and *C*-coordinates of one of the points into the slope *C*-intercept equation. The equation may be found by substituting the slope and *C*-intercept into the equation.

Accommodations

Visual—Let students use graphing calculators to understand coincident lines when graphing two lines.

Perceptual—Allow students to use visual cues such as highlighting for, colour coding, or bolding for the different forms used to calculate the slope when given two points.

Memory—Have students use graphing calculators when calculating the numerical values of slopes.

Student Success

Use the **think aloud** strategy to help students understand the thought processes involved in finding the equation of a line, given different information. (An example is included as **BLM 6.6.2 Student Success: Think Aloud**.)

Communicate Your Understanding Responses (page 341)

C1. a) Answers will vary. The equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

b) Answers will vary. Using $y = mx + b$, substitute slope for m, the values of the point for x and y and solve for b.

c) Answers will vary. Calculate the slope by using $m = \frac{y_2 - y_1}{x_2 \times x_1}$. Then, substitute using $y = mx + b$.

C2. Answers will vary.

a) If $m = 10$, $b = 7$
then $y = mx + b$
$y = 10x + 7$

b) If $m = 10$, P(1, 17)
then $y = mx + b$
$17 = 10(1) + b$
$b = 7$, Therefore, $y = 10x + 7$

c) $P_1(1, 17)$, $P_2(5, 57)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{57 - 17}{5 - 1}$$
$$m = \frac{40}{4}$$
$$m = 10$$
$$y = mx + b$$
$$17 = 10(1) + b$$
$$b = 7$$

Therefore, $y = 10x + 7$.

C3. a) It does not matter which point you substitute.

b) It is easier to use the point (1, 2) because the equation is easier to solve when there are smaller numbers in it but the final answer is the same.

C4. a) False. The walker started at a distance of 7 m from the sensor.

b) True. The point (3, 1) shows that the walker was 1 m from the sensor after 3 s.

c) True. The slope of the line is -2. This shows that the distance between the walker and the sensor decreases by 2 m each second. That is, the walker's speed is 2 m/s toward the sensor.

C5. a) Use the points to fine the slope. Substitute the slope and a point into the y-intercept equation and solve for b. Substitute the values for m and b into the y-intercept equation.

b) Use the points to find the slope. The y-intercept is given, so, substitute the values for b into the y-intercept equation and use the other point to solve for m.

c) Use the points to find the slope. The y-intercept is given, so, substitute the values for m and b into the y-intercept equation.

Practise

For questions 1 and 2, students will realize that whether points are given as ordered pairs or on a graph, the approach to finding the equation of a line is the same: start by using two known points to find the slope.

In question 3, the two points are implicitly given, for example, an x-intercept of 4 implies that (4, 0) is on the line.

Connect and Apply

Questions 5 to 7 require the application of the same set of skills, finding the equation of a linear relation given two points, in various contexts. Students should become comfortable with using different variables that match the situation at hand. Students with weak literacy skills may struggle with the heavy reading required. Consider having students work with a partner on some of these questions.

For question 8, remind students that there are many variables that may affect Anil's travel time. For example, highway distance signs are usually marked from the city's city hall. So, there is a significant uncertainty as to how far Anil's family actually is from their home, particularly the closer they get to the city. Other factors include traffic, time to unload the vehicle, etc.

Extend

Questions 9 and 10 lead in to Section 6.7 Linear Systems. Consider providing motion sensors and having students re-enact the situation.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1a), b), c), 2–7
Typical	1–8
Extension	9, 10

6.7 Linear Systems

Strand:
Analytic Geometry

Student Text Pages
344 to 351

Suggested Timing
80 min

Tools
- grid paper
- rulers

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- graphing calculators

Related Resources
BLM G16 Investigate Graph
BLM G10 Grid Paper
BLM 6.7.1 Ski Club Plan
BLM A20 Learning Skills Checklist
BLM 6.7.2 Practice: Linear Systems
BLM 6.7.3 Achievement Check Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.05 determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application.

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

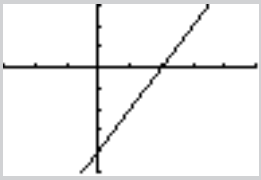
Review the processes and keystrokes for graphing lines on a graphing calculator.

