CHAPTER

7

Geometric Relationships

Vocabulary

polygon
vertex
interior angle
exterior angle
ray
equiangular
adjacent
supplementary
transversal
congruent
convex polygon
concave polygon
pentagon
hexagon
heptagon
octagon
regular polygon
midpoint
median
bisect
right bisector
centroid
similar

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Strand:
Measurement and Geometry

Overall Expectations

By the end of this course, students will:

MGV.01 determine, through investigation, the optimal values of various measurements;

MGV.02 solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

MGV.03 verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Investigating and Applying Geometric Relationships

By the end of this chapter, students will:

MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;

MG3.02 determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Strand:
Linear Relations

Overall Expectations

By the end of this course, students will:

LRV.01 apply data-management techniques to investigate relationships between two variables;

LRV.02 demonstrate an understanding of the characteristics of a linear relation;

LRV.03 connect various representations of a linear relation.

Specific Expectations

Understanding Characteristics of Linear Relations

RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Chapter Problem

The Chapter Problem introduced in the Chapter Opener leads students to find the centroid, or centre of mass, of a triangle. You may wish to have students try the experiment described. Triangles cut out of cardboard work very well. Students can take turns attempting to balance the triangle; you will find that some students come very close to succeeding through trial and error. Have students discuss their understanding of the topic as a class discussion.

You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 411. Alternatively, you may wish to assign only the Chapter Problem Wrap-Up when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment.

Chapter 7 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 7 Opener • 15 min	360–361			
Get Ready • 80 min	362–363	• BLM 7.GR.1 Using a MIRA With Isosceles Triangles • BLM 7.GR.2 Opposite, Supplementary, and Complementary Angles • BLM 7.GR.3 Practice: Get Ready	• BLM 7.GR.4 Get Ready Self-Assessment Checklist	
Section 7.1 Angle Relationships in Triangles • 80 min	364–373	• BLM G10 Grid Paper • BLM G4 Protractor • BLM 7.GR.3 Practice: Get Ready • BLM 7.1.1 Practice: Angle Relationships in Triangles • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM 7.1.2 Bisectors, Medians, and Altitudes • BLM 7.1.3 Word Origins and Plurals	• BLM A23 News Report Checklist	**Tools** • grid paper • rulers • protractors • scissors • globe • string **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators • Internet access
Section 7.2 Angle Relationships in Quadrilaterals • 80 min	374–383	• BLM G10 Grid Paper • BLM G4 Protractor • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals	• BLM A9 Communication General Scoring Rubric	**Tools** • grid paper • rulers • protractors • craft sticks • white glue **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators
Section 7.3 Angle Relationships in Polygons • 80 min	384–393	• BLM G10 Grid Paper • BLM G4 Protractor • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM 7.3.1 Practice: Angle Relationships in Polygons • BLM 7.3.2 Constructing the Gazebo	• BLM 7.3.3 Achievement Check Rubric • BLM A10 Observation General Scoring Rubric	**Tools** • grid paper • rulers • protractors • dollar coins (from students) **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators • Internet access
Section 7.4 Midpoints and Medians in Triangles • 80 min	394–400	• BLM 7.4.1 Practice: Midpoints and Medians in Triangles • BLM G10 Grid Paper • BLM G4 Protractor • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM 7.4.2 Using a MIRA to Investigate Lines of Symmetry • BLM 7.4.3 Sierpinski's Triangle	• BLM A9 Communication General Scoring Rubric	**Tools** • grid paper • rulers • protractors • MIRA **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators • Internet access

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Section 7.5 Midpoints and Diagonals in Quadrilaterals • 80 min	401–407	• BLM G10 Grid Paper • BLM G4 Protractor • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM 7.5.1 Constructing Parallelograms • BLM 7.5.2 Practice: Midpoints and Diagonals in Quadrilaterals	• BLM 7.5.3 Achievement Check Rubric	**Tools** • grid paper • rulers • protractors **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators
Chapter 7 Review • 80 min	408–409	• BLM 7.CR.1 Chapter 7 Review • BLM G10 Grid Paper • BLM G4 Protractor • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4		**Tools** • grid paper • protractors **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators
Chapter 7 Practice Test • 60 min	410–411	• BLM G10 Grid Paper • BLM G4 Protractor • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4	• BLM 7.PT.1 Chapter 7 Practice Test • BLM 7.CT.1 Chapter 7 Test	**Tools** • grid paper • protractors **Technology Tools** • *The Geometer's Sketchpad*® • computers • *Cabri*® *Jr.* • graphing calculators
Chapter 7 Problem Wrap-Up • 80 min	411	• BLM G10 Grid Paper • BLM G4 Protractor	• BLM A8 Application General Scoring Rubric • BLM A10 Observation General Scoring Rubric • BLM 7.CP.1 Chapter 7 Problem Wrap-Up Rubric • BLM A18 My Progress as a Problem Solver	**Tools** • cardboard triangles created in Chapter Opener • more heavy cardboard • grid paper • rulers • protractors • utility or craft knives • metre sticks **Technology Tools** • computers • Internet access

Chapter 7 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 7.GR.1	Using a MIRA With Isosceles Triangles	Student Support
	BLM 7.GR.2	Opposite, Supplementary, and Complementary Angles	Student Support
	BLM 7.GR.3	Practice: Get Ready	Practice
	BLM 7.GR.4	Get Ready Self-Assessment Checklist	Student Self-Assessment
7.1: Angle Relationships in Triangles			
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM 7.GR.3	Practice: Get Ready	Practice
	BLM 7.1.1	Practice: Angle Relationships in Triangles	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 7.1.2	Bisectors, Medians, and Altitudes	Student Support
	BLM 7.1.3	Word Origins and Plurals	Literacy
	BLM A23	News Report Checklist	Assessment Literacy
7.2: Angle Relationships in Quadrilaterals			
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 7.2.1	Practice: Angle Relationships in Quadrilaterals	Practice
	BLM A9	Communication General Scoring Rubric	Assessment
7.3: Angle Relationships in Polygons			
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 7.3.1	Practice: Angle Relationships in Polygons	Practice
	BLM 7.3.2	Constructing the Gazebo	Student Support
	BLM 7.3.3	Achievement Check Rubric	Assessment
	BLM A10	Observation General Scoring Rubric	Assessment

