Overview of *Principles of Mathematics 9*

The McGraw-Hill Ryerson *Principles of Mathematics 9* program has five components.

Student Text

The student text introduces topics in real-world contexts. In each numbered section, **Investigate** activities encourage students to develop their own understanding of new concepts. **Examples** present solutions in a clear, step-by-step manner, and then the **Key Concepts** summarize the new principles. **Communicate Your Understanding** gives students an opportunity to reflect on the concepts of the numbered section, and helps you assess students' grasp of the new ideas and readiness to proceed with the exercises.

Practise questions are single-step knowledge questions and assist students in building their understanding. **Connect and Apply** questions allow students to use what they have learned to solve problems and make connections among concepts. **Extend** questions are more challenging and thought-provoking. Answers to Practise, Connect and Apply, and Extend questions are provided at the back of the text. Most numbered sections conclude with a few **Math Contest** questions. **Chapter Tasks** are more involved problems that require students to use several concepts from the preceding chapters. Chapter Tasks are designed to help prepare students for the grade 9 EQAO test. Solutions to the Chapter Tasks are provided in the Teacher's Resource.

A **Chapter Review** of skills and concepts is provided at the end of each chapter. Questions are organized by specific numbered sections from the chapter. **Cumulative Reviews** are provided after Chapters 3, 6, and 9 and help prepare students for the Tasks.

The text includes a number of items that can be used as assessment tools:

- **Communicate Your Understanding** questions assess student understanding of the concepts
- **Achievement Checks** provide opportunities for formative assessment using the four Achievement Chart Categories, Knowledge/Understanding, Thinking, Communication, and Application
- **Practice Tests** contain multiple choice, short response, and extended response questions to help model both classroom testing practices and the grade 9 EQAO test
- **Chapter Problem Wrap-Ups** finish each chapter by providing a set of questions that involve all four Achievement Chart Categories
- **Chapter Tasks** are presented after Chapters 3, 6, and 9 and combine concepts from the preceding groups of chapters

Technology is integrated throughout the program and includes the use of scientific calculators, graphing calculators, spreadsheet programs, dynamic geometry programs, and the Internet.

Teacher's Resource

This Teacher's Resource provides the following teaching and assessment suggestions:

- **Teaching Suggestions** for all the sections
- **Practice** and chapter-specific blackline masters
- Answers to the **Investigate** questions
- Responses for the **Communicate Your Understanding** questions
- Responses for the **Chapter Problem Wrap-Up** and **Chapter Tasks**
- Students' **Common Errors** and suggested remedies
- Solutions and rubrics for the **Achievement Check** questions
- **Accommodations** for students with different needs

Computerized Assessment Bank CD-ROM

The Computerized Assessment Bank CD-ROM (CAB) contains questions based on the material presented in the student text, and allows you to create and modify tests. Questions are connected to the chapters in the student text. The question types include: True/False, Multiple Choice, Completion, Matching, Short Answer, and Problem. Each question in the CAB is correlated to the corresponding Achievement Chart Category, specific curriculum expectation, and curriculum strand from the Ontario Mathematics MPM1D Curriculum.

Solutions Manual

The Solutions Manual provides worked-through solutions for all questions in the numbered sections of the student text, except for Achievement Check questions, which are in the Teacher's Resource. In addition, the Solutions Manual provides worked-through solutions for questions in the Review, Practice Test, and Cumulative Review features.

Student Skills Book

The Student Skills Book is a "get ready for grade 9" package to help students fill in any gaps in their prior knowledge. It extends and supplements topics that are in the student text Get Ready pages, and adds other essential topics from earlier grades. The book includes worked-through solutions and a series of questions for each topic.

Web site

In addition to our McGraw-Hill Ryerson Web site, teachers can access the password protected site to obtain ready-made files for *The Geometer's Sketchpad®* activities in the text, further support material for differentiated learners, and many other supplemental activities.

To access this site go to:
http://www.mcgrawhill.ca/books/principles9
username: principles
password: math9

Structure of the Teacher's Resource

The teaching notes for each chapter have the following structure:

Chapter Opener

The following items are included in the Chapter Opener:
- **Mathematical Process Expectations**, **Overall Expectations**, and **Specific Expectations** that apply to the chapter, listed by strand
- **Vocabulary** items that will be introduced and defined in the chapter, listed in the margin
- Introduction to a **Chapter Problem** that includes questions designed to help students move toward the **Chapter Problem Wrap-Up** at the end of the chapter

Planning Chart

This table provides an overview of each chapter at a glance, and specifies:
- **Student Text Pages** references and **Suggested Timing** for numbered sections
- Related blackline masters available on the Teacher's Resource CD-ROM
- Assessment blackline masters for each section of the chapter
- Special tools and/or technology tools that may be needed

Get Ready

The following items are included in the margin:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The key items in this section include:
- **Teaching Suggestions** for how to use the Get Ready
- **Assessment** ideas on how to ascertain that students are ready for this chapter
- **Common Errors** and remedies to help you anticipate and deal with common errors that may occur
- **Accommodations** for students having difficulties or needing enrichment

Numbered Sections

The following items are listed in the margin:
- **Process Expectations** that are integrated in the section
- **Specific Expectations and Strand(s)** that the section covers in whole or in part
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The **Teaching Suggestions** include the following key elements:
- **Student Text Pages** references and **Suggested Timing**
- **Link to Get Ready** refers back to the relevant part of the Get Ready section (included in some numbered sections)
- **Warm-Up** and **Warm-Up Answers** provide a short check of the prerequisite skills needed for the section and often include a few Mental Math questions
- **Teaching Suggestions** give insights or point out connections on how to present the material from the text
- **Investigate Answers** and **Communicate Your Understanding Responses** let you know the expected outcomes of these activities

- Notes for the **Practise**, **Connect and Apply**, and **Extend** questions in the text provide: comments on specific questions to anticipate any difficulties; ways to deal with students' questions; and hints on how to help students answer the questions
- **Achievement Check Answers**
- **Literacy Connections** provide a way to link the math concepts in the section to literacy (included in some sections)
- **Common Errors** and remedies give you ideas on how to help students who make typical mistakes
- **Ongoing Assessment** suggestions give a variety of strategies that can be used to assess the students' learning
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Student Success** items provide suggestions for alternative ways to approach some key topics for at-risk students
- An **Exercise Guide** suggests the questions to be assigned—Minimum (essential questions for all students to cover the expectations), Typical, and Extension

End of Chapter Items

The **Chapter Review** and **Cumulative Reviews** (at the end of Chapters 3, 6, and 9) include the following items:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement
- **Using the Review** gives insights on how to present the information in the **Chapter Review**
- **Ongoing Assessment** suggestions give a variety of strategies you can use to assess the students' learning

The **Practice Test** has the following key features:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement
- **Study Guide** directs students who have difficulty with specific questions to appropriate examples to review
- **Summative Assessment** refers you to the **Chapter Test** to assess student performance
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Using the Practice Test** gives you insights on how to present the information in the Practice Test

The **Chapter Problem** includes the following elements:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** for extra practice and remediation, assessment, or enhancement
- **Using the Chapter Problem** includes teaching suggestions specific to the problem
- **Summative Assessment** refers you to the **Chapter Problem Rubric** to assess student achievement
- **Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 response

A series of **Chapter Tasks** occur at the end of Chapters 3, 6, and 9 and include:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** useful for extra practice or remediation, assessment, or enhancement
- **Mathematical Process Expectations** and **Specific Expectations** covered in the Chapter Tasks
- **Teaching Suggestions** with steps for you to follow
- **Prompts for Getting Started** provides a list of questions you can use to help students begin the Task
- **Hints for Evaluating a Response** provides a list of questions you should consider when assessing students' responses
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Ongoing Assessment** refers you to the **Chapter Task Rubric** to assess student achievement
- **Level 3 Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 answer

The **Teacher's Resource CD-ROM** provides various blackline masters, including:
- Generic Masters
- Tech Masters
- Practice Masters
- Assessment Masters
- Chapter-specific Masters
- Student Success Masters

Program Philosophy

Principles of Mathematics 9 is an exciting new resource for intermediate learners.

