CHAPTER

4

Equations

Vocabulary

equation
solution
root
constant term
formula

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Strand:
Number Sense and Algebra

Overall Expectations

By the end of this course, students will:

NAV.01 demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions.

NAV.02 manipulate numerical and polynomial expressions, and solve first-degree equations.

Specific Expectations

Manipulating Expressions and Solving Equations

By the end of this chapter, students will:

NA2.03 relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify and solve equations.

NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies);

NA2.08 rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement);

NA2.09 solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods.

Chapter Problem

The Chapter Problem is introduced in the Chapter Opener. The context is based on some popular reality television shows, which should appeal to students of this age. Have students discuss their understanding of the topic. Some students who are more familiar with these shows can describe them to other students who may not have seen them.

You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 233.

Alternatively, you may wish to assign the entire Chapter Problem when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment.

Chapter 4 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 4 Opener • 15 min	182–183			
Get Ready • 40–80 min	184–185	• BLM 4.GR.1 Practice: Get Ready	• BLM 4.GR.2 Get Ready Self-Assessment Checklist	
4.1 Solve Simple Equations • 80–160 min	186–195	• BLM 4.1.1 Practice: Solve Simple Equations • BLM 4.1.2 Use Algebra Tiles • BLM T7	• The Computer Algebra System (CAS) on the TI-89 Calculator • BLM A9 Communication General Scoring Rubric • BLM 4.1.3 Achievement Check Rubric	**Tools** • algebra tiles **Technology Tools** • *The Geometer's Sketchpad®* • computers • Computer Algebra System • TI-89 calculators • calculators
4.2 Solve Multi-Step Equations • 80–160 min	196–203	• BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator • BLM 4.2.1 Alternate Solution Method • BLM 4.2.2 Construct a Geometric Model With *The Geometer's Sketchpad®* • BLM T4 *The Geometer's Sketchpad®* 3 • BLM T5 *The Geometer's Sketchpad®* 4 • BLM 4.2.3 Practice: Solve Multi-Step Equations • BLM G10 Grid Paper	• BLM A7 Thinking General Scoring Rubric	**Tools** • grid paper **Technology Tools** • Computer Algebra System • TI-89 calculators • calculators • *The Geometer's Sketchpad®* • computers
4.3 Solve Equations Involving Fractions • 80 min	204–210	• BLM 4.3.1 Practice: Solve Equations Involving Fractions • BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator • BLM 4.3.2 Cross-Multiplication Solution Method • BLM T4 *The Geometer's Sketchpad®* 3 • BLM T5 *The Geometer's Sketchpad®* 4 • BLM 4.3.4 Student Success: Decision Tree	• BLM A8 Application General Scoring Rubric • BLM 4.3.3 Achievement Check Rubric • BLM A17 Teamwork Self Assessment	**Technology Tools** • Computer Algebra System • TI-89 calculators • calculators • *The Geometer's Sketchpad®* • computers • graphing calculators
4.4 Modelling With Formulas • 80 min	211–219	• BLM 4.4.1 Practice: Modelling With Formulas • BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator • BLM T4 *The Geometer's Sketchpad®* 3 • BLM T5 *The Geometer's Sketchpad®* 4 • BLM 4.4.2 Use a Computer Algebra System Directly • BLM 4.4.3 Student Success: Terrific Triangles • BLM G10 Grid Paper	• BLM A18 My Progress as a Problem Solver	**Tools** • grid paper • rulers or metre sticks **Technology Tools** • Computer Algebra System • TI-89 calculators • calculators • *The Geometer's Sketchpad®* • computers • graphing calculators
4.5 Modelling With Algebra • 80–160 min	220–229	• BLM 4.5.1 Investigate: Magic With Algebra • BLM T4 *The Geometer's Sketchpad®* 3 • BLM T5 *The Geometer's Sketchpad®* 4 • BLM 4.5.2 Practice: Modelling With Algebra • BLM G10 Grid Paper	• BLM 4.5.3 Achievement Check Rubric • BLM A22 Earning Money Report Checklist	**Tools** • grid paper **Technology Tools** • *The Geometer's Sketchpad®* • computers • Internet access

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 4 Review • 80 min	230–231	• BLM 4.CR.1 Chapter 4 Review	• BLM A14 Self-Assessment Recording Sheet • BLM A15 Self-Assessment Checklist	**Technology Tools** • Computer Algebra System • TI-89 calculators • calculators
Chapter 4 Practice Test • 60–80 min	232–233	•BLM T4 *The Geometer's Sketchpad*® 3 •BLM T5 *The Geometer's Sketchpad*® 4 •BLM G10 Grid Paper	•BLM 4.PT.1 Chapter 4 Practice Test •BLM 4.CT.1 Chapter 4 Test •BLM A8 Application General Scoring Rubric •BLM 4.P.1 Performance Task	**Tools** • grid paper **Technology Tools** • Computer Algebra System • TI-89 calculators • *The Geometer's Sketchpad*® • computers
Chapter 4 Problem Wrap-Up • 30 min	233		• BLM 4.CP.1 Chapter 4 Problem Wrap-Up Rubric	**Technology Tools** • Computer Algebra System • TI-89 calculators • calculators

Chapter 4 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 4.GR.1	Practice: Get Ready	Practice
	BLM 4.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
4.1: Solve Simple Equations			
	BLM 4.1.1	Practice: Solve Simple Equations	Practice
	BLM 4.1.2	Use Algebra Tiles	Student Support
	BLM T7	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM A9	Communication General Scoring Rubric	Assessment
	BLM 4.1.3	Achievement Check Rubric	Assessment
4.2: Solve Multi-Step Equations			
	BLM T7	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 4.2.1	Alternate Solution Method	Student Support
	BLM 4.2.2	Construct a Geometric Model with *The Geometer's Sketchpad*®	Student Support Technology
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 4.2.3	Practice: Solve Multi-Step Equations	Practice
	BLM A7	Thinking General Scoring Rubric	Assessment
	BLM G10	Grid Paper	Teacher Support
4.3: Solve Equations Involving Fractions			
	BLM 4.3.1	Practice: Solve Equations Involving Fractions	Practice
	BLM T7	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM 4.3.2	Cross-Multiplication Solution Method	Student Support
	BLM A8	Application General Scoring Rubric	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM A8	Application General Scoring Rubric	Assessment
	BLM 4.3.3	Achievement Check Rubric	Assessment
	BLM A17	Teamwork Self Assessment	Student Self-Assessment
	BLM 4.3.4	Student Success: Decision Tree	Student Success

	BLM	Title	Purpose
4.4: Modelling With Formulas			
	BLM 4.4.1	Practice: Modelling With Formulas	Practice
	BLM T7	The Computer Algebra System (CAS) on the TI-89 Calculator	Technology
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 4.4.2	Use a Computer Algebra System Directly	Student Support Technology
	BLM 4.4.3	Student Success: Terrific Triangles	Student Success
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM G10	Grid Paper	Student Support
4.5: Modelling With Algebra			
	BLM 4.5.1	Investigate: Magic With Algebra	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 4.5.2	Practice: Modelling With Algebra	Practice
	BLM 4.5.3	Achievement Check Rubric	Assessment
	BLM G10	Grid Paper	Student Support
	BLM A22	Earning Money Report Checklist	Assessment
Chapter 4 Review			
	BLM 4.CR.1	Chapter 4 Review	Practice
	BLM A14	Self-Assessment Recording Sheet	Student Self-Assessment
	BLM A15	Self-Assessment Checklist	Student Self-Assessment
Chapter 4 Practice Test			
	BLM 4.PT.1	Chapter 4 Practice Test	Diagnostic Assessment
	BLM 4.CT.1	Chapter 4 Test	Summative Assessment
	BLM 4.P.1	Performance Task	Performance Task
	BLM A8	Application General Scoring Rubric	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM G10	Grid Paper	Student Support
Chapter 4 Problem Wrap-Up			
	BLM 4.CP.1	Chapter 4 Problem Wrap-Up Rubric	Summative Assessment

Get Ready

Student Text Pages
184 to 185

Suggested Timing
40–80 min

Related Resources
BLM 4.GR.1 Practice: Get Ready

BLM 4.GR.2 Get Ready Self-Assessment Checklist

Common Errors

- Some students may collect like terms incorrectly (e.g., change the variable parts, or collect unlike terms).