	BLM	Title	Purpose
7.4: Midpoints and Medians in Triangles			
	BLM 7.4.1	Practice: Midpoints and Medians in Triangles	Practice
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 7.4.2	Using a MIRA to Investigate Lines of Symmetry	Student Support
	BLM A9	Communication General Scoring Rubric	Assessment
	BLM 7.4.3	Sierpinski's Triangle	Student Support
7.5: Midpoints and Diagonals Quadrilaterals			
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 7.5.1	Constructing Parallelograms	Student Support
	BLM 7.5.2	Practice: Midpoints and Diagonals in Quadrilaterals	Practice
	BLM 7.5.3	Achievement Check Rubric	Assessment
Chapter 7 Review			
	BLM 7.CR.1	Chapter 7 Review	Practice
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
Chapter 7 Practice Test			
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 7.PT.1	Chapter 7 Practice Test	Diagnostic Assessment
	BLM 7.CT.1	Chapter 7 Test	Summative Assessment

	BLM	Title	Purpose
Chapter 7 Problem Wrap-Up			
	BLM G10	Grid Paper	Student Support
	BLM G4	Protractor	Student Support
	BLM A8	Application General Scoring Rubic	Assessment
	BLM A10	Observation General Scoring Rubric	Assessment
	BLM 7.CP.1	Chapter 7 Problem Wrap-Up Rubric	Summative Assessment
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment

Get Ready

Student Text Pages
362 to 363

Suggested Timing
80 min

Related Resources
BLM 7.GR.1 Using a MIRA With Isosceles Triangles
BLM 7.GR.2 Opposite, Supplementary, and Complementary Angles
BLM 7.GR.3 Practice: Get Ready
BLM 7.GR.4 Get Ready Self-Assessment Checklist

Common Errors

- Some students may not know how to measure an angle.

R_x Assign **BLM 7.GR.3 Practice: Get Ready** as remediation or extra practice.

- Some students may not be familiar with some of the polygons.

R_x Ask students to look for different geometric shapes in their environment in order to make their application more relevant.

- Some students may not understand that some shapes can be classified in more than one way.

R_x As an example, challenge students to classify a square in as many ways as possible. They should be able see that a square is also a quadrilateral, a rectangle, a parallelogram, and a rhombus.

Accommodations

Gifted and Enrichment— Challenge students to investigate the other triangle centre points and the Euler line. Have them use dynamic geometry software to develop circle geometry properties and a variety of proofs of the Pythagorean theorem.

Student Success

Have students construct a **word wall** for the terms in this chapter as they arise. Have students phrase definitions in their own words and revise them as their understanding grows.

Teaching Suggestions

- The Get Ready provides a review of basic ideas and definitions from previous grades that students need to refer to throughout this chapter.
- Have students work with a partner or in small groups.
- You may wish to assign the entire Get Ready at the beginning of the chapter as an introduction, or assign relevant segments before starting each section.
- For further investigation or review of some interesting triangle properties, have students work on **BLM 7.GR.1 Using a MIRA With Isosceles Triangles** and/or **BLM 7.GR.2 Opposite**, **Supplementary**, **and Complementary Angles**.
- You may wish to use **BLM 7.GR.3 Practice: Get Ready** for remediation or extra practice.
- All BLMs referred to throughout this chapter can be found on the *Principles of Mathematics 9* Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; using **BLM 7.GR.4 Get Ready Self-Assessment Checklist**, students can place a checkmark beside topics in the Get Ready in which they feel confident with the necessary skills. Remedial action can be taken in small groups or with a whole class skill review.

7.1 Angle Relationships in Triangles

Strand:
Measurement and Geometry

Student Text Pages
364 to 373

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- scissors
- globe
- string

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators
- Internet access

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM 7.GR.3 Practice: Get Ready
BLM 7.1.1 Practice: Angle Relationships in Triangles
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.1.2 Bisectors, Medians, and Altitudes
BLM 7.1.3 Word Origins and Plurals
BLM A23 News Report Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Link to Get Ready

The Get Ready segments Classify Triangles, Classify Polygons, and Angle Properties provide the needed skills for this section. Have students complete or review Get Ready questions 1 to 6 before starting this section.

Warm-Up

Introduce this section by having students practise measuring angles. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity. Many students may not realize that a protractor can be used from either the left or right side depending upon the orientation of the angle being measured. You may want to distribute or review **BLM 7.GR.3 Practice: Get Ready**, if it was not already used in the Get Ready, and/or **BLM 7.1.1 Practice: Angle Relationships in Triangles**.

Teaching Suggestions

- There are three methods for the Investigate: Use Pencil and Paper, Use *The Geometer's Sketchpad*®, and Use a Graphing Calculator. The pencil and paper method takes about 10 min; the time needed for the technology approaches will vary depending upon the students' experience. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity. (10–20 min)
- The latest version of *Cabri*® *Jr.* can be downloaded from http://www.cabrijr.com.
- If you plan to conduct all three methods, have students work in small groups. Divide the groups equally so each method is covered by at least one group. For the Investigate and reflect questions, regroup the students so that each of the new groups includes at least one person who tried each of the three different approaches. Encourage group members to compare the methods that they used and their solutions.
- Prepared sketches for *The Geometer's Sketchpad*® and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson web site, at http://www.mcgrawhill.ca/books/principles9.

Common Errors

- Some students may draw every triangle as an isosceles or equilateral triangle.

R_x Have students practise drawing scalene triangles.

Ongoing Assessment

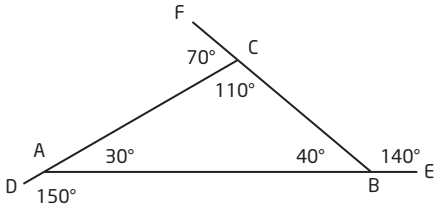
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- As a follow-up activity to the Investigate, have students draw a triangle, mark the interior angles, and cut out the triangle. Have them tear off the three vertices, then place the vertices so that the three angles are adjacent to each other on their desktop. They should form a straight angle, confirming the sum of the three interior angles. Have students then use a similar approach to investigate the sum of the exterior angles.
- Challenge the students to find a different sum of the angles by drawing unusually shaped triangles, such as a triangle with a very large obtuse angle.
- Assign the Examples. Have students work through the Examples, continuing with whatever methods they used in the Investigate.
- Discuss the Key Concepts and the Communicate Your Understanding questions. (10–15 min)
- Assign the Practise questions.
- You may wish to assign **BLM 7.1.2 Bisectors, Medians, and Altitudes,** in which students study these aspects of angles and look for patterns.
- You may wish to use **BLM 7.1.1 Practice: Angle Relationships in Triangles** for remediation or extra practice.