The Principles of Mathematics 9 program is designed to:

- provide full support in teaching the Ontario MPM1D mathematics curriculum
- enable and guide students' progress from concrete to representational and then to abstract thinking
- offer a diversity of options that collectively deliver student and teacher success

> *Given the changes occurring during adolescence, school administrators and teachers need to consider how best to match instruction to … the developing capabilities and varied needs of intermediate students…*
>
> *The (Principles of Mathematics 9) program is based on a view that all students can be successful in mathematics… [It] reflects principles of effective practice and research on how early adolescents learn, prerequisites for achieving a balanced approach to mathematics.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004*

During grades 7 to 9, most students are ready to progress from solely concrete thinking toward more sophisticated forms of cognition, as shown in the diagram:

Concrete Thinking

- typically work with physical objects
- focus of thinking is specific
- little or no reflection on thought processes
- able to solve very simple problems

Representative Thinking

- sometimes called "semi-concrete"
- typically work with diagrams
- thinking focus becoming more general and systematic
- meta-cognitive thinking about thought processes begins to develop
- explore hypothetical or "what-if" thinking, with support
- able to solve moderately challenging problems
- use problem-solving strategies effectively, with some guidance

Abstract Thinking

- able to work with or without materials or diagrams
- thinking focus instinctively general and systematic
- meta-cognitive thinking is well developed
- naturally explore hypothetical or "what-if" thinking
- able to solve problems that extend or deepen thinking
- confidently select and adapt problem-solving strategies

In *Principles of Mathematics 9*, students start with the concrete. Once they have experience with this, they move to the representative. Only when students are comfortable with the concrete and representative do they begin to move toward the abstract.

Approaches to Teaching Mathematics

> *Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.*
>
> *Ontario Ministry of Education and Training, 2004*

The concrete and abstract progression is exemplified in the following styles of mathematics teaching.

In grade 9, students learn best by using a concrete, discovery-oriented approach to develop concepts. Once these concepts have been developed, a connectionist approach helps students consolidate their learning.

Transmission-Oriented

- teaching involves "delivering" the curriculum
- focuses on procedures and routines
- emphasizes clear explanations and practice
- "chalk-and-talk"

▶

Connectionist-Oriented

- teaching involves helping students develop and apply their own conceptual understandings
- focuses on different models and methods and the connections among them
- emphasizes "problematic" challenges and teacher-student dialogue

▶

Discovery-Oriented

- teaching involves helping students learn by "doing"
- focuses on applying strategies to practical problems and using concrete materials
- emphasizes student-determined pacing
- "hands-on"

At this level, some transmission-oriented learning is also useful. This variety of approaches can be seen in the *Principles of Mathematics 9* program design.

Feature	Teaching Style(s) Supported
Chapter Problem	connectionist
Investigate, Reflect	discovery, connectionist
Examples	transmission, connectionist
Key Concepts	connectionist, transmission
Communicate Your Understanding	connectionist, discovery
Practise	transmission
Connect and Apply	connectionist, transmission
Extend	connectionist, transmission
Review	transmission, connectionist
Task	discovery, connectionist

Instructional Practice

The resources available in today's classroom offer opportunities and challenges. Indeed, the principal challenge—one that many teachers of mathematics are reluctant to confront—is to teach successfully to the opportunities available.

Grouping

> *Instructional practice that incorporates a variety of grouping approaches enhances the richness of learning for students.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004*

At one end of the scale, individual work provides an opportunity for students to work on their own, at their own pace. At the other extreme, class discussion of problems and ideas creates a synergistic learning environment. In between, carefully selected groups bring cooperative learning into play.

Manipulatives and Materials

> *Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.*
>
> *Ontario Ministry of Education and Training, 2003*

Although many teachers feel unsure about teaching with manipulatives and other concrete materials, many students find them a powerful way to learn. The *Principles of Mathematics 9* program supports the use of manipulatives, but also helps teachers adapt to this kind of teaching. The Teaching Suggestions sections in the Teacher Resource provide suggestions for developing student understanding using semi-concrete materials, such as diagrams and charts.

Technology

Although the extensive functions available on the modern scientific calculator are adequate for an introductory secondary school mathematics course, the added power and display capabilities of a graphing calculator can enhance and facilitate the learning experience for the student. In the *Principles of Mathematics 9* program, both scientific calculator keystrokes and graphing calculator instructions are provided in parallel with conventional calculations.

Special computer software designed for the classroom and licensed by the Ministry of Education for use in Ontario classrooms, such as *The Geometer's Sketchpad*® or *Fathom*™, provide powerful tools for teaching and learning. The *Principles of Mathematics 9* program supports the use of such software as an optional enhancement to the classroom experience. In addition, support for Computer Algebra Systems is included. Multiple solutions for worked-through examples in the text allow teachers to enjoy wide flexibility in lesson planning. As a result, you can plan activities using manipulatives, using software, or any combination of the two.

The Internet provides great opportunities for enhancing learning. As with many other sources of information, students must be protected from inappropriate content. The McGraw-Hill web site at *http://www.mcgrawhill.ca/books/Principles9* has been designed to offer only safe and reliable web site links for students to explore as an integrated part of the *Principles of Mathematics 9* program.

Literacy

Effective mathematics classrooms show students that math is everywhere in their world. For example, students should see that knowledge of probability is useful when learning about the electoral process in Social Studies class. Their work in graphing can be used in Science class. The written work they produce explaining their answers is also a language arts product. When connections such as these are made, students begin to see that math is not an isolated subject, but rather a vital part of everyday life. Contextual examples and problems can be linked to students' everyday experiences outside the classroom, as well.

Literacy Connections

There are Literacy Connections in every chapter. These features give students the help they may need to understand a symbol, a phrase, or a new word. They also provide suggestions for connecting mathematics to literacy, by connecting terms in mathematics to vocabulary used in other contexts.

In the early chapters, you might ask students to discuss the Literacy Connections items in small groups. After that, students can use the features to support their learning, when needed. Occasionally, the features could be a lead-in for discussing a concept. This feature provides one more way for students to feel successful in mathematics.

Writing and Mathematics

Being able to communicate ideas clearly is an important part of the *Principles of Mathematics 9* program. Students are asked to write about the mathematics they are learning, and communicate their understanding about what they are learning.

Take time to discuss the importance of being able to communicate understanding. The students' responses are meant to communicate with the teacher and are assessed as part of the mathematics work.

Cooperative Learning

Students learn effectively when they are actively engaged in the process of learning. Many of the sections in *Principles of Mathematics 9* include hands-on activities that foster this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other (Sternberg & Williams, 2002).

Teacher's Role

In classrooms where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, you will need to coach them in how to learn cooperatively. This may include:

- Making sure that the materials are at hand and directions are perfectly clear so that students know what they are doing before starting group work
- Carefully structuring activities so that students can work together
- Providing coaching in how to provide peer feedback in a way that allows the listener to hear and attend
- Constantly monitoring student progress and providing assistance to groups having problems either with group cooperation or the math at hand

Types of Groups

The size of group you choose to use may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class setting (Van de Walle, 2000). Research suggests that small groups are fertile environments for developing mathematical reasoning (Artz & Yaloz-Femia, 1999).