R_x Use algebra tiles to help consolidate students' understanding. Provide additional remediation, as needed.

- Some students may mix up integer signs when applying the distributive property.

R_x Review integer operations and the distributive property with students, as needed. Review the use of brackets as indicators of how far to distribute the monomial factor.

- Some students may simply multiply numbers together to find a common denominator, which is not generally the least common denominator.

R_x Review the methods as shown, using multiples and factors. Assign extra practice, as needed. You may wish to use **BLM 4.GR.1 Practice: Get Ready** as remediation.

- Some students may add or subtract fractions incorrectly.

R_x Review the methods for performing these operations. Use manipulatives (e.g., pattern blocks, fraction rings, and circles) to develop understanding. Assign extra practice, as needed. You may wish to use **BLM 4.GR.1 Practice: Get Ready** as remediation.

Accommodations

Memory—Encourage students to write out cue cards to remember the rules for multiplying positive and negative numbers. Let the students use visual clues when adding like terms.

Teaching Suggestions

- The selection of topics is eclectic, as dictated by the needs of the chapter contents. One approach is to use each relevant part just prior to the section being taught.
- Alternatively, assigning the entire Get Ready at the beginning of the chapter could provide an opportunity to generate student interest, e.g., How will all of these different topics be combined and used in the upcoming chapter? Whichever approach is taken, it is important for students to develop some appreciation of how the various branches of mathematics are connected to each other.
- All BLMs referred to throughout this chapter can be found on the *Principles of Mathematics 9* Teacher's Resource CD-ROM.
- You may wish to use **BLM 4.GR.1 Practice: Get Ready** for remediation or extra practice.

Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; students can place a checkmark beside topics in the Get Ready in which they feel confident with the necessary skills. You may wish to use **BLM 4.GR.2 Get Ready Self-Assessment Checklist** as a self-assessment for students. Remedial action can be taken in small groups or with a whole class skill review.

4.1 Solve Simple Equations

Strand:
Number Sense and Algebra

Student Text Pages
186 to 195

Suggested Timing
80–160 min

Tools
- algebra tiles

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- Computer Algebra System
- calculators
- TI-89 calculators

Related Resources
BLM 4.1.1 Practice: Solve Simple Equations
BLM 4.1.2 Use Algebra Tiles
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM A9 Communication General Scoring Rubric
BLM 4.1.3 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies).

Link to Get Ready

Students will need to recall geometric relationships and how to collect like terms for this section. Assign Get Ready questions 1, 2, 5, and 6 prior to teaching this section.

Warm-Up

Use this activity as a Warm-Up or as an alternate Investigate. You may wish to use *The Geometer's Sketchpad®* to support the manipulatives. By constructing a rectangle or using a pre-drawn image, students could manipulate the image to answer these questions in an organized way.

1. The Henderson's have a rectangular garden with a perimeter of 24 m. They want to change the dimensions (length and width) so that it looks better.

a) The length of the garden now is 8 m. What is the width? Explain in words the math operations that you used to find the width.

b) If the Henderson's told you that the width of the garden was 2 m, what would be the length? Explain in words the math operations you used to find the length. List the operations that match the language you described. Are the operations the same as those described in part a)?

Warm-Up Answers

1. a) $P = 2\ell + 2w$
$24 = 2(8) + 2w$
$24 = 16 + 2w$
$8 = 2w$
$4 = w$
The width of the garden is 4 m.

b) $P = 2\ell + 2w$
$24 = 2\ell + 2(2)$
$24 = 2\ell + 4$
$20 = 2\ell$
$10 = \ell$
The length of the garden is 10 m.

Teaching Suggestions

- A major focus of this section is to introduce the algebraic method. Some students may not appreciate the need for this because the equations they have encountered so far have not been very complicated. In grade 9 and beyond, however, the equations will become more complicated, and the ability to efficiently apply the algebraic method is vital. Emphasize the need for proper form (showing all steps to solutions).
- Have students work on the Investigate with a partner or in small groups. (5 min)
- Students will have solved simple equations in elementary school, however, they may not be familiar with formal algebraic methodologies. Use the Investigate to review the concept of variables as it relates to

Common Errors

- Some students may resist, or improperly apply, the formal algebraic method.

R_x Emphasize and model good form at all times. Express the need for clear presentation of solutions so that others can comprehend the student's work.

- Some students may apply the wrong operation when solving one-step equations.

R_x Remind students that the idea is to solve for the variable. Typically, the variable is "trapped" within some operation involving a numeric quantity. Have students think of "releasing the trap" by performing the opposite operation. Students will need to recall that addition and subtraction are opposite operations, as are multiplication and division.

- Some students may apply operations in the incorrect order when solving two-step equations.

R_x Remind students that they must isolate the variable term first, before solving for it. This typically requires that operations be done in the opposite order as BEDMAS suggests for simplifying expressions.

Ongoing Assessment

- Use Achievement Check question 17 to monitor student success. See Achievement Check Answers and **BLM 4.1.3 Achievement Check Rubric**.
- Chapter Problem question 14 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

solving an equation. Students should be able to come up with the answers, but not necessarily using a formal algebraic solution. You may wish to use **BLM 4.1.1 Practice: Solve Simple Equations** for remediation or extra practice.

- For Examples 1 and 2, conduct a discussion. (10–15 min)
- Examples 1 and 2 illustrate three methods for solving simple algebraic equations. Students will be familiar with inspection, and it appeals intuitively. This is a valid method for simple equations. The balance analogy segues to the formal algebraic approach (using the opposite operation). Note that the method of opposite operations is most likely new to students, and is the method of choice for further mathematics work.
- An excellent web site that supports the balance analogy and provides an interactive model of the balance method for solving equations can be found at: http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html.
- For Example 2 b), you may wish students to use algebra tiles as an alternate method of solving equations. Use **BLM 4.1.2 Use Algebra Tiles** to support this activity.
- Assign Communicate Your Understanding questions C1 a), b), and c), and follow up with a class discussion. (2–3 min)
- Assign Practise questions 1 to 5. (5–10 min)
- Debrief Practise answers with a class discussion. (2–3 min)
- Examples 3 and 4 use contextual situations to provide an opportunity to apply methods of solving equations, and extend into solving two-step equations. "Root" terminology is also introduced here, as is the use of Computer Algebra System (CAS) as an equation-solving tool. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- It is important that students not use CAS as a replacement for learning the algebraic method. The solution presented in the text requires students to select and apply each algebraic step, but lets the technology take care of the calculations. This approach may be of benefit for the student whose rudimentary operational skills with integers and fractions are still weak. It may also be helpful for students to use this as a means of checking their paper-and-pencil work.
- For Examples 3 and 4, have a class discussion. (15–20 min)
- Assign Communicate Your Understanding questions C1 d), 2, and 3. Follow up with a class discussion. (5 min)
- Assign the balance of the exercises as independent work. (balance of period)
- Depending on the progress of the class, you may wish to extend this lesson to 1.5 to 2 periods.