1. Graph the following using a graphing calculator.

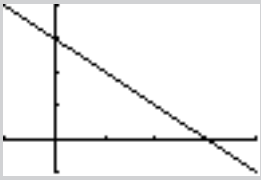
a) $y = 2x - 4$ **b)** $y = -x + 3$

Warm-Up Answers

1. a)



b)



Teaching Suggestions

- Assign the Warm-Up as individual work. You may wish to defer the Warm-Up until after students complete Example 1. (5–10 min)
- Assign the Investigate, and have students work individually or with a partner. You may wish to use **BLM G16 Investigate Graph** and/or **BLM G10 Grid Paper** to support this activity. (10 min)
- For the Investigate scenario, students should realize that the Frequent Extremist option would be preferable for a frequent skier. As students work through the activity, this is shown graphically. You may wish to prepare an overhead of the graph in which each rental plan is represented using a different colour, possibly using two layers, one with one graph and the second with the other; this allows projection of either graph separately or both superimposed.

Common Errors

- Some students may read only one coordinate as the solution to a linear system.

R$_x$ Remind students that all points on a line relate two variables, and that both coordinates are needed to give the point meaning

- Some students may substitute into only one equation when checking a linear system.

R$_x$ Remind students that a solution to a linear system must satisfy both equations. For two non-parallel lines, there are an infinite number of points that can satisfy either equation, but only one that satisfies both. Use an example or two to illustrate this concept.

Ongoing Assessment

- Use Achievement Check question 11 to monitor student success. See Achievement Check Answers and **BLM 6.7.3 Achievement Check Rubric**.
- Chapter Problem question 8 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Alternatively, you may wish to use **BLM 6.7.1 Ski Club Plan** and copy it onto an overhead. This BLM can also be used to take up the Investigate.
- After debriefing the Investigate, briefly discuss the three different types of linear systems, as presented just before Example 1. Point out that the single point of intersection type is the most common, and will be the type focused on for grade 9.
- Assign Example 1, and follow up with a class discussion. (10–15 min)
- Example 1 illustrates the graphical method, using paper-and-pencil, of solving and checking a linear system. The focus here is on the mechanical processes. By now, students should be comfortable applying various techniques for graphing lines, and should be encouraged to apply the most efficient method in each unique situation, as illustrated in the solution.
- It is important for students to see the connection between the geometric and algebraic interpretations of the solution to a linear system. In the former case, this is the point where the two lines cross. In the latter case, it is the coordinates of the point that satisfy both equations.
- Assign Example 2, and follow up with a class discussion. (10–15 min)
- In Example 2, the power of graphing technology is used to eliminate some of the tedium associated with generating the graphs. It is important to point out that one must apply number sense to obtain reasonable window settings in order to get a clear view of the linear system. You may wish to use **BLM T4/T5** to support this activity.
- Assign the exercises as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM A20 Learning Skills Checklist** to assist you in assessing your students.
- You may wish to use **BLM 6.7.2 Practice: Linear Systems** for remediation or extra practice.

Investigate Answers (page 344)

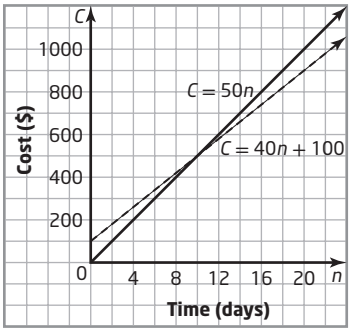
1. a) $C = 50n$

b) direct variation; There is no non-zero constant term.

2. a) $C = 40n + 100$

b) partial variation; There is a non-zero constant term.

3.



4. a) 10

b) Standard Rate. The cost value is lower than for the Frequent Extremist rate.

c) Frequent Extremist. The cost value is lower than for the Standard Rate.

d) $500. Mike should choose the Frequent Extremist rate if he plans on skiing more than 10 times throughout the winter. Otherwise, he should choose the Standard Rate.

5. Answers will vary. Each payment option has an advantage and a disadvantage. The Standard Rate option is cheaper for a small amount of skiing and the Frequent Extremist rate is cheaper for a large amount of skiing. Some additional information that a skier might need to know is rental costs or membership privileges associated with each rate.

Communicate Your Understanding Responses (page 348)

C1. a) Waverly Inn; $300 less **b)** Hotel Niagara; $200 less

c) Answers will vary. If you expect fewer than 80 guests, the Waverly Inn is cheaper, but if you expect more than 80 guests, the Hotel Niagara is cheaper.