Investigate Answers (pages 364–368)

Method 1

1–3. Answers will vary. Sample solution:



4. Sums should be approximately 360°. Students' measurements may not be precise enough to get exactly 360°.

5. Yes; The interior angle and the exterior angle at each vertex are supplementary, so the sum of all the interior and exterior angles is 540°. Since the sum of the interior angles in a triangle is 180°, the sum of the exterior angles is 540° − 180°, or 360°.

6. They are supplementary angles; their sum is 180°.

7. a) The exterior angle at each vertex is equal to the sum of the interior angles at the other two vertices.

b) Yes; the sum of the interior angles in a triangle is 180°, and each interior angle and the related exterior angle are supplementary.

Method 2

7. They are supplementary angles; their sum is 180°.

8. Answers will vary. Sample solution: The sum of the exterior angles is 360°.

9. The sum remains 360°.

10. The sum of the exterior angles is 360°. Measurements show that this sum remains 360° regardless of the shape of the triangle.

11. Answers will vary. Sample solution: The exterior angle is equal to the sum of the interior angles at the other two vertices.

13. Answers will vary. Sample solution: Measurements show that the exterior angle is equal to the sum of the interior angles at the other two vertices regardless of the shape of the triangle.

Accommodations

Gifted and Enrichment—Most students will create exterior angles by going around a figure consistently either clockwise or counter-clockwise. Some students may notice that there are two possible exterior angles at each vertex. Challenge students to consider whether it matters which side of the vertex they used to measure the exterior angle.

For an interesting extension in non-plane geometry, bring a globe into the classroom. Use a string to make a triangle with one vertex at the North Pole, another at the intersection of the Equator and longitude 0° (the Prime Meridian), and the third at the intersection of the Equator and longitude 90°W. Measure the interior angles of this triangle. Alternatively, you could find three locations with the same attributes, but using names instead of navigational lines.

ESL—Have students work with a partner or in small groups.

Student Success

Use a **jigsaw** approach to have students investigate and teach the concepts of this section.

Method 3

7. The sum of the three exterior angles is 360°.

8. Answers will vary. A sample hypothesis: The sum of the exterior angles is 360°.

10. Answers will vary. The sum of the exterior angles is 360°. Measurements show that this sum remains 360° regardless of the shape of the triangle.

11. They are supplementary angles; their sum is 180°.

12. Answers will vary. A sample hypothesis: The exterior angle is equal to the sum of the interior angles at the other two vertices.

14. Answers will vary. Measurements show that the exterior angle is equal to the sum of the interior angles at the other two vertices regardless of the shape of the triangle.

Communicate Your Understanding Responses (page 371)

C1. The sum of exterior angles is 360°. Therefore, the measure of the exterior angle at vertex X is 360° − 140° − 120°, or 100°.

C2. Yes, the exterior angle is equal to the sum of the interior angles at the other two vertices, so, x is 40° + 70°, or 110°.

Practise

The Practise questions are straightforward and involve the skills demonstrated in Example 1.

Connect and Apply

Before students begin questions 4, 5c), 5d), 5e), and 6, you may wish to review the properties of isosceles triangles.

Extend

For question 11, have students copy or trace the diagram, then remind students to work carefully through the two triangles and label values as they find them.

Question 12 requires students to apply their knowledge of ratios. Some students may need hints on how they can add up the terms in the ratio, divide the sum of the exterior angles (360°) by that total, and then multiply each term in the ratio by this result to get the angle measurement. Encourage students to check their results by simplifying the ratio of the angles.

Students will enjoy trying out question 13. There are many Web sites that provide information about hexaflexagons, such as http://www.maths.uq.edu.au/~infinity/Infinity%2012/hexaflex.html.

Literacy Connections

Plurals

Point out to students that to find the plural of a word in English, we usually just add an *–s*. Some words have irregular plurals, however, such as the word *vertex*—its plural is *vertices*. Challenge students to think of other words in mathematics that have unusual plurals. (They might try an Internet search.) Ask students if they can find a connection between the origins of words and unusual plurals. This particular connection can lead to many discoveries:

- the *–ix* and *–ex* endings in Latin become *–ices* in the plural (e.g., appendix/appendices, index/indices)
- the Latin ending of *–um* becomes *–a* in the plural (e.g., datum/data)
- the singular in Latin or Greek ending *–us* becomes *–i* in the plural (e.g., focus/foci, radius/radii)
- the Latin or Greek ending *–is* becomes *–es* in the plural (e.g., basis/bases, axis/axes, hypothesis/hypotheses, analysis/analyses)

You may wish to use **BLM 7.1.3 Word Origins and Plurals** to support this activity.

Conventions

Point out that mathematics involves many *conventions*. This means that everyone does something a certain way to make sure that we all understand the same thing. For example, if an angle is called ∠BAD, that means that it is formed by the lines between points B and A and points A and D, and that the vertex of the angle is at point A. Present the rule of order of operations as another convention; remind students of the acronym BEDMAS. Point out that such conventions mean that everyone working the same equation follows the same rules and will get the same answer. Challenge students to research information about mathematics in the time before we had such conventions, or about a famous mathematician who gave us a particular convention. Have them write a news report about their topic, remembering to talk about the who, what, where, when, why, and how. You may wish to use **BLM A23 News Report Checklist** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6
Typical	1–10
Extension	11–15

7.2 Angle Relationships in Quadrilaterals

Strand:
Measurement and Geometry

Student Text Pages
374 to 383

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- craft sticks
- white glue

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter example, with or without the use of dynamic geometry software.

Link to Get Ready

Before starting this section, refer students to Get Ready question 4 so that students are reminded of the different types of quadrilaterals.

Teaching Suggestions

- As in Section 7.1, there are three methods that can be used for the Investigate. Using dynamic software (Method 2 or Method 3) allows students to explore many possibilities, whereas using paper and pencil (Method 1) generates a static diagram that is useful for the single context. You may wish to have students using Method 1 compare results with others, so they can see the Investigate applied to several different quadrilaterals. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support Method 1. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support Method 2. (10–15 min)
- If you used all three methods in Section 7.1, have students work in new groups with one student in each group who experienced each method. Allow each group to choose which approach to use for this section. Alternatively, retain the groups that were formed at the start of Section 7.1, but rotate methods as much as possible.
- Prepared sketches for *The Geometer's Sketchpad*® and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site. Go to http://www.mcgrawhill.ca/books/principles9
- As an alternative to the three methods, have students draw a quadrilateral, mark the interior angles, and tear off the four vertices. Ask them to place the vertices (angles) adjacent to each other on their desktops; the four angles should form a full revolution. Then, suggest they use a similar approach to find the sum of the exterior angles.
- The Investigate and Reflect questions allow students to draw conclusions from the Investigate. Encourage them to write their own conclusions before summarizing the findings in a class discussion.
- If you plan to use the software to demonstrate the Examples, be aware that it is difficult to create a figure with sides of different lengths and angles of a specific degree measure. Instead, create a specific angle using a rotation. To get sides of different lengths, create new points on the reflected line segments.