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms (Kilpatrick, Swafford, & Findell, 2001). If the class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, you might pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. You might pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other and peers have a better chance of recognizing the value of working together.

Cooperative Learning Skills

When coaching students about cooperative learning, you may want to consider task skills and working relationship skills, as indicated in the table below.

Task Skills	Working Relationship Skills
• Following directions • Communicating information and ideas • Seeking clarification • Ensuring that others understand • Actively listening to others • Staying on task	• Encouraging others to contribute • Acknowledging and responding to the contributions of others • Checking for agreement • Disagreeing in an agreeable way • Mediating disagreements within the group • Sharing • Showing appreciation for the efforts of others

Class discussions, modelling, peer coaching, role-plays, and drama can be used to provide positive task skills. For example, you might role-play different ways to provide feedback and have a class discussion on which ones students like and why. You might discuss common group roles and how group members can use them. Students also need to understand that the same person can play more than one role.

Role	Math Connection	Sample Comment
Leader	• Makes sure the group is on task and everyone is participating • Pushes group to come to a decision	Let's do this. Can we decide...? This is what I think we should do...
Recorder	• Manages materials • Writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• Presents the group's results and conclusions	We feel that… These are our conclusions… Our group found…
Organizer	• Watches time • Keeps on topic • Encourages getting the job done	Let's get started. Where should we start? So far we've done the following... Are we on topic? What else do we need to do?
Clarifier	• Checks that members understand and agree	Does everyone understand? So, what I hear you saying is... Do you mean that...?

Types of Strategies

A number of different types of cooperative learning strategies can be used in the mathematics classroom, and many are suggested in the Student Success margin items in this Teacher's Resource. The *Principles of Mathematics 9* program includes blackline masters (BLMs) to use with some of these strategies.

Think-Pair-Share

Students individually think about a concept, and then pick a partner to share their ideas. For example, students might work on the Communicate Your Understanding questions, and then choose a partner to discuss the concepts with. Working together, the students could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Cooperative Task Group

Task groups of two to four students work on activities in the Investigate section. As a group, students share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw

Individual group members are responsible for researching and understanding a specific part of the information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied (BLM provided on the *Principles of Mathematics 9: Teacher's Resource* CD-ROM). For example, during data management, this type of group might have "experts" in making various types of graphs using technology. Group members could then coach each other in making each kind of graph.

Another way of using the Jigsaw method is to assign "home" and "expert" groups during a large project. For example, students researching the shapes of various sports' surfaces might have a home group of four in which each member is responsible for researching one of soccer, baseball, hockey, or basketball. Individual members then move to expert groups. Expert groups include all of the students responsible for researching one of the sports. Each of the expert groups researches their particular sport. Once the information has been gathered and prepared for presentation, individual members of the expert group return to their home group and teach other members about their sport.

Placemat

In groups of four, students individually complete their section of a placemat (BLM provided on the *Principles of Mathematics 9: Teacher's Resource* CD-ROM). The group then pools their responses and completes the centre portion of the placemat with group responses. This method can be used for pre-assessment (diagnostic), review, or to summarize a topic.

Concept Attainment

Based on a list of examples and non-examples of a concept, students identify and define the concept. Then, they determine the critical attributes of the concepts and apply their defined concept to generate their own examples and non-examples.

Think Aloud

Work through a problem in front of the class, verbalizing your thinking throughout. This method can help develop process thinking in students.

Decision Tree

Students use a graphic organizer flow chart to identify key decisions and consequences (BLM provided on the *Principles of Mathematics 9: Teacher's Resource* CD-ROM).

Carousel

Students at different stations display and explain topics or concepts to other classmates who rotate through the stations, usually in order.

Timed Retell

Students sit in pairs facing each other. After some preparation time, Student A has 30 s to tell what she or he knows about the topic to Student B. Student B then retells the talk for about 30 s and adds additional information. Both students then write a summary of the talk.

Frayer Model

Students complete four quadrants for a specified topic: definition, facts/characteristics, examples, and non-examples (BLM provided on the *Principles of Mathematics 9: Teacher's Resource* CD-ROM). Variation: Give students a completed model and ask them to identify the topic/concept.

Word Wall

Individually or in groups, students complete cards for words or symbols, and then post the cards to use during future studies. One side of the card has the word or symbol, while the other side has four quadrants: the word, definition, picture or diagram, and an example or application (BLM provided on the *Principles of Mathematics 9: Teacher's Resource* CD-ROM).

Blast Off

This strategy can be used to start a class in an energized way. Students are asked to record: **3** important things they learned last class; **2** questions they have about last class; **1** reflection on what they learned last class; Blast Off! (BLM provided on the *Principles of Mathematics 9: Teacher's Resource* CD-ROM).

Inside/Outside Circle

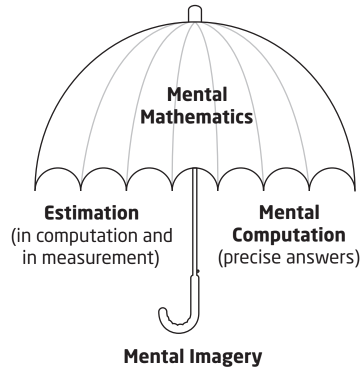
Students face each other in pairs, forming two concentric circles. Students take turns giving information to their partner, then the outside circle rotates one person to the right while the inside circle remains still. Students then share information with their new partners. The process continues until the students in the outside circle have rotated back to their starting point.

Three-Step Interview

In triads, label students A, B, and C. Have students individually compose interview questions. Assign roles to the three groups: A = Interviewer; B = Interviewee; C = Recorder. Student A interviews Student B, while Student C records the information. Then the students rotate roles. After all the interviews are complete, students share the recorded information in a Round Robin format.

Mental Mathematics

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding.



The diagram above shows the various components under the umbrella of Mental Mathematics. All three are considered mental activities and interact with each other to make the connections required for mathematics understanding.

Computational Estimation

Computational estimation refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all key in problem solving. Over 80% of out-of-school problem-solving situations involve mental computation and estimation (Reys & Reys, 1986).

Computational estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations are a check for reasonableness and provide learners with a strategy for checking their actual calculations.

Measurement Estimation

This skill relies on awareness of the measurement attributes (e.g., metre, kilometre, litre, kilogram, hour). Just as computational estimation enhances number sense, practice in measurement estimation enhances measurement sense.

A *referent* is a personal mental tool that students can develop for use in thinking about measurement situations. Tools could include the distance from home to school, a 100-km trip, the capacity of a can of juice, the duration of 30 min, and the area of the math textbook cover. These referents develop with measurement practice, and specifically with practice that encourages students to form these frames of reference. Students can compare other measurements to these referents. By doing so, they can gain a better understanding of what may be happening in a problem-solving situation.

You can help students develop referents by doing activities such as asking students to use their fingers or hands to show such measurements as: 6 cm, 260 mm, 0.4 m, a 60° angle, or 2000 cm^3.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is these mental representations, or conceptual knowledge, that need to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Asking, *What do you see in your mind's eye* when asked to visualize, as in the exercises below, for example, encourages students to think about the images they are using to help them solve problems. Students are often surprised when fellow students share their personal images; the discussion generated is very worthwhile.

Try these Mental Imaging Activities with your students.