Investigate Answers (page 186)

1. a) Let x represent the cost of the other magazine.
b) $5 + x = 11$
c) $x = 6$; To find the answer, subtract 5 from both sides of the equation and simplify.

2. a) Let x represent the cost of a mechanical pencil.
b) $4 + 2x = 10$
c) $x = 3$; Step 1: Subtract 4 from both sides. Step 2: Divide both sides by 2.

3. a) With the help of mathematical operations, you can isolate the unknown quantity. The value of the unknown quantity can then be found.
b) Answers can be verified by substituting the answer you found into the original equation. Then, independently, evaluate the left side and right side. If both sides have the same value then the answer is correct.

Communicate Your Understanding Responses (page 192)

C1. a) Add 5 to both sides.
b) Divide both sides by 3.
c) Multiply both sides by 5.
d) Subtract 75 from both sides, then divide both sides by 5.

C2. B. Substitute each possible solution into the L.S. of the equation. The result should be 21.

C3. B. Each jacket costs \$50 and n represents the number of jackets. Hence, the money raised, \$1000, should be equal to $50n$.

Accommodations

Gifted and Enrichment—In addition to the Extend questions, the Math Contest question provides additional challenges for talented students. Every section has some of these questions. Questions have been selected for their interest level as well as their level of difficulty.

Visual—The use of colour, bolding, or highlighting may help some students to more easily identify like terms.

Memory—Encourage students to write out cue cards to remember the rules for multiplying positive and negative numbers. Encourage students to use visual cues when adding like terms.

ESL—Be sure to stress that there are many terms that mean the same thing in mathematics. The root and the solution mean the same thing. So to solve or to find the root or to isolate the variable all have the same meaning and process.

Practise

The early questions are a good opportunity to get students using proper form and showing their steps.

For questions 9 and 10, some students may have difficulty in setting up the equations, based on given information. Consider having students work in pairs on some of the word problems.

Connect and Apply

For question 11, you may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

For question 12, some students may need a quick review on reducing fractions to lowest terms.

Question 13 requires students to add and subtract fractions, which is a common source of difficulty. Some remediation may be warranted. Later in the chapter, students will learn another technique for solving such equations that simplifies the fraction work.

Questions 14 to 16 require students to build and solve equations based on given information, and then make a decision. Ensure that students have answered all questions completely by looking back at the question.

In Achievement Check question 17, geometry and algebra are connected. Students will need to recall the geometric properties of an isosceles triangle. You may wish to use **BLM 4.1.3 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 195)

17. a) $a + 2(25°) = 180°$ $\qquad$ $a + 2(100°) = 180°$

$a = 130°$ $\qquad$ $a = -20°$

Note that the second situation is impossible.

b) $40° + 2b = 180°$ $\qquad$ $100° + 2b = 180°$

$2b = 140°$ $\qquad$ $2b = 80°$

$b = 70°$ $\qquad$ $b = 40°$

c) The maximum value of a is 178°. For a to be large, b must be as small as possible, say 1°. ($1° + 1° + 178° = 180°$). This assumes you are measuring to the nearest degree; otherwise, a is a value just less than 180°.

d) The maximum value of b is 89°. For b to be large, a must be as small as possible, say 2°. ($1° + 89° + 89° = 180°$). This assumes you are measuring to the nearest degree; otherwise, b is a value just less than 90°.

Extend

Question 19 is connected to linear systems in section 6.7 Linear Systems. Explain to students that the best choice of hall can change, depending on the particular circumstances.

Literacy Connections

Solutions

Remind students that when they are writing out a solution, they must be sure to make each line show what they are doing. Have them imagine that someone who has not studied this chapter is reading their solution. Explain that it needs to be very clear what they are doing so that someone can follow along and understand their solution. Remind students that it is important to only have one equal sign in any line.

Isolating the variable

Explain to students that *isolating the variable* in math has the same meaning as *isolating* in our English class. It means separating the variable from everything else so that it is alone.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6, 8–10, 12, 14
Typical	1–6, 8–12, 13, 15, 16
Extension	18–20

4.2 Solve Multi-Step Equations

Strand:
Number Sense and Algebra

Student Text Pages
196 to 203

Suggested Timing
80–160 min

Tools
- grid paper

Technology Tools
- Computer Algebra System
- computers
- *The Geometer's Sketchpad®*
- TI-89 calculators
- calculators

Related Resources
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM 4.2.1 Alternate Solution Method
BLM 4.2.2 Construct a Geometric Model With *The Geometer's Sketchpad®*
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.2.3 Practice: Solve Multi-Step Equations
BLM A7 Thinking General Scoring Rubric
BLM G10 Grid Paper

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies).

Link to Get Ready

Students will need to recall how to collect like terms, the distributive property, and geometric relationships for this section. Ensure that students have completed questions 1 to 6 of the Get Ready prior to starting this section.

Warm-Up

1. Solve.

a) $2x - 5 = 7$ **b)** $3 - 4x = -9$

2. Check your answers to question 1 using the left side/right side method.

Warm-Up Answers

1. a) $2x - 5 = 7$
$2x = 7 + 5$
$2x = 12$
$x = 6$

b) $3 - 4x = -9$
$3 + 9 = 4x$
$12 = 4x$
$3 = x$

2. a) LS $= 2(6) - 5$ RS $= 7$
$= 7$

b) LS $= 3 - 4(3)$ RS $= -9$
$= -9$

Teaching Suggestions

- Assign the Warm-Up as independent work. (5 min)
- Discuss the photo of the bridge in preparation for the Investigate, which provides an opportunity to draw connections between measurement and algebra. Triangles will be revisited from a geometric perspective in Example 3. It is important for students to appreciate that algebra is closely connected and applied to other branches of mathematics and used in the real world, and not simply a collection of irrelevant mechanical skills.
- Have students work on the Investigate with a partner or in small groups. (5–10 min)
- Assign the Examples, and follow up with a discussion. (20–25 min)
- Example 1a) introduces the method of collecting variable terms on the left side and constant terms on the right side.
- Example 1b) reinforces this technique with a slightly more complicated equation. Although movement of both terms is performed simultaneously, some students may prefer to perform these one step at a time, i.e., add $5k$ to both sides in the first step, and subtract 7 from both sides in the second step. A CAS method of solution is presented that still requires students to invoke each algebraic step. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.

Common Errors

- Some students may bring terms to the other side of an equation without changing their sign.

R_x Remind students that they must perform the opposite operation in order to remove a term from one side of the equation, and that identical operations must be performed on both sides of an equation for it to hold true.

- Some students may mix up signs when simplifying equations.

R_x This may be due to a lingering weakness in integer operations. Remediate as needed.

- Some students may handle fractions incorrectly, or not reduce fractional answers to lowest terms.

R_x This may be due to a lingering weakness in rational number operations. Remediate as needed.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Example 1c) shows that it does not matter which side of the equation the variables and constants are collected on. Show both methods to illustrate this.
- Example 2 introduces equations involving brackets. You may wish to use **BLM 4.2.1 Alternate Solution Method** to show students an alternate solution strategy.
- For Example 2b), consider using the following strategy to spark interest:
  - Tell students to close their textbooks.
  - Instruct the class to think about each step for a moment.
  - Then, select a volunteer to write the next step of the solution at the board and explain the step.
  - Allow for class discussion of each step. The steps may look slightly different from the solution presented in the textbook.
  - Have all students transcribe the complete solution and write their own explanation of each step.
- Example 3 applies equation solving to a geometric context. You may wish to present an alternate solution to students using an overhead projector. Use **BLM 4.2.2 Construct a Geometric Model With *The Geometer's Sketchpad*®**, and **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Alternatively, break up the lesson:
  - Assign Example 1, and Practise questions 1 to 3.
  - Debrief answers to Practise questions.
  - Have students do Examples 2 and 3, and assign the remaining exercises.
- Based on the progress of your students, you may wish to split the lesson over two days and provide more practice. You may wish to use **BLM 4.2.3 Practice: Solve Multi-Step Equations** for remediation or extra practice.
- Assign the Communicate Your Understanding questions, and consolidate with a class discussion. (5 min)
- Have students complete the balance of the exercises as independent work. (balance of period)

Investigate Answers (page 196)

1.