C2. **D**; Answers will vary. The equation $y = x - 2$ tells you that y is always two less than x. The only point where y is two less than x is **D**. $4 + 2 = 6$ so, point **D** works for the equation $x + y = 6$ too.

C3. Answers will vary. I would write the equations in slope y-intercept form and graph them. Then, I would look for the intersection point. Once I found the intersection point, I would substitute it back into both equations to see if it works.

Accommodations

Perceptual—Let students use graphing calculators to solve linear systems and to compare rates for the ski club plans.

Language—Encourage students to use visual cues to relate direct variation to the linear equation $y = mx$ and partial variation to the linear equation $y = mx + b$.

Practise

In question 1, remind students that they are required to identify the point of intersection only.

For question 2, consider providing access to graphing calculators to allow students to check their solutions, after they have done the questions using paper-and-pencil.

Connect and Apply

Questions 3 to 5 extend students' understanding of the scenario posed in the Investigate. The main points here are:

- The best choice of plan depends on how often you go skiing
- There is a "break-even point" at which point both plans work out to the same amount. This point corresponds to the solution of the linear system.

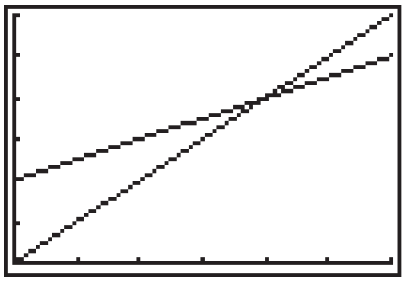
Questions 6 and 10 have students explore how the nature of a linear system can change when you change one of the parameters, in this case the vertical intercept/fixed part.

In question 8, the Chapter Problem, tell students that sometimes lines appear to be parallel when really they are not. Have students use a graphing calculator and experiment with various zoom settings to reveal this.

The context of question 11, the Achievement Check, may be of interest to students. Musicians and writers are often paid this way. What may be surprising is the small percent the artist typically receives. You may wish to use **BLM 6.7.3 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 350)

11. a) Graph, with units in 1000s on both axes.



b) The intersection point is (4000, 4000). The profit from both deals will be the same when the graphs intersect, that is, she will earn $4000 for the sale of 4000 CDs from either deal.

c) The artist's sales are rising with each new CD. It looks like she will sell more than 4000 (or 6000) CDs this time. The first offer that involves only royalties is the better deal at this level of sales.

Extend

Question 12 is an example of a non-linear system. Students are introduced to non-linear relations in grade 9, but will not study them in significant depth until later courses. It may be worthwhile for some students to see that the same types of analysis can often be performed in non-linear scenarios.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6
Typical	1–7, 9, 10
Extension	12–15

Chapter 6 Review

Student Text Pages
352 to 353

Suggested Timing
80 min

Tools
- grid paper

Related Resources
BLM G10 Grid Paper

BLM A16 My Progress as a Mathematician

BLM 6.CR.1 Chapter 6 Review

Ongoing Assessment

- Upon completing the Chapter Review, students can also answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related example in the text to help you with the questions? For which questions?*

Using the Chapter Review

Each Chapter Review question reviews different skills and concepts. Have students work independently to complete the Chapter Review, then, with a partner to compare solutions. You may wish to use **BLM G10 Grid Paper** to support the activities.

Alternatively, assign the Chapter Review for reinforcing skills and concepts in preparation for the Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult. You may wish to use **BLM 6.CR.1 Chapter 6 Review** for remediation or extra practice.

After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for the Chapter Test on the chapter's content. You may wish to have students use **BLM A16 My Progress as a Mathematician** to help them assess their learning.

Chapter 6 Practice Test

Student Text Pages
354 to 355

Suggested Timing
60–80 min

Tools
- grid paper

Related Resources
BLM 6.PT.1 Chapter 6 Practice Test
BLM 6.CT.1 Chapter 6 Test
BLM G10 Grid Paper

Summative Assessment

- After students complete **BLM 6.PT.1 Chapter 6 Practice Test**, you may wish to use **BLM 6.CT.1 Chapter 6 Test** as a summative assessment.

Accommodations

Motor—Give students extra time to complete these questions or allow them to do fewer questions, including examples of each type of question.