Example 1:
Draw the mental image you have for each of the following:
- $\frac{2}{3}$
- 243 100 in relation to a million
- 75% of the questions on the page
- a 175° angle
- 0.56 m
- 36 cm
- 280 mm
- a 6 m by 10 m garden
- a 6.3-kg fish
- a 6-g fish

Example 2:
Use mental imagery to answer the following:
1. How many edges does a cube have?
2. If I am facing east, what direction is to my left?
3. What is the perimeter of a 90 cm by 30 cm shelf?
4. How many sides does a hexagonal pyramid have?
5. Imagine a 5-cm cube. What is its volume?
6. You cut off one vertex on a cube. What shape is exposed?
7. You cut the top off a square pyramid. What shape is exposed?

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with others—for finding the exact answer. These mental calculations are often referred to as Mental Math.

As with computational estimation, strategies for mental computation develop in quantity and quality over time. Students need regular practice in these strategies.

Some Points Regarding Mental Mathematics

- Students must have knowledge of the basic facts (addition and multiplication) in order to estimate and calculate mentally. They learn the many strategies for fact learning in elementary school. With practice, they eventually commit these facts to memory. Without knowing the basic facts, it is unlikely that students will ever attempt to employ any estimation or mental math strategies, as these will be too tedious.
- The various estimation and mental calculation strategies must be taught; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Unlike the traditional paper and pencil algorithms, there are many mental algorithms to learn. With the learning, however, comes a greater

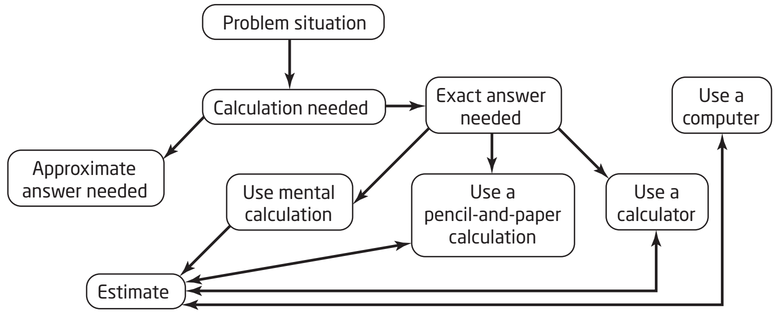
facility with numbers. Key to the development of skills in mental math is the understanding of place value and the number operations. This understanding is enhanced when students make mental math a focus when calculating.
- Mental math strategies are flexible; you need to select one that is appropriate for the numbers in the computation. Students should practise the strategy itself, select appropriate strategies for a variety of computation examples, and use the strategies in problem-solving situations.
- Although students should not be pressured with time constraints when first learning a mental math strategy, it is beneficial to provide timed tests once they have some facility at mental computation. If too much time is provided, many students will resort to the traditional algorithm, and will not use mental strategy.
- Mental math algorithms are used with whole numbers, fractions, and decimal numbers.
- Sometimes mental math strategies are used in conjunction with pencil and paper tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness, and to look for opportunities to calculate mentally.
- Encourage students to refer to the strategies by their name (for example, front-end strategy). Once the strategies have been taught, post them around the room for the students. Have students write problems in which a mental strategy would be the appropriate computation. Share these problems with the class.

Keep in Mind

Classroom practice has traditionally been in the form of asking students to write the answers to questions presented orally. This is particularly challenging for students who are primarily visual learners. Although we are sometimes faced with computations of numbers we cannot see, most often the numbers are written down. This makes it easier to select a strategy. In daily life, we see the numbers when solving written problems (e.g., when checking calculations on a bill or invoice, when determining what to leave for tips, when calculating discounted prices from a price tag). Provide students with mental math practice that is sometimes oral and sometimes visual.

Capable students of mathematics are comfortable with numbers. This comfort means that the students see patterns in numbers and intuitively know how they relate to each other and how they will behave in computational situations. Due to their comfort with numbers, these students have developed strong skills in estimation and mental math. Because of this, their understanding of numbers is further strengthened. We say they have "number sense." This sense of numbers develops gradually and varies as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

The position of the National Council of Teachers of Mathematics (NCTM) on how to proceed when faced with a problem that requires a calculation is best explained with this chart.



The chart tells us that, given a problem requiring calculation, students should ask themselves the following questions:

- Is an approximate answer adequate or do I need the precise answer?
- If an estimate is sufficient, what estimation strategy best suits the numbers provided?
- If an exact answer is needed, can I use a mental strategy to solve it?
- If the numbers don't lend themselves to a mental strategy, can I do the calculation using a paper-and-pencil method?
- If the calculation is too complex, I will use a calculator. What is a good estimate for the answer?

NCTM's Number and Operations Standard states that, "Instructional programs from kindergarten through grade 12 should enable all students to compute fluently and make reasonable estimates" (Principles and Standards for School Mathematics, 2000). Whether the students select an estimation strategy, a mental strategy, a paper-and-pencil method, or use the calculator, they must use their estimation skills to judge the reasonableness of any answer.

Mental Math Strategies

In *Principles of Mathematics 9*, mental math strategies are explicitly practised in some of the Warm-Up questions that are presented at the beginning of several sections in this Teacher's Resource. In addition, even though not always explicitly mentioned, students use mental math strategies in many parts of the text.

Problem Solving

> *Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.*
>
> *National Council of Teachers of Mathematics, 2000*

Problem solving is an integral part of mathematics learning. The National Council of Teachers of Mathematics recommends that problem solving be the focus of all aspects of mathematics teaching because it encompasses skills and functions, which are an important part of everyday life.

> *NCTM Problem-Solving Standard*
> Instructional programs should enable all students to—
> • Build new mathematical knowledge through problem solving
> • Solve problems that arise in mathematics and in other contexts
> • Apply and adapt a variety of appropriate strategies to solve problems
> • Monitor and reflect on the process of mathematical problem solving

Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill that can enhance logical reasoning. Individuals can no longer function optimally in society by just knowing the rules to follow to obtain a correct answer. They also need to be able to decide through a process of logical deduction what algorithm, if any, a situation requires, and sometimes need to be able to develop their own rules in a situation where an algorithm cannot be directly applied. For these reasons problem solving can be developed as a valuable skill in itself, a way of thinking, rather than just the means to an end of finding the correct answer.

However, true problem solving involves much more than solving word or story problems. True problem-solving tasks occur in a context where the solution path is not readily apparent; students have to identify the problem, decide on the solution method, and then implement it.

McGraw-Hill Ryerson has made the problem-based learning approach the focus of its program. In *Principles of Mathematics 9*, a variety of problem-solving opportunities are provided for students:

- The textbook opens with a chapter devoted to introducing the **Mathematical Process Expectations** (Problem Solving, Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing, and Communicating). These processes are then embedded throughout the remaining chapters. The Teacher's Resource identifies which Process Expectations are used in each numbered section.
- **Math Contest** questions are included at the end of many numbered section exercises to give students more opportunities to solve problems.
- Each chapter begins with an investigation of a real-life problem. The **Chapter Problem** is then revisited throughout the chapter through **Chapter Problem** questions, and ends with the **Chapter Problem Wrap-Up**.
- At the end of every three chapters, students are presented with **Chapter Tasks** where the solution path is not readily apparent and where solving the problem requires more than just applying a familiar procedure. These cross-curricular tasks require students to apply what they have learned in the current chapter and the two previous chapters to solve real-life, broad-based problems.

Mathematical Processes

The seven expectations presented at the start of the mathematics curriculum in Ontario describe the mathematical processes that students need to learn and apply as they investigate mathematical concepts, solve problems, and communicate their understanding. Although the seven processes are categorized, they are interconnected and are integrated into student learning in all areas of the *Principles of Mathematics 9* program.