Solution	Explanation
$5x + 3 = 3x + 7$	Given equation.
$5x + 3 - 3 = 3x + 7 - 3$	Apply opposite operations: subtract 3 from both sides.
$5x = 3x + 4$	Simplify by adding integers.
$5x - 3x = 3x + 4 - 3x$	Apply opposite operations: subtract $3x$ from both sides.
$2x = 4$	Simplify by collecting like terms.
$\frac{2x}{2} = \frac{4}{2}$	Apply opposite operations: divide both sides by 2.
$x = 2$	Simplify.

2.

Solution	Explanation
$5x + 3 = 3x + 7$	Given equation.
$5x + 3 - 7 = 3x + 7 - 7$	Apply opposite operations: subtract 7 from both sides.
$5x - 4 = 3x$	Simplify by adding integers.
$5x - 4 - 5x = 3x - 5x$	Apply opposite operations: subtract $5x$ from both sides.
$-4 = -2x$	Simplify by collecting like terms.
$\frac{-4}{-2} = \frac{-2x}{-2}$	Apply opposite operations: divide both sides by -2.
$2 = x$	Simplify.

Accommodations

Gifted and Enrichment—Challenge students to show more than one solution for solving the equations in this section.

Spatial—Have students use grid paper to organize their solutions when solving equations. You may wish to use **BLM G10 Grid Paper**.

Motor/Memory—For some students, it may be beneficial to pose a multi-step equation and provide (in multiple choice format) a number of possible choices for the first step. Have students pick which is correct with justification. This requires less writing and the choices presented may provide cues for students with weak memory functions.

Memory—Encourage students to solve the equations in sequential steps reminding them to highlight the like terms.

Student Success

Provide **extrinsic rewards** for solving equations (e.g., secret code decoders, patterns in matching exercises, etc.).

3. a) The results from steps 1 and 2 are the same.

b) Answers will vary. Use opposite operations to collect the variable terms on one side. Do the same for constant terms to collect them on the other side. Then, simplify the equation.

4. Replace every x in the expressions for the side lengths with 2 and simplify to find the actual side lengths of each triangle. The lengths are 3, 4, and 6 for the first triangle, and 2, 5, and 6 for the second triangle. This checks as both have perimeter of 13.

Communicate Your Understanding Responses (page 200)

C1. a) Add 9 to both sides, to collect the like constant terms on the left side.

b) Add 8 to both sides, to collect the like constant terms on the left side.

c) Expand the brackets on both sides.

C2. C. If, after substituting $p = -2$ into both sides of the equation, the left side has the same value as the right side, then we know that $p = -2$ is the correct solution for that equation.

Practise

Remind students that it does not matter to which side of the equation they move the variables. For example, in 1c), it is easier to work with the variables on the right side.

For question 5, encourage students to use the left side/right side method of checking solutions, as opposed to simply substituting into the equation and simplifying. Working with LS/RS independently becomes particularly important in future mathematics chapters when non-linear equations can lead to extraneous roots. Mastering the technique now will benefit students later.

In question 6, students may struggle in setting up the initial equation. Encourage them to start with a diagram and to label it using the given information.

Connect and Apply

When working with CAS in questions 11 and 12, students may get error messages due to syntax problems. Watch in particular for missing parentheses when nested brackets are needed.

For question 14, students may struggle in setting up the initial equation. Encourage them to start with a diagram, and to label it using the given information.

Extend

Some students may have trouble understanding three-term ratio notation in questions 15 and 16. They also may struggle in setting up the equation. Provide support, as needed. You may wish to use **BLM A7 Thinking General Scoring Rubric** for question 15 to assist you in assessing your students.

Question 17 requires facility in fraction operations. Students will learn another method for solving such equations in the next section. Plodding through these questions first may help students appreciate the technique of clearing fractions.

Literacy Connections

Constant

Explain to students that a constant is something that does not change. The same is true in math. A constant term in an equation or expression is a value that will not change; it is a number.

BEDMAS

Tell students that the term *BEDMAS* is a mnemonic. A mnemonic is a device to aid the memory. Explain that the mnemonic BEDMAS helps us to remember the order of operations. There are many mnemonics in mathematics. Ask students, *Can you think of any others that you know already?*

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1a), c), 2a), c), 3a), c), 4a), c), 5a), c), 6, 9a), c), 10, 13
Typical	1–10, 13, 14
Extension	15–21

4.3 Solve Equations Involving Fractions

Strand:
Number Sense and Algebra

Student Text Pages
204 to 210

Suggested Timing
80 min

Technology Tools
- Computer Algebra System
- *The Geometer's Sketchpad®*
- TI-89 calculators
- calculators
- graphing calculators
- computers

Related Resources
BLM 4.3.1 Practice: Solve Equations Involving Fractions
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM 4.3.2 Cross-Multiplication Method
BLM A8 Application General Scoring Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.3.3 Achievement Check Rubric
BLM A17 Teamwork Self Assessment
BLM 4.3.4 Student Success: Decision Tree

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies).

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Students should have completed all of these prior to working on this section.

Warm-Up

Have students work alone or in pairs.

1. a) What is a trapezoid?
b) Draw a trapezoid to support your explanation.

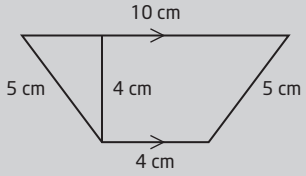
2. a) What is the formula for the area of a trapezoid?
b) Take measurements of the trapezoid you drew in 1b) and calculate its area.

3. Trade your work with another classmate or group and check each other's work for accuracy and correctness.

Warm-Up Answers

1. a) A trapezoid is a four-sided figure with two parallel sides. The other two sides may or may not have the same length.

b) Answers may vary. Sample solution:



2. a) $A = \frac{1}{2}h(a + b)$

b) Answers may vary. Sample solution based on diagram in 1 b).

$A = \frac{1}{2}h(a + b)$
$= \frac{4}{2}(4 + 10)$
$= 2(14)$
$A = 28$
The area is 28 cm^2.

Teaching Suggestions

- Have students complete the Warm-Up independently. (5–10 min)
- After the Warm-Up, discuss the problem posed in the Investigate. Discuss which variables are given, and which is unknown. Then, let students try to solve the equation for the unknown with a partner or in small groups. (5–10 min)
- There are different ways that this equation can be solved, without clearing fractions: guess and check, convert to decimals, divide both sides by $\frac{1}{2}$,

Common Errors

- Some students may multiply the side with the fraction twice when multiplying to clear fractions, for example,

$$2 \times \frac{(x+3)}{2} = 2 \times 5$$
$$2x + 6 = 10$$

R_x Have students clearly show the dividing out of the factor and what is left after dividing (in this case, 1s on the left hand side where the 2s are).

- Some students may forget to multiply the side without the fraction when multiplying to clear fractions, for example,

$$3 \times \frac{(y+1)}{3} = 7$$

R_x Remind students that any operation performed on one side of an equation must be performed identically on the other side in order for the equation to hold true.