Common Errors

- Some students may have forgotten the properties that arise from a set of parallel lines and a transversal.

R_x Have students review these properties, highlighting the letters (F, C, and Z) that are used as reminders of the properties.

- Some students may struggle with vocabulary.

R_x Encourage students to use correct mathematical language. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

- Some students who are unable to construct an accurate diagram may have difficulties with these exercises.

R_x While it is best if students have their own equipment, have extra protractors and rulers available, or provide transparent copies of **BLM G4 Protractor**.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Summarize the Investigate for students. For the questions, have students use the pencil and paper method to allow students' understanding of the properties of angles guide them in their solutions.
- Review the new vocabulary items, and add them to the word wall. You may also wish to add all the variations of the Z, C, and F patterns to the wall.
- You may wish to use **BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals** for remediation or extra practice.

Investigate Answers (pages 374–378)

Method 1

3-4. Sums should be about 360°. Students' measurements may not be precise enough to get exactly 360°.

5. Answers will vary. A sample hypothesis: The sum of the interior angles of any quadrilateral is 360°. Draw an example of each type of quadrilateral and measure the interior angles.

6. The sum of the interior angles of the two triangles equals the sum of the interior angles of the quadrilateral.

7. Answers will vary. Sample solution: Since a quadrilateral can be divided into two triangles, the sum of its interior angles is $2 \times 180°$, or 360°.

8. Sums should be about 360°. Students' measurements may not be precise enough to get exactly 360°.

9. Answers will vary. A sample hypothesis: The sum of the exterior angles of any quadrilateral is 360°. You use the sum of the interior angles sum to calculate the sum of the exterior angles.

Method 2

6. Answers will vary. A sample hypothesis: The sum of the interior angles of any quadrilateral is 360°.

8. The sum of the interior angles of a quadrilateral is 360°. Measurements show that the sum remains 360° regardless of the shape of the quadrilateral.

9. Answers will vary. A sample hypothesis: The sum of the exterior angles of any quadrilateral is 360°.

12. Answers will vary. Sample solution: Measurements show that the sum of the exterior angles is 360° regardless of the shape of the quadrilateral.

Method 3

5. Answers will vary. A sample hypothesis: The sum of the interior angles of any quadrilateral is 360°.

7. The sum of the interior angles of a quadrilateral is 360°. Measurements show that the sum remains 360° regardless of the shape of the quadrilateral.

8. Answers will vary. A sample hypothesis: The sum of the exterior angles of any quadrilateral is 360°.

12. Answers will vary. Sample solution: Measurements show that the sum of the exterior angles is 360° regardless of the shape of the quadrilateral.

Communicate Your Understanding Responses (page 380)

C1. Since the sum of the interior angles of any quadrilateral is 360°, the measure of $\angle P$ is $360° - 70° - 90° - 90°$, or 110°.

C2. Yes; Since the sum of the exterior angles of any quadrilateral is 360°, x represents an angle measure of $360° - 90° - 95° - 125°$, or 50°.

Accommodations

Gifted and Enrichment—As with triangles (see Section 7.1), challenge students to consider if it matters which side of the vertex is used for the exterior angle. Extend the ideas of a cyclic quadrilateral using dynamic geometry software.

Student Success

Have students construct **Frayer Models** for the concepts in this section.

Practise

If you ask students to draw diagrams for questions 2 and 3, be sure they use the proper instruments with accurate degree measures. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity.

Students might need more assistance with question 5, which requires them to apply the properties of parallel lines. You may wish to assign **BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals** for more reinforcement.

Connect and Apply

For question 10, encourage students to create diagrams and explain their thinking. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity.

Question 12c) will challenge students. You may wish to make it into a poster to add to the classroom. Consider having students create a bridge similar to the one shown using craft sticks and white glue. Once it dries, load it with increasingly heavy weights to see how much stress it will tolerate before collapsing.

Extend

Encourage students to create diagrams and add explanations to show their thinking for all questions. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support these activities.

Question 16 will again call on students' skills involving ratios (refer them to question 12 in Section 7.1 to extrapolate a similar method for quadrilaterals).

The idea of a cyclic quadrilateral, introduced in question 17, will be new to students and can provide opportunities for gifted students to explore angle properties.

Literacy Connections

Prefixes

Remind students of the discussion of prefixes from Chapter 3. Explain that many of the prefixes we use in mathematics come from Greek. For example, the terms *pentagon*, *hexagon*, *heptagon*, and *octagon* are derived from the Greek words for five, six, seven, and eight respectively. Point out that the prefix *poly-* means "many," so, when we refer to a polygon, we are talking about a many-sided object. By convention, we use *polygon* to signify a shape with four sides or more (we saw a similar convention in Chapter 3: monomial, binomial, trinomial, and polynomial—one, two, three, and then many).

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 5, 7, 9
Typical	1–12
Extension	13–19

7.3 Angle Relationships in Polygons

Strand:
Measurement and Geometry

Strand:
Linear Relations

Student Text Pages
384 to 393

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- dollar coins (from students)

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators
- Internet access

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.3.1 Practice: Angle Relationships in Polygons
BLM 7.3.2 Constructing the Gazebo
BLM 7.3.3 Achievement Check Rubric
BLM A10 Observation General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Understanding Characteristics of Linear Relations

RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Link to Get Ready

The Get Ready section Classify Polygons provides students a chance to consolidate their previous experiences with polygons before starting this section. You may wish to assign question 3.

Teaching Suggestions

- Assign Investigate A. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity.
- In a class discussion, summarize the Investigate into angle relationships in pentagons. (5–10 min)
- Similar to Sections 7.1 and 7.2, Investigate B can be done with pencil and paper (Method 1), or on computers with *The Geometer's Sketchpad*® (Method 2). A worksheet for Method 3: Using a Graphing Calculator with *Cabri*® *Jr.* is available at www.mcgrawhill.ca/links/principles9.
- Divide the class into small groups, and assign each group a method. (If technology is not available, have all groups use Method 1). If you have used all three methods in the previous two sections, assign students a method they have not yet used in the previous activities. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor**, and/or **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4**.
- Prepared sketches for *The Geometer's Sketchpad*® and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site. Go to http://www.mcgrawhill.ca/books/principles9.
- For Investigate and Reflect questions, re-form the groups so that each group includes someone who has done each of the methods, and encourage them to compare notes and results.
- Encourage students to compare answers from the different groups and different approaches that they may have seen. Follow up with a class discussion, and conclude with the resulting formula for calculating the

Common Errors

- As with triangles and quadrilaterals, some students may try to draw each polygon as a regular polygon.