Problem Solving

Problem solving is the basis of the *Principles of Mathematics 9* program. Students can achieve the expectations by using this essential process, and it is an integral part of the mathematics curriculum in Ontario. Useful problem-solving strategies include: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

Reasoning and Proving

Critical thinking is an essential part of mathematics. As the students investigate mathematical concepts in *Principles of Mathematics 9*, they learn to: employ inductive reasoning; make generalizations based on specific findings; use counter-examples to disprove conjectures; use deductive reasoning.

Reflecting

Students are given opportunities to regularly and consciously reflect on their thought processes as they work through the problems in *Principles of Mathematics 9*. As they reflect, they learn to: recognize when the technique they are using is not helpful; make a conscious decision to switch to a different strategy; rethink the problem; search for related knowledge; determine the reasonableness of an answer.

Selecting Tools and Computational Strategies

Students are given many opportunities to use a variety of manipulatives, electronic tools, and computational strategies in the *Principles of Mathematics 9* program. The student text provides examples of and ways to use various types of technology, such as calculators, computers, and communications technology, to perform particular mathematical tasks, investigate mathematical ideas, and solve problems. These important problem-solving tools can be used to: investigate number and graphing patterns, geometric relationships, and different representations; simulate situations; collect, organize, and sort data; extend problem solving.

Connecting

Principles of Mathematics 9 is designed to give students many opportunities to make connections between concepts, skills, mathematical strands, and subject areas. These connections help them see that mathematics is much more than a series of isolated skills and concepts. Connecting mathematics to their everyday lives also helps students see that mathematics is useful and relevant outside the classroom.

Representing

Throughout the *Principles of Mathematics 9* program, students represent mathematical ideas in various forms: numeric, geometric, graphical, algebraic, pictorial, and concrete representations, as well as representation using dynamic software. Students are encouraged to use more than one representation for a single problem, seeing the connections between them.

Communicating

Students use many different ways of communicating mathematical ideas in the *Principles of Mathematics 9* program, including: oral, visual, writing, numbers, symbols, pictures, graphs, diagrams, and words. The process of communication helps students reflect on and clarify ideas, relationships, and mathematical arguments.

Using Mathematical Processes

You can encourage students to use the mathematical processes in their work by prompting them with questions such as the following:

- *How can you tell whether your answer is correct/reasonable?* This promotes reasoning and reflection.
- *Why did you choose this method?* This promotes reflection, reasoning, selecting tools and computational strategies, and communication.
- *Could you have solved the problem another way?* This promotes reasoning, reflection, selecting tools and computational strategies, representing, and communication.
- *In what context have you solved a problem like this before?* This promotes connecting.

You can also encourage students to use a Think-Pair-Share approach to problem solving (see the **Cooperative Learning** section in this Program Overview). They will benefit greatly from brainstorming ideas and comparing methods of approach. A useful life skill is willingness to try different methods of solving a problem, learning from methods that perhaps do not reach the final goal, and being able to change their approach to reach the solution.

Technology

> *The use of technology in instruction should further alter both the teaching and the learning of mathematics. Computer software can be used effectively for class demonstrations and independently by students to explore additional examples, perform independent investigations, generate and summarize data as part of a project, or complete assignments. Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings.*
>
> *In this setting, the teacher encourages experimentation and provides opportunities for students to summarize ideas and establish connections with previously studied topics.*
>
> *Curriculum and Evaluation Standards for School Mathematics, NCTM, 1989*

Principles of Mathematics 9 taps the full power of today's interactive technologies to engage students in math inquiry, research, and problem solving. Technology is a major focus in several of the chapters, providing students with hands-on experience in using spreadsheets, creating graphs, and constructing and manipulating geometric figures. If at all possible, a classroom environment should be in place in which students are encouraged to reach for and apply technology whenever they feel the situation calls for it. In such an environment, the ongoing use of technology becomes another tool in the student's problem-solving tool kit, rather than a discrete event.

The *Principles of Mathematics 9* program includes opportunities for students to do research in the library or on the Internet. Consider having a class discussion on Internet web sites and appropriate sources. Remind students that anyone can create a web site on any topic on the Internet. Ask students to raise their hands if they have a personal web site or keep an Internet journal (a *blog*). Explain that web sites like these contain personal opinions and information contained on them should be looked at critically. This also may provide an opportunity to remind students that personal information should never be revealed over e-mail, in an on-line journal, or a chat-room, and that anything that makes them uncomfortable should be reported immediately to their parent or guardian.

Types of Programs

Several types of software programs are used in *Principles of Mathematics 9*:

Spreadsheet Programs

- Corel® *Quattro Pro*® 8
- Corel® *Quattro Pro*® 10
- Microsoft® *Excel*

Interactive Geometry Programs

- *The Geometer's SketchPad*®
- *Cabri Jr.*

Interactive Statistics Program

- *Fathom*™

Technology BLMs are also available, providing students with step-by-step directions on how to use technology, such as software and Computer Algebra System calculators, to explore the mathematical concepts of the lesson. These BLMs include:

- BLM T1 Corel® *Quattro Pro*® 8

- BLM T2 Corel® *Quattro Pro*® 10
- BLM T3 Microsoft® *Excel*
- BLM T4 *The Geometer's Sketchpad*® 3
- BLM T5 *The Geometer's Sketchpad*® 4
- BLM T6 *Fathom*™
- BLM T7 CAS TI-89

The **Technology Appendix**, on pages 524–536, of the student text provides clear step-by-step instruction in the basic functions of the TI-83+/84 graphing calculator and the basic features of *The Geometer's Sketchpad*®.

Assessment

The main purpose of assessment is to improve student learning. Assessment data helps you determine the instructional needs of your students during the learning process. Some assessment data is used to evaluate students for the purpose of reporting.

Assessment must be purposeful and inclusive for all students. It should be varied to reflect learning styles of students and be clearly communicated with students and parents. Assessment can be used diagnostically to determine prior knowledge, formatively to inform instructional planning, and in a summative manner to determine how well the students have achieved the expectations at the end of a learning cycle.

Diagnostic Assessment

Assessment for diagnostic purposes can determine where individual students will need support and will help to determine how the classroom time needs to be spent. *Principles of Mathematics 9* provides you with diagnostic support at the start of the text and the beginning of every chapter.

- The **Get Ready** section at the beginning of each chapter provides coaching on essential concepts and skills needed for the upcoming chapter. **Get Ready Self-Assessment** blackline masters are also provided for each chapter.
- For students needing support beyond the Get Ready, **Practice Masters** are provided in this Teacher's Resource that both develop conceptual understanding and improve procedural efficiency.

Diagnostic support is also provided at the start of every section.

- Each section begins with an introduction to facilitate open discussion in the classroom.
- Each activity starts with a question that stimulates prior knowledge and allows you to monitor students' readiness.

Formative Assessment

Formative assessment tools are provided throughout the text and Teacher's Resource. Formative assessment allows you to determine students' strengths and weaknesses and guide your class towards improvement. *Principles of Mathematics 9* provides blackline masters for student use that complement the text in areas where formative assessment indicates that students need support.

The **Chapter Opener**, visual, and the introduction to the **Chapter Problem** at the beginning of each chapter in the student book provide opportunities for you to do a rough formative assessment of student awareness of the chapter content.

Within each lesson:

- **Key Concepts** can be used as a focus for classroom discussion to determine the students' readiness to continue.
- **Communicate Your Understanding** questions allow you to determine if the student has developed the conceptual understanding and/or skills that were the goal of the section.
- **Connect and Apply** offers you an opportunity to determine students' understanding of concepts through conversations and written work. It also allows you to monitor students' procedural skills, their application of procedures, their ability to communicate their understanding of concepts, and their ability to solve problems related to the section's Key Concepts.
- **Achievement Check** questions allow students to demonstrate their knowledge and understanding and their ability to apply, think of, and communicate what they have learned.