Memory—Provide students with the formulas for quizzes and tests.

ESL—Allow students to use a dictionary or translator when completing the questions in the Chapter Review and Chapter Test.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	6.1	Example 1 (page 298)
2	6.3	Investigate (page 315)
3	6.4	Example (page 327)
4	6.4	Example (page 327)
5	6.7	Example 1 (page 345)
6	6.1	Example 3a) (page 301)
7	6.2	Example 1, 2 (pages 309, 310)
8	6.3	Example 2 (page 317)
9	6.5	Example 1 (page 330)
10	6.6	Example (page 339)
11	6.5	Example 3 (page 334)
12	6.7	Example 2 (page 347)

Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?
- Identify the slope and y-intercept of a linear graph
- Identify the slope and y-intercept given a linear equation
- Write the equation of a line, given the slope and y-intercept
- Rearrange the equation of a line to express it in slope y-intercept form
- Identify the x- and y-intercepts of a linear graph
- Find the x- and y-intercepts given the equation of a line
- Graph a line using various methods (e.g., table of values, slope and y-intercept, x- and y-intercepts)
- Solve contextual problems involving the equation and graph of a line
- Identify whether two lines are parallel, perpendicular, or neither, given their equations
- State the slope of a line parallel or perpendicular to a given line
- Find the equation of a line given its slope and a point on the line
- Find the equation of a line given two points on the line
- Solve and check a linear system of two equations using the graphical method
- Interpret the solution of a linear system in problem-solving contexts

Chapter 6 Problem Wrap-Up

Student Text Pages
355

Suggested Timing
60–80 min

Technology Tools
- Internet
- computers

Related Resources
BLM 6.CP.1 Chapter 6 Problem Wrap-Up Rubric

Summative Assessment

- Use **BLM 6.CP.1 Chapter 6 Problem Wrap-Up Rubric** to assess student achievement.

Using the Chapter Problem

The Chapter Problem Wrap-Up is an excellent review of the skills introduced in this chapter. Encourage students to include at least as many skills as were introduced in the text.

Review the various problems leading up to the Chapter Problem Wrap-Up to ensure that students understand the skills involved in each one. Caution students to keep their problems simple so that they can be easily solved. It may present a challenge for many students to create problems that have the answers they intend but are still simple.

If students are to exchange problems, it will be more interesting if they choose a variety of cities on which to base their problems. Provide access to the Internet for students to search for interesting facts to provide a context for their problems.

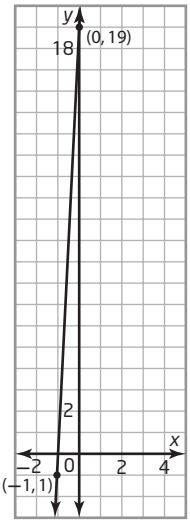
You may wish to assign the Chapter Problem Wrap-Up as a project with a number of days to complete. Allow sufficient time for students to be able to polish their work, providing richer contexts and more complete solutions to the algebra. Note that the original problem does not ask students to provide solutions to their algebra problems. You may decide to add this requirement (and allow additional time for completion).

You may also consider having students work with a partner or in small groups for this Chapter Problem Wrap-Up. The task is sufficiently rich that the parts of the problem could be shared. This will also encourage more checking and polishing of work before it is submitted.

Level 3 Sample Response

I decided to use my hometown for this problem. I used the same hints as in the text. Hints: The town is in Ontario. Each number that you find corresponds to a letter in the alphabet, in sequence. For example, 1 means A, 5 means E, and so on.

1. This town in Ontario is located on a river whose banks form a very steep slope. Two of the letters in the name of this city can be found by determining the slope and y-intercept of the graph shown. [Answer: 18 and 19]



2. There are many retail stores in my hometown. The equation $n - E + 8 = 0$ relates how much a worker earns, E, dollars per hour, to the number of years experience, n. Find the hourly earnings of a beginning retail worker, and of a worker with 5 years experience, to find two more letters in the name of this town. [Answer: 8 and 13]

3. Most of the streets in my hometown run parallel or perpendicular to each other. To find two more letters in the name of this city, find the x- and y-intercepts of the line that is perpendicular to $y = \frac{1}{5}x + 3$, and passes through the point $(2, -5)$. [Answer: 1 and 5]

4. Fort Malden was famous for its role in the war of 1812. It lies at the intersection of two roads. Solve this linear system to find two more letters.