R_x Have students spend time specifically practising drawing non-regular polygons.

Ongoing Assessment

- Use Achievement Check question 15 to monitor student success. See Achievement Check Answers and **BLM 7.3.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

interior angles of a polygon.
- Use the resulting formula to work through the Examples with the class. (10–15 min)
- For a list of polygon names, visit: http://www.math.com/geometry/polygons.htm.
- You may wish to assign **BLM 7.3.1 Practice: Angle Relationships in Polygons** for remediation or extra practice.

Investigate Answers (pages 384–388)

A.

1. Answers will vary. A sample hypothesis: The sum of the interior angles of a pentagon is 540°.

3. Sums should be about 540°. Students' measurements may not be precise enough to get exactly 540°.

4. Answers will vary. Sample solution: Since the class finds the sums to be about 540° for a variety of pentagons, it is likely that the sum is the same for all pentagons.

5. These diagonals create three triangles.

6. The interior angles of the triangles make up the interior angles of the pentagon, so their sums are equal. This relationship holds for all pentagons.

7. Answers will vary. The relationship shows that the sum of interior angles of pentagon is 180° × 3, or 540°.

9. Sums should be about 360°. Students' measurements may not be precise enough to get exactly 360°.

10. Answers will vary. Sample solution: Since the class finds the sums to be about 360° for a variety of pentagons, it is likely that the sum is the same for all pentagons.

B.

Method 1

1.

Polygon	Number of Sides	Number of Diagonals from One Vertex	Number of Triangles in the Polygon	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	0	1	180°	360°
Quadrilateral	4	1	2	360°	360°
Pentagon	5	2	3	540°	360°
Hexagon	6	3	4	720°	360°
Heptagon	7	4	5	900°	360°

2. 720°

3. 360°

4. 900°

5. 360°

6. The sum of the exterior angles of any polygon is 360°

7. Number of diagonals = number of sides − 3

8. Number of triangles = number of sides − 2

9. Answers will vary. Sample solution: The sum of the interior angles increases by 180° when the number of sides increases by 1. So, the sum of the interior angles is $180°(n - 2)$. You can find the sum of the interior angles of any polygon by substituting the appropriate value for n into the equation.

Accommodations

Gifted and Enrichment—Use Math Contest question 21 as the starting point for a discussion of how you can add up a series of numbers. Although this reaches forward to grade 11 material, a strong student might like to know that there is a formula for finding the value of $n + (n - 1) + (n - 2) + \ldots + 3 + 2 + 1$.

Student Success

Have students construct **collages** of angle relationships in polygons that they find in real life. Post the collages in class.

Method 2.

1.

Polygon	Number of Sides	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	180°	360°
Quadrilateral	4	360°	360°
Pentagon	5	540°	360°
Hexagon	6	720°	360°
Heptagon	7	900°	360°

5. Moving the vertices does not affect the sum of the interior angles.

6. Answers will vary. Sample solution: Measurements show that the sum of interior angles in any hexagon is 720° regardless of the specific shape of the hexagon.

8. 360°; Moving the vertices does not affect the sum of the exterior angles.

9. Answers will vary. Sample solution: Measurements show that the sum of exterior angles in a hexagon is 360° regardless of the specific shape of the hexagon.

11. 900°; Moving the vertices does not affect the sum of the exterior angles.

12. Answers will vary. Sample solution: The sum of interior angles in any heptagon is 900° since the sum does not change when the vertices are moved.

14. 360°; The sum is not affected by moving the vertices.

15. Answers will vary. Sample solution: Measurements show that the sum of exterior angles in a heptagon is 360° regardless of the specific shape of the heptagon.

16. The sum of exterior angles in any convex polygon is 360°.

17. Answers will vary. Sample solution: The sum of the interior angles increases by 180° when the number of sides increases by 1.
So, the sum of interior angles is $180°(n - 2)$. You can find the sum of the interior angles of any polygon by substituting the appropriate value for n into the equation.

Communicate Your Understanding Responses (page 390)

C1. Answers will vary. Sample solution: The number of sides is 7, so the sum of the interior angles is $180°(7 - 2)$ or 900°.

C2. The pentagon is not regular; its interior angles are not equal.

Practise

For question 5, ensure that students discuss interior and exterior angles, number of diagonals, etc.

Connect and Apply

Ask students in advance to each bring in a one-dollar coin for question 10. Alternatively, you may wish to make enlarged photocopies of a dollar coin or a tracing of a dollar coin. For homework, ask them, *What about a two-dollar coin?*

For question 13, you may wish to use **BLM 7.3.2 Constructing the Gazebo** to support this activity. It provides detailed instructions on constructing these diagrams using either *The Geometer's Sketchpad*® or *Cabri*® *Jr.* Point out to students that some gazebos have 8 struts, while others have 12. You may wish to have them to research gazebo designs on the Internet.

For question 15, the Achievement Check, you may wish to use **BLM 7.3.3 Achievement Check Rubric** to assist you in assessing your students.

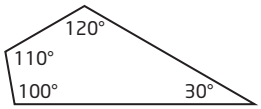
Achievement Check Answers (page 393)

15. a) Draw any obtuse angled triangle.

b) The triangle is impossible as the third angle of the triangle would be 0°.

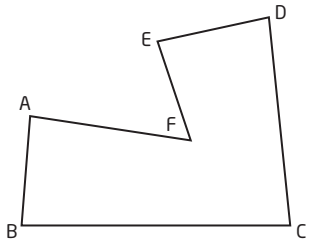
c) Draw any rectangle.

d) Answers will vary. One example is a quadrilateral with interior angles of 100°, 110°, 120°, and 30°.



e) Answers will vary. The sum of the five angles of a pentagon is $5(180°) - 360°$, or 540°. The greatest size of an acute angle is slightly less than 90°, so the three acute angles total less than 270°. Therefore, the two obtuse angles must total at least $540° - 270°$, or 270°, but less than $2 \times 180°$, or 360°. This condition is easily met. For example, the obtuse angles could measure 150° each with the acute angles measuring 80° each.

f) The hexagon is not possible. The sum of the six interior angles of a convex hexagon is $180°(6 - 2)$, or 720°. The greatest measure for an acute angle is slightly less than 90°, say 89°. If each of the acute angles measures 89°, the sum of the five acute angles is $5 \times 89°$, or 445°. Then, the sixth angle of the hexagon must $720° - 445°$, or 275°, which is a reflex angle. Therefore, any hexagon with five acute interior angles must be concave.