- **Chapter Problem** questions provide opportunities to verify that students are developing the skills and understanding they need to complete the **Chapter Problem Wrap-Up** questions.
- **Extend** questions are more challenging and thought-provoking, and are aimed at Level 3 and 4 performance.
- **Chapter Reviews** and **Cumulative Reviews** provide an opportunity to assess Knowledge/Understanding, Thinking, Communication, and Application.

Summative Assessment

Summative data is used for both planning and evaluation.

- A **Practice Test** and a **Chapter Test** in each chapter assess students' achievement of the expectations in the areas of Knowledge/Understanding, Thinking, Communication, and Application.
- The **Chapter Problem** provides a problem-solving opportunity using an open-ended question format that is revisited in the **Chapter Problem Wrap-Up** questions. The **Chapter Problem** can be used to evaluate students' understanding of the expectations under the categories of Knowledge/ Understanding, Thinking, Communication, and Application.
- **Chapter Tasks** are open-ended investigations with rubrics provided. They are presented at the end of Chapters 3, 6, and 9. The Tasks require students to use and make connections among several concepts from the preceding chapters.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios:

- Help teachers assess students' growth and mathematical understanding
- Provide insight into students' self-awareness about their own progress
- Help parents understand their child's growth

Principles of Mathematics 9 has many components that provide ideal portfolio items. Inclusion of all or any of these chapter items provides insight into students' progress in a non-threatening, formative manner. These items include:

- Students' responses to the **Chapter Opener**
- Students' responses to the **Chapter Problem Wrap-Up** assignments
- Responses to **Communicate Your Understanding** questions, which allow students to explore their initial understanding of concepts
- Answers to **Achievement Check** questions, which are designed to show students' mastery of specific expectations
- **Chapter Task** assignments, which show students' understanding across several chapters

Assessment Masters

Principles of Mathematics 9 provides a variety of assessment tools with the chapter-specific blackline masters, such as Chapter Tests, Chapter Problem Wrap-Up rubrics, and Task rubrics. In addition, the program has a wide variety of generic assessment blackline masters. These BLMs will help you to effectively monitor student progress and evaluate instructional needs.

Generic Assessment BLM	Type	Purpose
BLM A1 Assessment Recording Sheet	Chart	Organize comments for assessment of student observations, portfolios, and presentations
BLM A2 Attitudes Assessment Checklist	Checklist	Assess students' attitude as they work on a task
BLM A3 Portfolio Checklist	Checklist	Assess students' portfolios
BLM A4 Presentation Checklist	Checklist	Assess students' oral and written presentations
BLM A5 Problem Solving Checklist	Checklist	Assess students' problem-solving skills
BLM A6 Knowledge/ Understanding General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Knowledge/ Understanding category
BLM A7 Thinking General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Thinking category
BLM A8 Application General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Application category
BLM A9 Communication General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Communication category
BLM A10 Observation Assessment General Scoring Rubric	Rubric	Assess students' understanding of the expectations under all four categories
BLM A11 Group Work Assessment Recording Sheet	Worksheet	Record comments as students work on group tasks
BLM A12 Group Work Assessment General Scoring Rubric	Rubric	Assess students' group-related work
BLM A13 How I Work	Worksheet	Students self-assess independent and group work
BLM A14 Self-Assessment Recording Sheet	Worksheet	Students self-assess their understanding of chapter material
BLM A15 Self-Assessment Checklist	Checklist	Students self-assess their understanding of chapter material
BLM A16 My Progress as a Mathematician	Checklist	Students self-assess their understanding of mathematics, in general
BLM A17 Teamwork Self Assessment	Worksheet	Students evaluate their work as part of a team
BLM A18 My Progress as a Problem Solver	Checklist	Students self-assess their ability at solving problems
BLM A19 Assessing Work in Progress	Worksheet	Student groups assess their progress as they work to complete a task
BLM A20 Learning Skills Checklist	Checklist	Assess students' work habits and learning skills
BLM A21 Opinion Piece Checklist	Checklist	Assess students' work on an opinion piece
BLM A22 Earning Money Report Checklist	Checklist	Assess students' work on a report
BLM A23 News Report Checklist	Checklist	Assess students' work on a news report

Intervention

Principles of Mathematics 9 accommodates a broad range of needs and learning styles, including those students requiring accommodations, at-risk students, students with limited proficiency in English, and gifted learners. This Teacher's Resource provides support in addressing multiple intelligences and learning styles through a variety of strategies.

- Excellent visuals and multiple representations of concepts and instructions support visual learners, ESL students, and struggling readers
- **Literacy Connections** and bolded, highlighted, and defined key terms in the margin support struggling readers and promote mathematics literacy for all learners
- Relevant contexts, including multicultural examples, engage students and provide a purpose for the mathematics being learned
- **Extend** questions and **Math Contest** questions provide additional challenge for gifted learners
- **Making Connections** activities provide additional opportunities for hands-on and minds-on learning
- **Accommodations** in the margin provide suggestions for students having difficulties or needing enrichment
- **Student Success** items in the margin provide suggestions for alternative ways to approach some key topics for at-risk students

Reaching all Students

Students may experience difficulty meeting provincial standards for a variety of reasons. General cognitive delays, social-emotional issues, behavioural difficulties, health-related factors, and extended or sporadic absences from instruction underlie the math difficulties experienced by some students. These factors do not explain the challenges other students encounter, however. For these students, math difficulties are usually related to three key areas: language, visual/perceptual/spatial/motor, or memory.

Language

Students with language learning difficulties demonstrate difficulty reading and understanding math vocabulary and math story problems, and determining saliency (e.g., picking out the most important details from irrelevant information). Processing information that is presented using oral or written language is often difficult for these students, who may be more efficient learners when information is presented in a non-verbal, visual format. Diagrams and pictorial representations of math concepts are usually more meaningful to these students than lengthy verbal or written descriptions.

Visual/Perceptual/Spatial/Motor

Some students demonstrate difficulties understanding and processing information that is presented visually and in a non-verbal format. Language support to supplement and make sense of visually presented information is often beneficial (e.g., verbal explanation of a visual chart). Visual, perceptual, spatial, and motor difficulties may be evident in students' written output, as well as in their ability to process visually inputted information. Difficulties with near and far point copying, accurately aligning numbers in columns, properly sequencing numbers, and illegible handwriting are examples of output difficulties in this area.

Memory (Short-Term, Working, and Long-Term Memory)

Students with short-term memory difficulties find it hard to remember what they have just heard or seen (e.g., auditory short-term memory, visual short-term memory). A weak working or active memory makes it difficult for students to hold information in their short-term memory and manipulate it (e.g., hold what they have just heard and then perform a mathematical operation with that information). For others, the retrieval of information from long-term memory (e.g., remembering number facts and previously taught formulae) is difficult. Students with long-term memory difficulties may also have difficulty storing information in their long-term memory, as well as retrieving it.

Modifications, Individual Education Plans (IEP), and Accommodations

A modification changes what is being taught by reaching well below or well above grade level, or by reducing the number of curriculum expectations. Students with a modified math program have an Individual Education Plan (IEP) describing how their program differs from classmates in their grade. An IEP also describes strategies, resources, and how the student will be evaluated. Modifying a student's program is a well-defined process involving the principal, teachers, parents, and student. Addressing a student's need for program modification falls outside the scope of this Teacher's Resource.