$x - y + 2 = 0$

$10x - 9y = 0$ [Answer: 18 and 20]

5. This town is also famous for its role in the Underground Railroad, which helped to settle African Americans fleeing slavery. Many secret codes were used to hide these refugees. Rearrange the equation $y = -\frac{2}{7}x - 3$ into the form $Ax + By + C = 0$. The values of A, B, and C give the final three letters of the name of this town. [Answer: 2, 7, 21]

1, 2, 5, 7, 8, 13, 18, 18, 19, 20, 21 translates to A, B, E, G, H, M, R, R, S, T, U. These letters form the name Amherstburg.

Note: This level of detail is really a level 4 response. A level 3 response would provide a complete puzzle to solve, but some of the parts would lack context or would be very simple mathematically.

Level 3 Notes

Look for the following:

- Complete set of problems that has the intended solution possibly with minor errors
- No letters are missing from the city name, but repeated letters may appear only once
- Problems will involve a variety of analytic geometry skills
- Problems may be closely patterned on those of the text
- Understanding of the analytic geometry concepts in this chapter

What Distinguishes Level 2

At this level, look for the following:

- Partial set of problems working toward an intended solution; major errors may be present
- Letters may be missing from the city name
- Problems may only involve a small set of analytic geometry skills; some problem contexts may be repeated
- Problems may lack a coherent context or may copy those of the text without appropriate adjustments
- Partial understanding of the analytic geometry concepts in this chapter

What Distinguishes Level 4

At this level, look for the following:

- Complete set of problems that has the intended solution with no errors
- All letters are provided in problems including repetitions
- Problems will involve a variety of analytic geometry skills and may extend the set used in the text
- Problem context are rich and connected to each other (and the city in question)
- Thorough understanding of the analytic geometry concepts in this chapter

Chapters 4 to 6 Review

Student Text Pages
356 to 357

Suggested Timing
80 min

Related Resources
BLM A14 Self-Assessment Recording Sheet

BLM A15 Self-Assessment Checklist

BLM G10 Grid Paper

Ongoing Assessment

- This is an opportunity for students to assess themselves by completing selected questions and checking their answers against the answers in the back of the student text. They can then revisit any questions with which they had difficulty.
- Upon completing the Chapters 4 to 6 Review, students can also answer questions such as the following:
  - Did you work by yourself or with others?
  - What questions did you find easy? Difficult? Why?
  - How often did you have to check the related Example in the student text to help you with the questions? For which questions was this necessary?

Using the Chapters 4 to 6 Review

Each question reviews different skills and concepts. Have students work independently to complete the Chapters 4 to 6 Review, then with a partner to compare solutions. Alternatively, assign the Chapters 4 to 6 Review to reinforce skills and concepts in preparation for the specific chapter Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult.
After they complete the Chapters 4 to 6 Review, provide an opportunity for students to discuss any questions, consider alternative strategies, and ask about questions with features they find difficult. You may wish to use **BLM G10 Grid Paper** or **BLM A14 Self-Assessment Recording Sheet** or **BLM A15 Self-Assessment Checklist** to help students assess their understanding.

Task

Salary and Commission

Strand:
Linear Relations

Strand:
Analytic Geometry

Student Text Pages
358

Suggested Timing
20—40 min

Tools
- grid paper

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad*®
- computers

Related Resources
BLM G10 Grid Paper
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 6.T1.1 Task: Salary and Commission Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Part A

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.04 compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

Investigating the Properties of Slope

AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation);

AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Part B

Manipulating Expressions and Solving Equations

RE2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies);

RE2.09 solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods;

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

Connecting Various Representations of Linear Relations

RE3.01 determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;

Strand:
Analytic Geometry

Accommodations

Gifted and Enrichment—Have students create multiple-choice questions for the class, based on the skills needed for this Task.

Visual—Have students use a highlighter to emphasize key words as they read through the question.

Perceptual—Have another students read the questions aloud.

ESL—Use peer coaching, partnering a stronger student paired with a student needing extra support.

Ongoing Assessment

- Use **BLM 6.T1.1 Task: Salary and Commission Rubric** to assess student achievement.

Using the Properties of Linear Relations to Solve Problems

AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550);

AG3.05 determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application.