Extend

For question 19, assure students that they will not have to look far to find polygons, as many buildings have polygons as a prominent part of the design—including a number of newer schools. A good Web resource on buildings can be found at http://www.greatbuildings.com/buildings.html. Alternatively, ask students to look around the classroom for polygons. They may find them in flags, desks, cabinets, speakers, computers, books, pictures, boxes, windows, doors, etc. You may wish to use **BLM A10 Observation General Scoring Rubric** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 6–8, 10
Typical	1–14
Extension	16–21

7.4 Midpoints and Medians in Triangles

Strand:
Measurement and Geometry

Student Text Pages
394 to 400

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- MIRA

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators
- Internet access

Related Resources
BLM 7.4.1 Practice: Midpoints and Medians in Triangles
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.4.2 Using a MIRA to Investigate Lines of Symmetry
BLM A9 Communication General Scoring Rubric
BLM 7.4.3 Sierpinski's Triangle

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.02 determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Warm-Up

Review terminology with a few hand-drawn constructions involving midpoints and medians. Or, use examples from **BLM 7.4.1 Practice: Midpoints and Medians in Triangles** as a Warm-Up activity.

Teaching Suggestions

- As in Sections 7.1 to 7.3, begin by creating groups and deciding which method, or combinations of methods, students will use. If all methods are available to you, you may wish to consult with students; their preference may surprise you. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor**, and/or **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Have the groups work through the Investigate. (10–15 min)
- Prepared sketches for *The Geometer's Sketchpad* and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site.
  Go to http://www.mcgrawhill.ca/books/principles9.
- Follow with a summary of students' findings and a discussion of the Reflect questions. As in the earlier sections in this chapter, if students have used different methods, provide an opportunity for them to compare results. (5–10 min)
- Work through the Examples with the class. As an extension of Example 2, ask students to use a MIRA or paper folding to establish that the two halves of an isosceles triangle are congruent. Refer to **BLM 7.4.2 Using a MIRA to Investigate Lines of Symmetry** for more exercises.
- You may wish to use **BLM 7.4.1 Practice: Midpoints and Medians in Triangles** for remediation or extra practice.

Common Errors

- Some students may draw all triangles as either isosceles or equilateral.

R_x Remind students to include some triangles that are scalene.

Ongoing Assessment

- Chapter Problem question 10 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Student Success

Have students conduct an **Internet search** for information on relationships among triangle constructions, such as altitudes, right bisectors, angle bisectors, etc.

Investigate Answers (pages 394—395)

Method 1

2. BC is twice as long as DE.

3. DF is parallel to BC.

4. Vertex A will touch side BC unless ∠B or ∠C is obtuse; vertex A will touch an extension of BC if ∠B or ∠C is obtuse.

5. The height of △ABC is twice the height of △ADE. The height of △ADE is equal to the height of quadrilateral BCED.

7. Answers will vary. Sample solution: Since the class found that the relationships apply to a variety of triangles, these relationships likely apply to all triangles.

Method 2

3. BC is twice as long as DE.

6. The height of △ABC is twice the height of △ADE.

7. The height of △ADE is equal to the height of quadrilateral BCED.

8. The length ratios remain constant as does the sum of ∠EDB and ∠DBC.

9. The line segment is parallel to the third side and half its length. The segment also bisects the height of the triangle.

Communicate Your Understanding Responses (page 398)

C1. The height of △ADF is equal to the height of quadrilateral DBCF, so the three triangles in DBCF all have heights equal to that of △ADF. Since DF is half the length of BC, all four triangles have equal bases. Since the four triangles have equal heights and equal bases, the areas are also equal. Therefore, the area of △ADF is one quarter of the area of △ABC.

C2. Any scalene obtuse or scalene right triangle is a counter-example.

Practise

The Practise questions are straightforward and follow the pattern of the Examples.

Connect and Apply

Assign question 5 and review students' responses as a way of assessing whether students are ready to move on to the rest of the Connect and Apply questions. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing students' responses.

Suggest students use MIRAs, or paper folding, in question 9 to see triangles reflected in the bisectors.

In the Chapter Problem, question 10, have students consider the various properties that are present in the diagram. Encourage students to construct the diagram using either *The Geometer's Sketchpad*® or *Cabri*® *Jr.* to see if the three medians always intersect at a single point. Have students attempt to balance their cardboard triangle (from the Chapter Opener) along one of the median lines. Ask them why this might work.

Extend

Assign as many of the Extend exercises as students' abilities and time permit; they focus on the special points where angle bisectors, medians, altitudes, and perpendicular bisectors intersect. They provide a rich resource for student investigations.

You may wish to use **BLM 7.4.3 Sierpinski's Triangle** to guide students through constructing this interesting geometric phenomenon using *The Geometer's Sketchpad*®.

Literacy Connections

Counter-Examples

Explain to students that using a counter-example is useful both in mathematics and in English. If you can think of an example to refute a statement or "rule," then that is a good way to disprove it, or to limit how generally it can be applied. As an example, use the spelling "rule" of "*i* before *e*." Challenge students to recall the counter-examples that limit the generality "except after *c* or when sounding like *a* as in *neighbour* and *weigh."* Then, ask if they can find a counter-example to the counter-example (e.g., *seize*).