Ongoing Assessment

- Use Achievement Check question 10 to monitor student success. See Achievement Check Answers and **BLM 4.3.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills (see Achievement Chart for levels)

Accommodations

Gifted and Enrichment—Encourage students to solve other simple, higher degree equations including quadratic, cubic, and quadratic equations.

Visual—Let students use a graphing calculator to graph linear relations related to the linear equations to be solved and relate the *x*-intercept to the root of an equation.

Motor/Memory—Give a multi-step equation. Then, provide each step to the solution on randomly ordered separate slips of paper. The student's task is to arrange the slips of paper in the correct order to present a coherent solution. Have students work with a partner and take turns explaining each step of the solution. You may wish to use **BLM A17 Teamwork Self Assessment** to assist you in assessing your students.

Language—Encourage students to determine how equations are formed in order to help them use a CAS (Computer Algebra System) when solving equations.

apply the distributive property with the fraction, etc. Ask students to share which approach or approaches they tried. Each of these methods can work, however the calculations can become difficult. After having wrestled with one or more of these approaches, students should be in a position to appreciate the techniques of clearing fractions, as presented in the Examples. You may wish to use **BLM 4.3.1 Practice: Solve Equations Involving Fractions** as remediation or extra practice.

- Assign the Examples, and follow up with a discussion. (15–20 min)
- Example 1 introduces a technique for clearing a single fraction, which is to multiply both sides of the equation by the fraction's denominator. Notice the two forms that the equations can appear in: either with a rational coefficient multiplied by a polynomial, as in part a), or with the entire side of the equation appearing as a fraction, as in part b).
- A CAS solution method is provided for b). Note that while the initial fraction is cleared in the same way as the paper-and-pencil method, the distributive property can be avoided by dividing by the coefficient 3. Because the resulting fraction poses no difficulty for the CAS to handle, the result is a shorter, more elegant solution. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- Example 2 addresses equations involving more than one fraction. **BLM 4.3.2 Cross-Multiplication Method** provides an alternate technique, called cross-multiplication. Note that the technique of cross-multiplication can only be used if you have two rational expressions equal to each other. If you have more than two expressions, you must clear denominators using the lowest common denominator.
- Assign the Communicate Your Understanding questions and consolidate with a class discussion. (5 min)
- Assign the balance of the exercises as independent work. (balance of period)

Investigate Answers (page 204)

1. $50 = \frac{(8 + b)5}{2}$

2. $b = 12$; The garden is 12 m wide at the back.

3. Answers will vary. A sample answer: To work with the fraction $\frac{5}{2}$ was somewhat difficult. If the denominator, 2, could be eliminated, it would be easier to find the solution.

Communicate Your Understanding Responses (page 207)

C1. a) Multiply both sides by 4.
b) Multiply both sides by 3.

C2. A and C

C3. a) 30; The lowest common multiple of 5 and 6 is 30.
b) 9; The lowest common multiple of 9 and 3 is 9.

Practise

For question 1, remind students that the entire side of an equation is multiplied. For example in 1a), some students may be tempted to multiply the $\frac{1}{3}$ by 3, the x by 3, and the -2 by 3. Some will forget to also multiply the 5 by 3.

When working with CAS in question 2, especially when entering complicated equations, ensure that students watch for the correct number and placement of open and closed parentheses to avoid syntax errors or incorrect equations.

Student Success

Construct a **decision tree** for solving equations with the class (see **BLM 4.3.4 Student Success: Decision Tree** for an example). Use a large roll of chart paper and lots of colours. Or, have students construct their own copies on ledger paper.

Connect and Apply

Question 6 provides practice in trouble-shooting skills. Encourage students to try to diagnose their own errors, as much as possible. Remind students that this requires they clearly show all steps to their solutions.

An interesting extension to question 7 is to have students rearrange this equation to express F in terms of C. Talented students may explore this for enrichment. The skill of rearranging formulas is the focus of the next section. You may wish to use **BLM A8 Application General Scoring Rubric** for question 7 to assist you in assessing your students.

Question 8 draws another connection between measurement and algebra. Students will need to recall the formula for the area of a triangle.

Students need to synthesize various concepts for question 9: solving equations, area, perimeter, and the Pythagorean theorem.

For question 10, some students may benefit from exploring this problem using *The Geometer's Sketchpad*®. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Achievement Check Answers (page 209)

10. a) If a square has perimeter of 144 m, then each side is 36 m.

b) The perimeter is given by $w + 3w + w + 3w = 144$, or $8w = 144$.
So, $w = \frac{144}{8}$. The rectangle is 18 m wide and 54 m long.

c) The perimeter is $x + x + \frac{x - 2}{3} = 144$.
$\frac{7x}{3} = 144 + \frac{2}{3}$
$7x = 3(144) + 2$
$7x = 434$
$x = 62$
Therefore, the sides of the triangle are 62 m, 62 m, and $\frac{62 - 2}{3}$ m, or 20 m.

d) The areas of the three figures are:
Square: $36 \times 36 = 1296$
Rectangle: $18 \times 54 = 972$
Triangle: The isosceles triangle has a base of 20 m, and the height (altitude) bisects the base. Using the Pythagorean theorem to find the height gives
$h = \sqrt{62^2 - 10^2}$
$= \sqrt{3744}$
$\doteq 61.2$
So, the area is $\frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(20 \times 61.2)$ or 612.
Therefore, the square has the greatest area ($1296\ \text{m}^2$) and the triangle the least area ($612\ \text{m}^2$).

Extend

The same technique from Example 2 can be used for question 11. The extension is that you must find the lowest common denominator of three or more fractions.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1, 3, 5, 7
Typical	1, 3, 5–9
Extension	11–13

4.4 Modelling With Formulas

Strand:
Number Sense and Algebra

Student Text Pages
211 to 219

Suggested Timing
80 min

Related Resources
BLM 4.4.1 Practice: Modelling With Formulas
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.4.2 Use a Computer Algebra System Directly
BLM 4.4.3 Student Success: Terrific Triangles
BLM A18 My Progress as a Problem Solver
BLM G10 Grid Paper

Tools
- grid paper
- rulers or metre sticks

Technology Tools
- calculators
- graphing calculators
- Computer Algebra System
- TI-89 calculators
- *The Geometer's Sketchpad®*
- computers

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.03 relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify and solve equations.

NA2.08 rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement).

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of these prior to working on this section.

Warm-Up

1. Use a calculator to evaluate. Round to one decimal place, if necessary.

a) $\sqrt{400}$ **b)** $\sqrt{225}$ **c)** $\sqrt{200}$ **d)** $\sqrt{1250}$

2. Substitute, then solve for the unknown variable.

a) $A = P + I$ $P = 500; I = 75$

b) $y = mx + b$ $m = \frac{1}{2}; x = 4; b = -9$

Warm-Up Answers

1. a) 20 **b)** 15 **c)** 14.1 d) 35.4

2. a) 575 **b)** −7

Teaching Suggestions

- Students may struggle with this section initially because the level of abstract symbolic reasoning is taken to a new level. Students may be used to finding a numeric "answer" to a math question, and now they are starting and finishing with an algebraic expression. It is important for students to gain some rationale and comfort level for learning such skills.
- They have some initial exposure in Chapter 3 where they were required to simplify an expression, however, here the "answer" often looks no simpler than the "question." They increasingly will need to apply this kind of reasoning in the future study of mathematics (e.g., factoring, etc.).
- Students may feel that they are simply performing arbitrary manipulations that serve no useful purpose, and thus lose interest. Try to connect the activities to real-world examples as often as possible. You may wish to use **BLM 4.4.1 Practice: Modelling With Formulas** for remediation or extra practice.
- Have struggling students create cue cards for various mathematical processes that list the steps involved in the process. Encourage students to keep their cue cards at hand and use them when needed.
- Have students complete the Warm-Up as independent work. (5 min)
- For the Investigate, have students work with a partner or in small groups. (5–10 min)

Common Errors

- Some students may apply the wrong operation to isolate the variable. For example, in Practise question 1a), they might subtract 4 from both sides, instead of divide by 4.