Teaching Suggestions

- Part A of this Task models an EQAO task. (20 min)
- Review drawing linear graphs, slope, and intercepts, substituting in formulas, and solving linear equations before assigning this Task.
- Have students work with a partner to review a part of the work, and have them report/teach another small group or pair.
- Alternately, conduct a short review of related problem exercises from Chapters 4 to 6 (direct and partial variation, reviewing equation solution skills drawing graphs, with or without technology).
- Provide grid paper and a pre-drawn table to assist students. You may wish to use **BLM G10 Grid Paper** to support this activity.
- Graphing technology will be helpful but is not essential for this assignment.
- Part B of this Task is more extensive and will take longer. Allow students to work with a partner, and provide graphing calculators/graphing technology to assist in the problem analysis. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- If students have difficulty getting started, ask them if there is another representation (e.g., using graphs or charts) of the data given in the question.
- The Task could be assigned as a take-home performance assessment and added to the student's portfolio of mathematics assessments.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

Part A

- What parts of the task did the student complete/not complete?
- How much assistance did the student need to get started with the graphing?
- How much assistance did the student need to complete the task?
- Is the table completely filled in?
- Was an equation for the data created?
- Does the student show an understanding of the meaning of the slope and T-intercept?

Part B

- How much assistance did the student need to get started with the task?
- How much assistance did the student need to complete the task?
- Did the student translate the given information into graphs or charts?
- Was the information combined into one graph (or chart)?
- Were calculations performed correctly? (e.g., percent)
- What parts of the task did the student complete/not complete?
- Did the student provide clear and complete reasoning for their answers?

Level 3 Sample Response

Part A

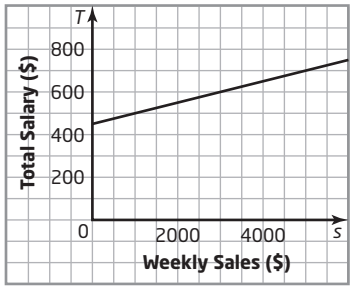
a) Carmella's base salary is $37.5 \times \$12 = \$450$ per week.

b)

Weekly Sales ($)	Commission ($)	Total Salary ($)
0	0	450
1000	50	450 + 50 = 500
2000	100	450 + 50 = 500
3000	150	450 + 50 = 500
4000	200	450 + 50 = 500

c)

Carmella's Salary



The relation is linear.

d) The equation of the relation is $T = 0.05S + 450$ Carmella's total salary is her base salary ($450) added to her commission ($0.05 \times S$).

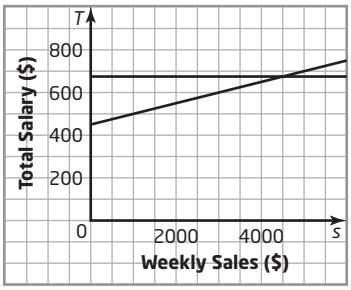
e) The slope of the graph is 0.05. It represents the rate of commission—5 cents out of every dollar of sales, or 5% commission.

f) The T-intercept is the amount that Carmella earns without commission or her base salary of $450.

Part B

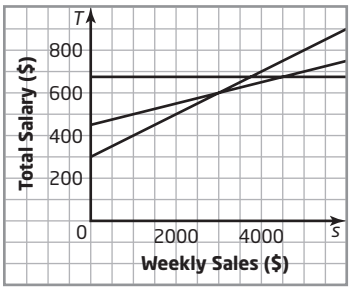
a) The equation for Carmella's total salary from part d) above is $T = 0.05S + 450$. Similarly, the equation for Sam's total salary is $T_2 = 37.5 \times 18$ or $675. By graphing, equation solving, or trial and error students can find an answer. The friend's total salaries are equal for $S = \$4500$, i.e., when they both earn $675.

Carmella and Sam Salary



b) Students should draw graphs for all three Total salary relations, as follows:
$T_1 = 0.05x + 450$; $T_2 = 675$; $T_3 = 0.10x + 300$;

Carmella-Maria-Sam Salaries



Students found in part a) that for sales up to $4500 Sam earned more than Carmella. From examining the graphs above or from trial and error, Maria earned less than Sam for sales up to $3750. So, for sales between $3750 and $4500, Sam earned more than Carmella and less than Maria. Maria's salary range is $675 to $750; Carmella's is $637.50 to $675 and Sam's is $675.

c) Answers will vary. Some students may note that factors such as working conditions, benefits and interpersonal relations are as important as salary. However, if all three friends have weekly sales around $5000 then, Carmella earns about $\$(0.05 \times 5000 + 450) = \$700$; Maria earns about $\$(0.10 \times 5000 + 300) = \$800$ and Sam earns $675. So, Maria seems to have the best job.