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 6, 7
Typical	1–9
Extension	11–16

7.5 Midpoints and Diagonals in Quadrilaterals

Strand:
Measurement and Geometry

Student Text Pages
401 to 407

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.5.1 Constructing Parallelograms
BLM 7.5.2 Practice: Midpoints and Diagonals in Quadrilaterals
BLM 7.5.3 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.02 determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Teaching Suggestions

- The skills developed now will help students who go on to take the grade 12 geometry and discrete mathematics course.
- As in the rest of the chapter, begin by creating groups and deciding what method, or combination of methods, students will use. If all methods are available, consult with students on their preference.
- Outline what students will be doing in the Investigate, and ask them if they have any idea of what might happen.
- Have the groups work through the Investigate. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor**, and/or **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity. (10–15 min)
- Prepared sketches for *The Geometer's Sketchpad*® and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site. Go to http://www.mcgrawhill.ca/books/principles9.
- Follow up with a summary of students' findings and a discussion of the Reflect question. As in the earlier sections in this chapter, if students have used different methods, provide an opportunity for them to compare results. If a student asks "Does this always happen?" point out that asking and investigating such questions is the attitude of a mathematician! (5–10 min)
- Assign the Examples. Stress that making conjectures, then testing it to see if it is true, or finding a counter-example are fundamental to establishing properties of geometric figures. (10–15 min for each)
- Example 1 and several of the exercises deal with parallelograms. If your students are using technology, you may wish to use **BLM 7.5.1 Constructing Parallelograms**, which gives detailed instructions for *The Geometer's Sketchpad*® and *Cabri*® *Jr.*
- Discuss the Communicate Your Understanding questions. Following question 2, you could have students create parallelograms from paper and confirm their responses. Alternatively, you could have them use a MIRA.
- You may wish to use **BLM 7.5.2 Practice: Midpoints and Diagonals in Quadrilaterals** for remediation or extra practice.

Common Errors

- Some students may encounter a problem with interpreting the questions, either due to reading difficulties or a lack of understanding.

R_x Prepare some comments about each question to assist students in their understanding.

- As in previous sections, some students may tend to draw regular figures.

R_x Be sure to stress in all examples that a figure is only regular if that information is provided.

Ongoing Assessment

- Use Achievement Check question 12 to monitor student success. See Achievement Check Answers and **BLM 7.5.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to research Euclidean proofs at the library or on the Internet.

Investigate Answers (pages 401—402)

Method 1

2. EFGH is a parallelogram. Students may produce a rhombus, rectangle, or square if they start with some type of parallelogram.

3. HE = GF and EF = HG

5. Opposite sides are parallel.

6. Answers will vary. Sample solution: Opposite sides are parallel and equal so, the quadrilateral is a parallelogram.

8. Answers will vary. Sample solution: Since the class produced parallelograms with many different quadrilaterals, it is likely that joining the midpoints of the sides of any quadrilateral produces a parallelogram.

Method 2

3. EFGH is a parallelogram. Students may produce a rhombus, rectangle, or square if they start with some type of parallelogram.

4. Opposite sides are equal in length.

5. No. The relationships do not change if you change the location of the vertices.

7. Opposite sides are parallel. Moving a vertex of the original quadrilateral does not change the angle sums.

8. Answers will vary. Sample solution: Opposite sides are parallel and equal so the quadrilateral is a parallelogram. Since EFGH remains a parallelogram regardless of the location of the vertices of ABCD, joining the midpoints of the sides of any quadrilateral always produces a parallelogram.

Communicate Your Understanding Responses (page 404)

C1. Answers will vary. Sample solution: If the co-interior angles between two sides are supplementary, the sides are parallel.

C2. Answers will vary. Sample solution: Fold the parallelogram so that one pair of opposite vertices touch, and then, fold it so that the other pair of opposite vertices touch. The intersection of the two fold lines is at the intersection of the diagonals. Therefore, the intersection of the diagonals is an equal distance from opposite vertices.

Practise

Questions 1 to 3 are straightforward.

Connect and Apply

Question 4 is not difficult, but students may not know what a jack is or how it is used. Be prepared to explain this tool to them.

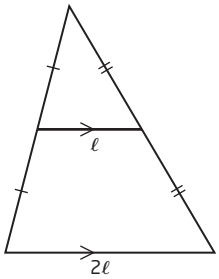
If possible, suggest students approach question 6 using *The Geometer's Sketchpad*®, so that they can easily test their conjecture. You may wish to use **T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

For question 9, students may not know what a kite-shaped quadrilateral is. Be aware that investigating any properties of a kite may be new to some students.

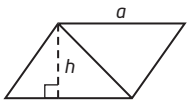
For question 12, the Achievement Check, you may wish to use **BLM 7.5.3 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 407)

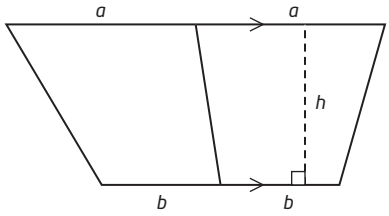
12. a) False. Any triangle can be used as a counter-example. The line segment joining the two midpoints divides the triangle into a smaller triangle and a trapezoid with equal heights. Of these two sections, the trapezoid clearly has the larger area. In the Communicate Your Understanding question C1 in Section 7.4, students show that the area of the smaller triangle is one quarter that of the larger triangle. Therefore, a line segment joining the midpoints of two sides of a triangle does not bisect the area of the triangle.



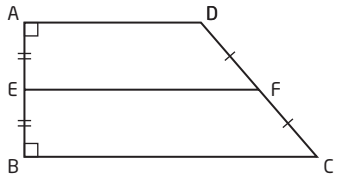
b) True. The two triangles formed by the diagonal have equal bases and equal altitudes. Therefore, the areas of the two triangles are equal. Students may use *The Geometer's Sketchpad*® to show that the areas are equal regardless of the shape of the parallelogram.



c) True. The line segment divides the original trapezoid into two smaller trapezoids. Both of the smaller trapezoids have the same height as the original trapezoid, and both have parallel sides half the length of those in the original trapezoid. Since the area of a trapezoid is $\frac{(a + b)\ h}{2}$, the two smaller trapezoids have equal areas. Students may use *The Geometer's Sketchpad*® to show that the areas are equal regardless of the shape of the trapezoid.



d) False. Many counter-examples are possible. Here is one of the simplest:
In this quadrilateral AD is much shorter than BC, so the area of AEFD is clearly less than the area of EBCF.
Students may use *The Geometer's Sketchpad*® to show that a line segment joining the midpoints of opposite sides of a quadrilateral often forms two smaller quadrilaterals with unequal areas.



Extend

These questions may pose difficulties for some students. Be prepared to discuss them in detail if used. To show that a quadrilateral is a parallelogram, students should establish that adjacent angles are supplementary.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 5, 7, 8
Typical	1–10
Extension	13–17

Chapter 7 Review

Student Text Pages
408 to 409

Suggested Timing
80 min

Related Resources
BLM 7.CR.1 Chapter 7 Review
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4

Tools
- grid paper
- protractors

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators

Ongoing Assessment

- Upon completing the Chapter Review, students can also answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related example in the text to help you with the questions? For which questions?*

Using the Chapter Review

Each question in **BLM 7.CR.1 Chapter 7 Review** reviews different skills and concepts. Have students work independently to complete the Chapter Review, then with a partner to compare solutions. Alternatively, the Chapter Review could be assigned for reinforcing skills and concepts in preparation for the Practice Test. Provide an opportunity for students to discuss any questions about strategies or any features they find difficult. Use **BLM 7.CR.1 Chapter 7 Review** for the Chapter Review.