Accommodations

Accommodations do not change what is being taught. Rather, an accommodation to a student's program alters the "how," "when," or "where" the student is taught or assessed without changing curriculum expectations. This Teacher's Resource provides suggested accommodations based on the student's identified area of difficulty. Three types of accommodations are provided.

- Instructional accommodations refer to changes in teaching strategies that allow the student to access the curriculum.
- Environmental accommodations refer to changes that are required to the classroom and/or school environment.
- Assessment accommodations refer to changes that are required in order for the student to demonstrate learning.

The following three charts provide accommodations for the three key areas underlying math difficulties. Accommodations have been grouped under the headings of instructional, environmental, and assessment.

Chart I: Accommodations for Students with Language Difficulties

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Give concise, step-by-step directions • Teach students to look for cue words, highlight these words • Use visual models • Use visual representations to accompany word problems • Encourage students to look for common patterns in word problems	• Provide reference charts with operations and formulae stated simply • Post reference charts with math vocabulary • Reinforce learning with visual aids and manipulatives • Using a visual format, post strategies for problem solving • Use a peer tutor or buddy system	• Read instructions/word problems to students on tests • Extend time lines

Chart II: Accommodations for Students with Visual/Perceptual/Spatial/Motor Difficulties

Instructional	Environmental	Assessment
• Reduce copying • Provide worksheets • Provide graph paper • Provide concrete examples • Allow use of a number line • Provide a math journal • Encourage and teach self-talk strategies • Chunk learning and tasks	• visual bombardment • a work carrel or work area that is not visually distracting • rest periods and breaks	• Provide graph paper for tests • Extend time lines • Provide consumable tests • Reduce the number of questions required to indicate competency • Provide a scribe when lengthy written answers are required

Chart III: Accommodations for Students with Memory Difficulties

Instructional	Environmental	Assessment
• Regularly review concepts • Activate prior knowledge • Teach mnemonic strategies (e.g., BEDMAS) • Teach visualization strategies • Allow use of multiplication tables • Colour-code steps in sequence • Teach functional math concepts related to daily living	• Provide reference charts with commonly used facts, formulae, and steps for problem-solving • Allow use of a calculator • Use games and computer programs for drill and repetition	• Allow use of multiplications charts • Allow use of other reference charts as appropriate • Allow use of calculators • Extend time lines • Present one concept-type of question at a time

Accommodations for Enrichment

Some students benefit from having their programs enriched by extending their learning and emphasizing higher-order thinking skills. For the purposes of this resource manual, the term "enrichment" will be applied to activities that enrich and extend a student's program. This form of enrichment differs from acceleration. Acceleration involves reaching well above grade level expectations and thereby modifying a student's program. Students whose programs have been modified in this way are often identified as "Intellectual-Gifted" by an Identification Placement and Review Committee (IPRC). Modifying a student's program falls beyond the scope of this Teacher's Resource.

Accommodations for Enrichment

Instructional	Environmental	Assessment
• Structure learning activities to develop higher-order thinking skills (analysis, synthesis, and evaluation) • Provide open-ended questions • Value learner's own interests and learning style, and allow for as much student input into program options as possible • Encourage students to link learning to wider applications • Encourage learners to reflect on the process of their own learning • Encourage and reward creativity • Avoid repetitive tasks and activities	• Encourage a stimulating environment that invites exploration of mathematical concepts • Display pictures of role models who excel in mathematics • Provide access to computer programs that extend learning	• Reduce the number of questions to allow time for more demanding ones • Allow for opportunities to demonstrate learning in non-traditional formats • Pose more questions that require higher-level thinking skills (analysis, synthesis, and evaluation) • Reward creativity

Accommodations for ESL Students

For ESL students, language issues are pervasive throughout all subject areas, including math. Non-math words are often more problematic for ESL students because understanding the meaning of these words is often taken for granted. Everyday language is laden with vocabulary, comparative forms, figurative speech, and complex language structures that are not explained. By contrast, key words in math are usually highlighted in the text and carefully explained by the teacher. Accommodations to the programs of ESL students do not change the curriculum expectations.

Accommodations for ESL Students

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Explain colloquial expressions and figurative speech • Review comparative forms of adjectives	• Display reference charts with mathematical terms and language • Encourage personal math dictionaries with math terms and formulae	• Allow access to personal math dictionaries • Read instructions to students and clarify terms • Allow additional time

Accommodations for Learning-Disabled Students

A student with a learning disability usually suffers from an inability to think, listen, speak, write, spell, or calculate that is not obviously caused by any mental or physical disability. There seems to be a lag in the developmental process and/or a delay in the maturation of the central nervous system. Providing simplified presentations, repetitions, more specific examples, or breaking content blocks into simpler sections may help in minor cases of learning disability.

Accommodations for At-Risk Students

Each chapter of the Teachers' Resource has several margin items labelled **Student Success**. Students learn in different ways. For all students to have the opportunity to succeed, we need to have alternative ways of delivering program. For example, a student whose dominant learning modality is kinesthetic/tactile needs active, hands-on investigations. A student with strong social/emotional intelligence benefits more from interpersonal interactions and needs instructional strategies like Jigsaw or Think-Pair-Share to optimize their chances of acquiring the skills and knowledge in the curriculum (see the **Cooperative Learning** section in this Teacher's Resource). These students underachieve and become at-risk not because they have acquired concepts imperfectly (and need remediation), but because they have not become engaged in their own learning, and often have failed to acquire concepts at all. At-risk students are in danger of completing their schooling without adequate skills development to function effectively in society. Risk factors include low achievement and retention, behaviour problems, poor attendance, and low socio-economic status.

Student Success items are suggestions for alternative ways to approach some key topics. By addressing these topics in a new or different way, teachers can provide at-risk students with the opportunity to learn in a manner that may engage them and increase their chances of success.

Neither failing such students nor putting them in pullout programs has produced much gain in achievement, but there are certain approaches that do help.

- Allow students to proceed at their own pace through a well-defined series of instructional objectives.
- Place students in small, mixed-ability learning groups to master the material first presented by the teacher. Reward teams based on the

individual learning of all team members.

- Have students serve as peer tutors, as well as being tutored. This helps raise their self-esteem and makes them feel they have something to contribute.
- Involve students in learning about something that is relevant to them, such as money management or wise shopping.
- Get parents involved in their child's learning as much as possible.

Curriculum Correlation between McGraw-Hill Ryerson *Principles of Mathematics 9* and The Ontario Curriculum (MPM1D)

This course enables students to develop an understanding of mathematical concepts related to algebra, analytic geometry, and measurement and geometry through investigation, the effective use of technology, and abstract reasoning. Students will investigate relationships, which they will then generalize as equations of lines, and will determine the connections between different representations of a linear relation. They will also explore relationships that emerge from the measurement of three-dimensional figures and two-dimensional shapes. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

Mathematical Process Expectations

The mathematical processes are integrated into student learning in all areas of this course. Throughout this course, students will:

Problem Solving **MPS.01**	• develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving **MPS.02**	• develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting **MPS.03**	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies **MPS.04**	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical idea and to solve problems;
Connecting **MPS.05**	• make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing **MPS.06**	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representation to solve problems;
Communicating **MPS.07**	• communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

The process expectations are introduced in Chapter 1 of *Principles of Mathematics 9* and then integrated throughout Chapters 2 to 9.