Level 3 Notes

Look for the following:

- Appropriate solutions for all parts of the problems with minor errors
- Understanding of concepts of slope and y-intercept
- Ability to set up a linear relation from a real-life situation
- Understanding of equation solving techniques
- Knowledge of analytic geometry techniques (e.g., drawing lines, finding and interpreting intersections of lines)
- Organized justifications for responses

What Distinguishes Level 2

Look for the following:

- Some appropriate solutions for all parts of the problems with some significant errors
- Some understanding of concepts of slope and *y*-intercept
- Some ability to set up a linear relation from a real-life situation; work has some errors
- Some understanding of equation solving techniques, but some difficulty in applying the techniques
- Some knowledge of analytic geometry techniques (e.g., drawing lines, finding and interpreting intersections of lines)
- Somewhat organized justifications for responses

What Distinguishes Level 4

Look for the following:

- Appropriate and detailed solutions for all parts of the problems with very few minor errors
- Thorough understanding of concepts of slope and *y*-intercept
- Highly effective planning and ability to set up and analyse a linear relation based on a real-life situation
- Thorough understanding of equation solving techniques
- Thorough knowledge of analytic geometry techniques (e.g., drawing, finding and interpreting intersections of lines)
- Clear, accurate, and detailed justifications for responses

Task

Cod Fish Catches

Strand:
Linear Relations

Strand:
Analytic Geometry

Student Text Pages
359

Suggested Timing
20—40 min

Tools
- grid paper

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G10 Grid Paper
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 6.T2.1 Task: Cod Fish Catches Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using Data Management to Investigate Relationships

RE1.03 design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs);

RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);

Understanding the Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.04 compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

Investigating the Properties of Slope

AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation);

Using the Properties of Linear Relations to Solve Problems

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Teaching Suggestions

- Review drawing linear graphs, slope, and intercepts before assigning this Task.
- You may wish to use **BLM G10 Grid Paper** to support this activity.
- Have students work with a partner to review a part of the work, and have them report/teach another small group or pair.
- Alternately, conduct a short review of related problem exercises from Chapters 4 to 6 (using graphing technology, interpreting slope, intercepts, research skills).

Accommodations

Gifted and Enrichment—Have students investigate the current state of the fishing industry in Canada and report their findings to the class.

Motor—Provide appropriate graphing software or graphing calculators for student having difficulty.

Memory—Provide examples of graphs with lines of best fit on a bulletin board, and create large posters in the classroom that visually summarize the steps in using available technology to draw and interpret a scatter plot or broken line graph and line of best fit.

Ongoing Assessment

- Use **BLM 6.T2.1 Task: Cod Fish Catches Rubric** to assess student achievement.

- You may wish to provide graphing calculators and/or graphing technology to assist in the problem analysis. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- The complete Task is long for a practice EQAO task. You may wish to assign parts a), b), d), and e) in class with a 20-min time limit. Assign part c) and additional research on the state of the fishing industry in Canada as an extended performance assessment that could be added to the student's portfolio of mathematics assessments.
- Historical data on cod fishing can be found at: http://www.statcan.ca/english/freepub/11-516-XIE/sectionn/sectionn.htm. Since the data is in Imperial units (pounds), students will need to convert to metric units.

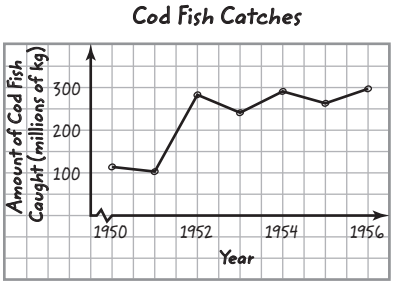
Hints for Evaluating a Response

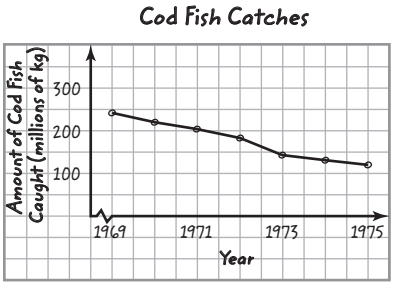
Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

- What parts of the task did the student complete/not complete?
- How much assistance did the student need to get started with the graphing?
- How much assistance did the student need to complete the task?
- How well did the student use graphing software or a graphing calculator?
- How well was the student able to read the graphs and interpret them?
- Does the student show an understanding of slope and intercept?
- Can the student find a line of best fit and interpret its slope?