R_x Have students ask themselves, *What operation is being performed between the variable of interest and the other variables or constants? What is the opposite operation required to "undo" this?*

- Some students may perform operations in the incorrect order. For example, in Practise question 2a) some students may divide by t before first subtracting b.

R_x Remind students that they must isolate the term containing the variable of interest first, before dividing or multiplying to isolate the variable itself. This is similar to the approach used earlier in the chapter when actually solving for the root of an equation in one variable. Review a simple example of this, if necessary, such as: $3x + 4 = 10$ (you must subtract 4 first, before dividing by 3).

Ongoing Assessment

- Chapter Problem question 7 also can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Use the photo of the view from the CN Tower to generate a class discussion. If your school is far from the CN Tower, you may wish to use an interesting local alternative (e.g., Skylon Tower in Niagara Falls, any nearby rural fire towers, etc.). You may need to research some height data to adjust the Investigate.
- The purpose of the Investigate is to pose a problem that will prepare students to see the desirability of rearranging a formula into a different form. Students should have little difficulty in solving Investigate question 1, except possibly for how to handle the square root. Suggest substituting and simplifying the expression inside the square root symbol first.
- Have graphing calculators or software available for students to use for the Investigate activity. There are different ways that the students may try to solve 2b) of the Investigate, such as: guess and check, substitute and rearrange to solve the equation, and using a graphing calculator to enter a function and reading its graph. The latter may require some support, as students thus far have had limited or no experience in solving equations involving square roots. Remind students of the Pythagorean theorem, and the opposite relationship of squaring a number and taking its square root. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator**, **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Assign the Examples, and follow up with a class discussion. (15–20 min)
- After debriefing the results of the Investigate, use Example 1 to illustrate the various techniques for rearranging simple formulas. These examples require only one step, to apply the opposite operation in order to isolate the variable of interest. Students should be able to transfer skills learned earlier in the chapter to working with and possibly shifting more than one variable.
- Example 2 involves rearranging formulas in more than one step. The context chosen is an important one, as it connects to future work in Chapter 6. Operations typically are done in the opposite order, according to BEDMAS. The main idea is to isolate the term containing the variable of interest first, and then solve for the variable.
- A Computer Algebra System (CAS) can be useful for rearranging formulas, and could be presented as an alternate approach to Example 2. In Method 2, students enter each step, and CAS produces the result of that step. CAS is also capable of rearranging a formula directly, without the need for entering intermediate steps. You may wish to have students use this method for checking their work, however, it is important for the academic student to be able to master the basic skills involved here and not use the CAS as a substitute for pencil and paper. Use **BLM 4.4.2 Use a Computer Algebra System Directly** for this activity. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- Communicate Your Understanding: class discussion (5 min)
- Assign the exercises as independent work. (balance of period)
- You may wish to use **BLM A18 My Progress as a Problem Solver** at any point in this lesson to assist you in assessing your students.

Accommodations

Gifted and Enrichment—Challenge students to research geostationary satellites and share their new knowledge with their classmates.

Visual—Encourage students to highlight the variables in the formulas in different colours.

Perceptual—Allow students to use visual cues, such as the distance, speed, time triangle, to help them when they are manipulating formulas.

Memory—Encourage students to review how to use the functions on their scientific and graphing calculators.

Student Success

After teaching this section, introduce students to **formula triangles**. Emphasize the mathematical operations behind why the triangles work. Use **BLM 4.4.3 Student Success: Terrific Triangles** to support this activity.

Investigate Answers (page 211)

1. a) 67 km **b)** 76 km

2. a) 781 m

b) Substitute $d = 100$ into the formula and solve for h.

Step 1: Substitute $d = 100$: $100 = 2 \times \sqrt{3.2 \times h}$

Step 2: Divide both sides by 2: $50 = \sqrt{3.2 \times h}$

Step 3: Square both sides: $50^2 = 3.2 \times h$

Step 4: Divide both sides by 3.2: $h = \frac{50^2}{3.2}$

Step 5: Simplify: $h = 781.25$

You would have to go up 781 m, to the nearest metre.

3. Formulas can be rearranged to isolate one of the variables. Then, the known values can be substituted in the equation to calculate the isolated variable.
For example: $N = 7x + 12$, evaluate x when $N = 82$.

$$N = 7x + 12$$
$$N - 12 = 7x$$
$$\frac{N - 12}{7} = x$$

Substitute $N = 82$ in the rearranged equation to evaluate x.

$$\frac{82 - 12}{7} = x$$
$$10 = x$$

Communicate Your Understanding Responses (page 214)

C1. a) Multiply both sides by t.

b) Subtract b from both sides.

c) Divide both sides by π.

C2. The probability of errors is reduced. Rearranging the formula helps to simplify the expression and make the substitution easier.

C3. Answers may vary. Sample solution:

Step 1: Divide both sides by 2.

$$\frac{p}{2} = \frac{2(\ell + w)}{2}$$

Step 2: Divide the 2s on the right side.

$$\frac{p}{2} = \frac{\cancel{2}(\ell + w)}{\cancel{2}}$$

Step 3: Subtract w from both sides.

$$\frac{p}{2} - w = \ell + w - w$$

Step 4: Simplify.

$$\frac{p}{2} - w = \ell$$

Yes, there is more than one way to isolate ℓ. You could expand the expression on the right side and then subtract $2w$ from the both sides. Next, you would have to divide both sides by 2 to isolate ℓ. However, the results of both expressions will be equivalent.

Practise

Students may have difficulty getting started because they are uncertain what to do. Reinforce that they first need to identify the variable of interest. Then, have them consider the operation that will allow them to isolate that variable. This will usually require applying the opposite operation to numbers and/or other variables.

Connect and Apply

Have graphing calculators or **BLM G10 Grid Paper** available as a number of the questions ask students to compare algebraic and graphic models. For questions 3 and 4, use a ruler or metre stick (that also shows inches) to show students where this relationship comes from.

Question 4 connects to linear relations, a key theme of the chapter, as well as section 5.2 Direct Variation.

Question 6 connects solving equations and future work in section 5.3 Partial Variation, as well as linear and non-linear relations.

Question 8 connects measurement relationships and equations. Some students may benefit from exploring this problem using *The Geometer's Sketchpad*® or a graphing calculator. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator**, **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Questions 10 to 12 connect to future work in senior chemistry and physics. Although a thorough understanding of the scientific principles is not necessary here, students will benefit from seeing how mathematics is required for manipulating these formulas.

Extend

Questions 13 and 14 extend the idea of inverse operations applied to formulas into the idea of inverse functions, which students will study in depth in senior mathematics. At this level however, students should be able to simply identify some similar and distinguishing features between the curves, without any formal analysis. They should also realize that you could use either curve (e.g., A versus ℓ, or ℓ versus A) to find specific information about the relationship between the two variables.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–7
Typical	1–8, 10–12
Extension	13–19

4.5 Modelling With Algebra

Strand:
Number Sense and Algebra

Student Text Pages
220 to 229

Suggested Timing
80–160 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- Internet access

Related Resources
BLM 4.5.1 Investigate: Magic With Algebra
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.5.2 Practice: Modelling With Algebra
BLM 4.5.3 Achievement Check Rubric
BLM G10 Grid Paper
BLM A22 Earning Money Report Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.09 solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods.

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of these prior to working on this section.