The codes for the curriculum expectations are consistent with the codes used in the PDF document Grade 9 Math Principles (Reference; Principles of Mathematics Expectations MPM 1D), which is based on The Ontario Curriculum, Grades 9 and 10, Mathematics, 2005. The document is available on-line from The Ontario Curriculum Unit Planner (OCUP), in the section Grade by Grade PDFs of Ontario Curriculum Expectations. <http://www.ocup.org>

Number Sense and Algebra

Overall Expectations

By the end of this course, students will:

NAV.01 demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions

NAV.02 manipulate numerical and polynomial expressions, and solve first-degree equations

Specific Expectations

	Chapter/Section	Pages
Operating with Exponents		
By the end of this course, students will:		
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and $(9.83)^3$ by using a calculator]);	**3.1, 3.2** **Throughout Chapters 8, 9**	104–118 412–519
NA1.02 describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by $(x)(x)$ or x^2; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or x^3];	**3.1, 3.2**	104–118
NA1.03 derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;	**3.3**	119–129
NA1.04 extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.	**3.3**	119–129
Manipulating Expressions and Solving Equations		
By the end of this course, students will:		
NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;	**Throughout all chapters**	
NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion;	**Throughout all chapters**	
NA2.03 relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations;	**3.1, 3.2** **4.4**	104–118 211–219
NA2.04 add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $(3x^2y + 2xy^2) + (4x^2y - 6xy^2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);	**3.4, 3.5, 3.6**	130–159
NA2.05 multiply a polynomial by a monomial involving the same variable [e.g., $2x(x + 4)$, $2x^2(3x^2 - 2x + 1)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil);	**3.7**	160–169
NA2.06 expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);	**3.7** **Throughout Chapter 6**	160–169 292–355
NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies);	**4.1, 4.2, 4.3**	186–210

NA2.08 rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement);	**4.4** **Throughout Chapter 6 and Chapter 8**	211–219 292–355 412–473
NA2.09 solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods;	**4.5**	220–229

Linear Relations

Overall Expectations

By the end of this course, students will:

REV.01 apply data-management techniques to investigate relationships between two variables;

REV.02 demonstrate an understanding of the characteristics of a linear relation;

REV.03 connect various representations of a linear relation.

Specific Expectations

	Chapter/Section	Pages
Using Data Management to Investigate Relationships		
By the end of this course, students will:		
RE1.01 interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant (e.g., on a scatter plot of height versus age, interpret the point (13, 150) as representing a student who is 13 years old and 150 cm tall; identify points on the graph that represent students who are taller and younger than this student);	**2.3** **2.5, 2.6**	56–67 77–94
RE1.02 pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;	**2.1** **2.5, 2.6**	42–47 77–94
RE1.03 design and carry out an investigation or experiment involving relationships between two variables, including the collection and organization of data, using appropriate methods, equipment, and/or technology (e.g., surveying; using measuring tools, scientific probes, the Internet) and techniques (e.g., making tables, drawing graphs);	**2.2, 2.3** **2.6**	48–67 88–94
RE1.04 describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses (e.g., describe the trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your hypothesis? Identify and explain any outlying pieces of data. Suggest a formula that relates the variables. How might you vary this experiment to examine other relationships?);	**2.1** **2.3, 2.4, 2.5, 2.6**	42–47 56–94
Understanding Characteristics of Linear Relations		
By the end of this course, students will:		
RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;	**Throughout Chapters 2 and Chapters 4, 5, 6**	38–99 182–359
RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);	**2.3, 2.4, 2.5, 2.6** **5.1, 5.2** **5.5, 5.6**	56–94 238–253 272–287

RE2.03 identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;	**2.5** **5.4, 5.5, 5.6**	77–87 264–287
RE2.04 compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);	**5.1, 5.2**	238–253
RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).	**2.5** **5.1** **5.5, 5.6** **7.3**	77–87 238–245 272–287 384–393
Connecting Various Representations of Linear Relations		
By the end of this course, students will:		
RE3.01 determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;	**2.4** **Throughout Chapter 5**	68–76 234–291
RE3.02 describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;	**2.6** **5.1, 5.2** **5.4** **6.1, 6.2**	88–94 238–253 264–271 294–314
RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);	**2.5, 2.6** **5.1, 5.2** **5.5, 5.6**	77–94 238–253 272–287
RE3.04 describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).	**5.1, 5.2**	238–253

Analytic Geometry

Overall Expectations

By the end of this course, students will:

AGV1.01 determine the relationship between the form of an equation and the shape of its graph with respect to linearity and non-linearity;

AGV1.01 determine, through investigation, the properties of the slope and y-intercept of a linear relation;

AGV1.01 solve problems involving linear relations.

Specific Expectations

	Chapter/Section	Pages
Investigating the Relationship between the Equation of a Relation and the Shape of its Graph		
By the end of this course, students will:		
AG1.01 determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of nonlinear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation);	**5.5** **6.1**	272–278 296–307

AG1.02 identify, through investigation, the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$;	**6.1, 6.2**	296–314
AG1.03 express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$.	**6.2**	308–314
Investigating the Properties of Slope		
By the end of this course, students will:		
AG2.01 determine, through investigation, various formulas for the slope of a line segment or to determine the slope of a line segment or a line; (e.g., $m = \frac{\text{rise}}{\text{run}}$, $m = \frac{\text{the change in } y}{\text{the change in } x}$ or $m = \frac{\Delta y}{\Delta x}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$ and use the formulas to determine the slope of a line segment or a line;	**5.3, 5.4** **5.6**	254–263 279–287
AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;	**6.1, 6.3 Use Technology, 6.4**	296–307 323–329
AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation);	**5.1, 5.2** **5.4** **5.6**	238–253 264–271 279–287
AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.	**5.3** **6.1** **6.4**	254–262 296–307 323–329
Using the Properties of Linear Relations to Solve Problems		
By the end of this course, students will:		
AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);	**Throughout Chapter 6**	292–355
AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);	**6.5, 6.6**	330–343
AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);	**5.1, 5.2** **5.4** **5.6** **Throughout Chapter 6**	238–253 264–271 279–287 330–343
AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550);	**5.1, 5.2** **6.2, 6.3** **6.7**	238–253 308–322 344–351
AG3.05 determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application.	**6.7**	344–351

Measurement and Geometry

Overall Expectations

By the end of this course, students will:

MGV.01 determine, through investigation, the optimal values of various measurements;

MGV.01 solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

MGV.01 verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Investigating the Optimal Values of Measurements		
By the end of this course, students will:		
MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;	**9.1, 9.2**	478–490
MG1.02 determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant;	**9.1, 9.2**	478–490
MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];	**9.3** **9.6**	491–497 510–515
MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);	**9.3, 9.4, 9.5, 9.6**	491–515
MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).	**Throughout Chapter 9**	474–519
Solving Problems Involving Perimeter, Area, Surface Area, and Volume		
By the end of this course, students will:		
MG2.01 relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$;	**8.1**	418–425
MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);	**8.3, 8.4, 8.5**	436–456
MG2.03 solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles);	**8.2**	426–435

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is $\frac{1}{3}$ the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{pyramid} = \frac{V_{prism}}{3}$ or $V_{pyramid} = \frac{(\text{area of base})(\text{height})}{3}$;	**8.5, 8.6, 8.7**	451–469
MG2.05 determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);	**8.3**	436–443
MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.	**8.3, 8.4, 8.5, 8.6, 8.7**	436–469
Investigating and Applying Geometric Relationships		
By the end of this course, students will:		
MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;	**7.1, 7.2, 7.3**	364–393
MG3.02 determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);	**7.4, 7.5**	394–407
MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);	**Throughout Chapter 7**	360–411
MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.	**Throughout Chapter 7**	360–411