Level 3 Sample Response

a)





b) The graph for the period 1950–1956 is non-linear but shows a strong tendency towards a steep increase in catches every second year.

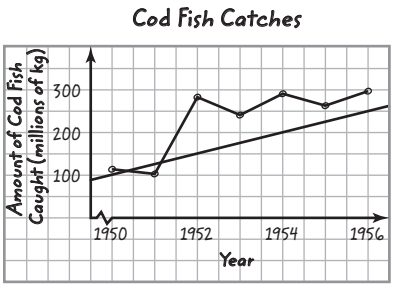
The graph for the period 1969–1975 is almost linear and represents a major decline the amount of cod being landed each year. This indicates a major problem with over-fishing and resulted in the ban on fishing for cod that is still in place.

c) Answers will vary. Several important factors should be included in responses. Prior to World War II, the norm was to use hand line and trawl (both hook and bait) fishing methods where little damage was done to either the stocks or the fishing grounds. After the war, the move was to use gill nets and bottom dragging from larger fishing vessels. These advancements in technology had a devastating effect on the cod, especially bottom dragging, which caught enormous amounts of fish and damaged the bottom of the ocean particularly affecting the spawning grounds.

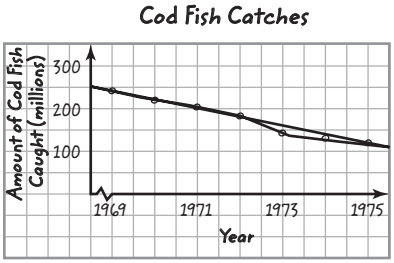
Factory freezer ships were not a problem until the 1970s. Their biggest impact was to reduce the numbers of fish coming ashore for processing, therefore putting a strain on the land-based fish plants, and causing unemployment in parts of eastern Canada. Another problem caused by the large factory freezer ships was policing the numbers of fish that were caught. Policing was done at sea, as most of the factory ships did not visit Canadian ports.

d) The graph for the period 1969–1975 has a negative slope and the decline in cod fish landed is almost a linear relation in that catches declined by approximately 20 million kg each year. During that time, although fishing was banned within a 166 km radius around Canadian fishing grounds by the Canadian government, international fishing continued outside this boundary. The result was that the spawning grounds where the cod gathered to breed became the main target of both international and Canadian fishers, resulting in severe damage to the ecosystem where cod were nurtured.

e)



The slope is positive and indicates a significant increase in the amounts of cod caught over the period of seven years. The average yearly increase was $(297 - 113) \div 7$ or 26.3 million kg annually.



The slope is negative and indicates a significant decline in the amount of cod caught during the seven-year period. The average annual decline was $(120 - 246) \div 7$ or -18 million kg annually, which is close to the values of the slope in the lines of best fit, reflecting that the decline from 1969–1975 was almost a linear relation.

Level 3 Notes

Look for the following:

- Understanding of concepts of slope and *y*-intercept
- Ability to set up a linear relation from a real-life situation
- Knowledge of analytic geometry techniques (e.g., drawing lines, finding a line of best fit)
- Organized justifications for responses

What Distinguishes Level 2

At this level, look for the following:

- Some understanding of concepts of slope and *y*-intercept
- Some ability to set up a linear relation from a real-life situation; work has some errors
- Some knowledge of analytic geometry techniques (e.g., drawing lines, finding a line of best fit)
- Somewhat organized justifications for responses

What Distinguishes Level 4

At this level, look for the following:

- Thorough understanding of concepts of slope and *y*-intercept
- Highly effective planning and ability to set up and analyse a linear relation based on a real-life situation
- Thorough knowledge of analytic geometry techniques (e.g., drawing, finding and discussing a line of best fit)
- Clear, accurate and detailed justifications for responses