Warm-Up

1. Solve for x.
$x + 2x + 3x - 100 = 20$

2. The formula for the area of a trapezoid is:
$A = \frac{1}{2}h(a + b)$
Find the area of a trapezoid whose parallel sides are 8 cm and 12 cm, and whose height is 10 cm.

3. Rearrange the following formula in terms of p.
$a = 5p + q$

Warm-Up Answers

1. 20 **2.** 100 cm^2 **3.** $\frac{a - q}{5} = p$

Teaching Suggestions

- Assign the Warm-Up as independent work. (5 min)
- Have students work on the Investigate as a class or in small groups. (10–15 min)
- The purpose of the Investigate is to provide students with an insight into the power of algebraic reasoning, in this case to debunk a magic trick. Conduct this as a class activity. The secret number can be posted on the board out of view of the magician. You may wish to be the magician, or choose a student volunteer. You may wish to have the student dress up as a magician for this class. Instruct students to leave books closed, except as needed. Use **BLM 4.5.1 Investigate: Magic With Algebra** for the designated magician.
- After groups have discussed question 5 of the Investigate for a few minutes, you may wish to give the class a hint: Simplify this expression: $[3(2x + 50) - 100] \div 2$
- Compare the original expression to the audience's steps in question 2. Compare the simplified expression to the magician's secret steps in question 3.
- Assign the Examples, and follow up with a class discussion. (25–35 min)
- Example 1 illustrates how an algebraic equation can be built to model and solve a payroll problem, using one variable. It is important for students to be able to build the expressions for the different workers' pay using a single variable. To do this, they must translate the given information into

Common Errors

- Some students may have trouble setting up an algebraic equation, based on given information.

R_x Have students look for key words and what operations they imply, for example, total, difference, times, etc. Provide practice with just translating phrases before delving into full problems.

- Some students may have incomplete solutions. For example in question 9, the student may solve for x, which represents one of the integers, and stop there, without stating the other integers.

R_x Typically, these problems require a number of steps. Have students look back to the question and verify that they have answered it completely.

Ongoing Assessment

- Use Achievement Check question 15 to monitor student success. See Achievement Check Answers and **BLM 4.5.3 Achievement Check Rubric**.
- Chapter Problem question 8 also can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

algebraic expressions. Ensure that students practise this skill alone before assigning full problems.

- In Example 2, a formula model is built involving several variables. The formula is then rearranged in terms of one of the other variables. This is done in two different ways. Method 1 is more straightforward conceptually, but results in a rather complicated formula, involving a fraction and a decimal. Method 2 produces a more elegant result but requires an insightful manipulation that clears the need for any fractions or decimals. In a class discussion, have students debate the merits of each approach.
- Example 3 requires synthesizing equation-solving concepts with area relationships and the Pythagorean theorem. Two approaches are presented and compared. Method 1 is algebraic, although it requires some geometric reasoning. Method 2 is a dynamic geometry approach that is far more visual. You may wish to demonstrate Method 2 to the class. In a class discussion, have students compare the different approaches to solving the problem. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Have students work with a partner or in small groups for the Communicate Your Understanding questions. Have students share their ideas with the class in a class discussion. (5 min)
- Assign the balance of the exercises and have students work independently. (balance of period)
- Encourage diverse approaches to problem solving and encourage students to share their approaches as they work independently or in small groups to tackle the exercise problems.
- Alternatively, break up the lesson:
  - Assign Example 1, and assign Practise questions 1 to 5.
  - Debrief answers to Practise questions.
  - Have students do Examples 2 and 3, and assign the remaining exercise questions.
- Based on the progress of your students, you may wish to split the lesson over two days and provide more practice. You may wish to use **BLM 4.5.2 Practice: Modelling With Algebra**.

Investigate Answers (page 220)

2. Answers will vary. Sample solution: Choose the number 2.
a) 4 **b)** 54 **c)** 162 **d)** 62 **e)** 31 **f)** 31

3. Abracadabra
a) 6 **b)** 2 **c)** 2

4. Answers will vary. Sample Solution: Choose the number 9. Results are: 18, 68, 204, 104, 52, 52. Magician: 27, 9.

6. a) No; Pick a number x and follow the magician's instructions to the audience.
Step 1: $2x$
Step 2: $2x + 50$
Step 3: $6x + 150$
Step 4: $6x + 50$
Step 5: $3x + 25$
Step 6: Final Result is $3x + 25$
Now, consider the magician's secret steps to find the number.
Step 1: $3x$
Step 2: x
So, after doing a series of calculations, one arrives at the number itself. The initial changes in the number introduced by addition and multiplication are countered by subtraction and division. In simple terms, these operations represent a circle. One starts at a certain point and performs some calculations (travels along the circle) and finds the number (arrives at the starting point).

b) Yes, this trick will work for both numbers greater than 10 and for negative numbers. As above, the calculations are independent of the number someone chooses. You can substitute any value for x and the steps, described above, still hold.

c) Answers will vary. You can use algebra to work backwards from the desired answer. For example, if you want x then, you can use algebraic equations to develop a series of steps that will result in x.

Communicate Your Understanding Responses (page 226)

C1. D. Let Hanna's age be h. Rufio is 5 years older than Hanna, so, Rufio's age is $(h + 5)$. Their ages add up to 37, so $h + (h + 5) = 37$.

C2. B. p is the amount earned delivering papers; Brittany earned twice as much babysitting $(2p)$ for a total of \$800: $800 = p + 2p$

C3. C. $12h$ represents the \$12 Asraf earns per hour plus the 10% (0.1) commission on sales (s): $12h + 0.1s$

Accommodations

Gifted and Enrichment—Challenge students to create magic puzzles that can be solved by their classmates using algebra and to research the work of the scientists Kepler and Hershel on the Internet.

Perceptual—Encourage students to create diagrams when solving problems.

Language—Have students use highlighting, bolding, or colour to help them identify and extract key pieces of information. For example, change this question:

Cheryl is three years older than Joanne is. The sum of their ages is 56. How old is each girl?

to look like this:

Cheryl is **three** years **older** than Joanne is. The **sum** of their ages is **56**. **How old** is **each** girl?

Discuss with the students what each key word implies, in order to build the equation model and solve the problem.

Memory—Allow students to use words instead of variables when developing formulas.

ESL—Encourage students to work together with their classmates when using *The Geometer's Sketchpad®* and to use translators to understand the new words in this section.

Student Success

Have students construct a **journal of number puzzles**, similar to the ones in this section, and then present the puzzles to the class, challenging the class to explain why they work.

Practise

Use the Practise questions as a means to develop students' skills at translating information given in prose form into algebraic expressions. Often, this is one of the most challenging aspects of algebraic modelling for students. If students struggle with this step, consider assigning additional questions in this area prior to assigning the Practise, or have students work on the Practise questions with a partner. Use **BLM 4.5.2 Practice: Modelling With Algebra** for additional practice.

Connect and Apply

Have students work with a partner on some of these problems to help build confidence.

Students may solve questions 9 and 10 using guess and check. This is a perfectly valid strategy as long as students can demonstrate and communicate how their method works.

For questions 11 to 13, encourage students to start with a diagram and to label any information that they are given.

You may wish to have students use **BLM G10 Grid Paper** for Achievement Check question 15.

Achievement Check Answers (page 228)

15. a) An algebraic expression for Paloma's total earnings is $9h + 12m$, where h is the time worked, in hours, and m is the number of memberships sold.