Another approach would be to form groups of five students and assign each section to one student in the group. Then, have students teach each other. Access to *The Geometer's Sketchpad*® or TI-83 Plus or TI-84 graphing calculators with *Cabri*® *Jr.* would be helpful. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor** and/or **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support these activities.

As students complete the questions, encourage them to keep notes of important results. Their ability to define ideas becomes more critical as the mathematics challenges them more.

After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content.

Chapter 7 Practice Test

Student Text Pages
410 to 411

Suggested Timing
60 min

Tools
- grid paper
- protractors

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- *Cabri® Jr.*
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 7.PT.1 Chapter 7 Practice Test
BLM 7.CT.1 Chapter 7 Test

Summative Assessment

- After students complete **BLM 7.PT.1 Chapter 7 Practice Test**, you may wish to use **BLM 7.CT.1 Chapter 7 Test** as a summative assessment.

Accommodations

Visual—Provide opportunities for students to have the questions read to them.

Perceptual—If possible, provide students with opportunities to work with an educational assistant or to work with a partner so students understand exactly what is being asked in the questions.

Motor—Allow students extra time to complete the questions in this section.

Language—Encourage students who are having difficulty with language-processing to work with a scribe and/or a partner who will read the questions to them.

ESL—Encourage students to use a translator to ensure that they understand the meaning and the context of the words being used.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	7.1	Example 1 (page 369)
2	7.1	Example 2 (page 370)
3	7.3	Example 1 and 2 (page 389)
4	7.3	Example 2 (page 389)
5	7.4	Example 2 (page 396)
6	7.3	Example 1 (page 389)
4	7.5	Example 1 (page 403)
5a)	7.1	Example 1 (page 369)
5b)	7.2	Example 1 (page 379)
5c)	7.1	Example 2 (page 370)
5d)	7.2	Example 2 (page 379)
6	7.5	Example 1 (page 403)
7	7.5	Example 2 (page 404)

Using the Practice Test

This Practice Test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?

- Identify different types of triangles
- Find the degree measure of missing interior and exterior angles of triangles
- Identify different types of quadrilaterals
- Find the degree measure of missing interior and exterior angles of quadrilaterals
- Use *The Geometer's Sketchpad®* and/or *Cabri® Jr.* to construct sketches
- Find the sum of the interior angles of a polygon
- Find the sum of the exterior angles of a polygon
- Create a geometric diagram from given information
- Find the degree measure of each angle of a regular polygon and construct it
- Draw a regular polygon by constructing its "spokes" first
- Find the midpoints of line segments
- Construct the medians of a triangle
- Test a property of a geometric figure using dynamic geometry
- Provide a counter-example for a conjecture that is incorrect

Chapter 7 Problem Wrap-Up

Student Text Page
411

Suggested Timing
80 min

Tools
- cardboard triangles created in Chapter Opener
- more heavy cardboard
- grid paper
- rulers
- protractors
- utility or craft knives
- metre sticks

Technology Tools
- computers
- Internet access

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM A8 Application General Scoring Rubric
BLM A10 Observation General Scoring Rubric
BLM 7.CP.1 Chapter 7 Problem Wrap-Up Rubric
BLM A18 My Progress as a Problem Solver

Summative Assessment

- Use **BLM 7.CP.1 Chapter 7 Problem Wrap-Up Rubric** to assess student achievement.

Using the Chapter Problem

- The Chapter Problem is introduced at the beginning of the chapter and revisited in Section 7.4. It takes the concept of a median from an abstract geometric idea to a context that students can understand and engage in.
- Introduce the concept of a centre of mass. Hopefully, students will get the idea that in order for something to balance there must be an equal distribution of mass on either side of the ruler. That concept can then be extended to the intersection of the medians as the balance point for the triangle.
- Have students work individually or with a partner. Alternatively, you may wish to assign the Chapter Problem as a homework activity.
- The difficulty of this Chapter Problem depends on how much discussion about the concepts has preceded its assignment, and how much experience students have. Some students may find it quite easy. You may wish to extend the problem for these students by suggesting that they also explore the same properties with other geometric shapes (e.g., square, parallelogram, rhombus, kite, arbitrary quadrilateral, pentagon, etc.). Some stronger students may complete this extension without prompting.
- You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor**.
- Alternatively, you could use this activity as part of a year-end performance assessment package to be done by students in class. You can observe and record student confidence and work habits as they tackle this type of activity. You may wish to use **BLM A8 Application General Scoring Rubric** and/or **BLM A10 Observation General Scoring Rubric**, in addition to **BLM 7.CP.1 Chapter 7 Problem Wrap-Up Rubric**. Or, have students self-assess using **BLM A18 My Progress as a Problem Solver**.
- As an extension, suggest that students use the Internet to investigate Winnipeg's claim that it is the centre of mass for North America. How would city officials there have arrived at this conclusion? Do any other cities make this or similar claims?

Level 3 Sample Response

1. The student will hand in two or more cardboard triangles of varying shapes. The three medians should be drawn on each cardboard triangle.

a) Yes, the cutout balanced if I carefully placed it so that one of the medians lay along the edge of a metre stick.

b) The centre of mass must lie along any median since my experiment in a) shows that there is equal mass on either side of the median. The only point that is on all three medians is their point of intersection.

c) This point where the three medians meet seems to be the balance point, although it was difficult to put my finger at this exact point.

Level 3 Notes

Look for the following:
- One or two cardboard triangle created with proficient use of geometric tools (e.g., rulers, cutting tools, etc.)
- Summary report that states conclusions simply for each part of the experiment
- An understanding of geometric properties involved in the question

What Distinguishes Level 2

At this level, look for the following:

- One cardboard triangle submitted, may be of poor quality workmanship
- Summary report may be missing or contain results for only one part of the experiment
- Some understanding of the geometric properties involved

What Distinguishes Level 4

At this level, look for the following:

- More than two well-constructed, labelled cardboard triangles submitted
- Summary report is complete and detailed in its explanations
- Report may include discussion about problems with the activities and possible future remedies
- A high degree of proficiency with geometry software
- An understanding of geometric properties involved in the question and connections between the ideas from the two activities