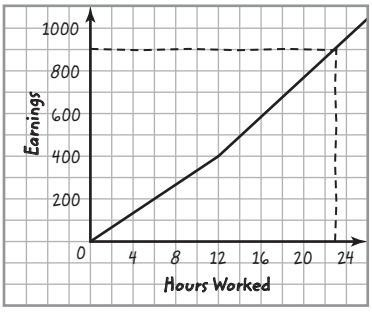
b) $9(8) + 12(7) = 156$ Paloma earned \$156.

c) Substituting and solving,

$$\begin{aligned} 9(24) + 12m &= 600 \\ 12m &= 600 - 216 \\ 12m &= 384 \\ m &= 32 \end{aligned}$$

Paloma must sell 32 memberships in order to earn \$600.

d) In the first 12 h, Paloma earns $9 + 2(12) = 33$, or \$33/h.
In the next 12 h, Paloma earns $9 + 3(12) = 45$, or \$45/h.
In order to graph this, students need to draw a line that rises at \$33/h for the first 12 h. From that point, they must draw a line that rises at \$45/h for the next 12 h. the graph will intersect the \$900 line at the 23 h mark.



Extend

For question 16, the Pythagorean theorem is required to set up the equation for the unknown side. Then, the area of a square relationship is applied. The Pythagorean theorem is also required for question 17.

Literacy Connections

Wages

Ask students, *Have you ever been paid for shovelling someone's driveway? If so, were you paid an hourly wage for the time you spent, or were you given*

a flat fee for completing the driveway? Explain that there are many different ways to be paid for work done. Sometimes people are paid an hourly wage and sometimes the amount is based on how much salespeople sell. Often in retail, the more people sell the more they make. Have students check on the Internet and with their friends and family about the variety of ways to earn money. Instruct students to write a brief report comparing and contrasting two methods of earning money at a job. You may wish to use **BLM A22 Earning Money Report Checklist** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–7
Typical	1–7, 9–14
Extension	16–20

Chapter 4 Review

Student Text Pages
230 to 231

Suggested Timing
80 min

Technology Tools
- Computer Algebra System
- TI-89 calculators
- calculators

Related Resources
BLM 4.CR.1 Chapter 4 Review
BLM A14 Self-Assessment Recording Sheet
BLM A15 Self-Assessment Checklist

Ongoing Assessment

- Upon completing the Chapter Review, students can also answer questions such as the following:
  - *Did you work by yourself or with others?*
  - *What questions did you find easy? Difficult? Why?*
  - *How often did you have to check the related teach example in the text to help you with the questions? For which questions?*
- You may wish to use **BLM A14 Self-Assessment Recording Sheet** and/or **BLM A15 Self-Assessment Checklist** to assist you in assessing your students.

Using the Chapter Review

Each question reviews different skills and concepts. Have students work independently to complete the Chapter Review, then with a partner to compare solutions. Alternatively, assign the Chapter Review for reinforcing skills and concepts in preparation for the Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with features they find difficult. You may wish to assign **BLM 4.CR.1 Chapter Review** to assist students with their review of the chapter.

Place vocabulary on a word wall, create a crossword puzzle, to review terminology in this chapter. After they complete the Chapter Review, encourage students to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content.

Chapter 4 Practice Test

Student Text Pages
232 to 233

Suggested Timing
60–80 min

Tools
- grid paper

Technology Tools
- Computer Algebra System
- TI-89 calculator
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM 4.PT.1 Chapter 4 Practice Test
BLM 4.CT.1 Chapter 4 Test
BLM 4.P.1 Performance Task
BLM A8 Application General Scoring Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM G10 Grid Paper

Summative Assessment

- After students complete **BLM 4.PT.1 Chapter 4 Practice Test**, you may wish to use **BLM 4.CT.1 Chapter 4 Test** as a summative assessment.
- The end of the chapter is a good place for a performance task. **BLM 4.P.1 Performance Task** targets process expectations to a higher degree, particularly problem solving where students must develop, select, apply, and compare a variety of problem-solving strategies. You may wish to use **BLM A8 Application General Scoring Rubric** to assist you in assessing your students for the Performance Task.

Accommodations

Gifted and Enrichment—Challenge students to prepare an extra Chapter Test for their classmates.

Memory—Encourage students to solve each of the equations using more than one method. For example, by inspection, by showing steps using pencil and paper, and by using a CAS.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	4.1	Example 3 (page 190)
2	4.1	Example 3 (page 190)
3	4.4	Example 1 (page 212)
4	4.5	Example 1 (page 221)
5a)	4.1	Example 1 (page 187)
5b)	4.1	Example 2 (page 189)
5c)	4.1	Example 3 (page 190)
5d)	4.2	Example 1 (page 197)
5e), f)	4.2	Example 2 (page 198)
6a)	4.3	Example 1 (page 205)
6b)	4.3	Example 2 (page 206)
7a)	4.4	Example 1 (page 212)
7b)	4.4	Example 2 (page 213)
7c)	4.4	Example 1c) (page 212)
8	4.5	Example 1 (page 221)
9	4.3	Example 2b) (page 207)
10	4.5	Example 2 (page 222)

Using the Practice Test

This Practice Test may be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.

Can students do each of the following?

- solve simple equations
- solve equations involving several steps, including application of the distributive property
- solve equations involving fractions
- check solutions to equations
- rearrange formulas
- apply formulas to solve problems
- build algebraic equation models
- apply equation models to solve problems, and compare solutions to alternate solution methods

Chapter 4 Problem Wrap-Up

Student Text Page
233

Suggested Timing
30 min

Related Resources
BLM 4.CP.1 Chapter 4 Problem Wrap-Up Rubric

Technology Tools
- calculators
- Computer Algebra System
- TI-89 calculators

Summative Assessment

- **Use BLM 4.CP.1 Chapter 4 Problem Wrap-Up Rubric** to assess student achievement.

Using the Chapter Problem

- The Chapter Problem can be done individually. Some students may not understand the concept of the minimum rating required to ensure a second season. Allow students to discuss the problem in groups before they begin.
- This Chapter Problem assumes that calculators with CAS will not be used. However, some students will benefit from the availability of a simple calculator for numerical computations.
- You may wish to add a second part to the problem after students have completed the Chapter Problem. Have the students solve the problem with CAS and compare the solutions. Since CAS calculators can "step though" a solution, students could also assess the completeness and effectiveness of their pencil and paper solutions.

Level 3 Sample Response

Yes, I think that the show was successful enough to earn a second season. Both the adults and the girls have ratings greater than 8 according to the question with the girls rating being much greater than 8.1. With two groups representing 70% of the vote being above 8, I think the average will be above 8.

b) Suppose the average rating for girls was x. Solving for x,

$$8 = \frac{5x + 3(7.4) + 2(8.1)}{10}$$

Multiply both sides by 10 and simplify.

$$80 = 5x + 22.2 + 16.2$$
$$80 - 22.2 - 16.2 = 5x$$
$$41.6 = 5x$$
$$\frac{41.6}{5} = x$$
$$x = 8.32$$

A girls' average rating of 8.32 would be the minimum required for success.

Level 3 Notes

Look for the following:
- Clear justification for whether there will be a second season (can be negative if justified)
- Clear solution to the equation with possibly minor errors
- Solution has a strategy for clearing fractions from the equation
- Appropriate concluding sentence

What Distinguishes Level 2

At this level, look for the following:
- Statement that there will be a second season but with little or no justification
- Student substitute values into the formula but with errors
- Equation solution has some logic but contains significant errors
- Solution may not include a correct method for clearing fractions
- Concluding sentence may be missing

What Distinguishes Level 4

At this level, look for the following:
- Clear, detailed justification for whether there will be a second season (can be negative if justified). There may be a statement that there is no definitive way to decide this question without solving the equation in the later part of the question.
- Possible justification of both points of view for the first question
- Equation solution is efficient with detailed justification and no errors
- May show a formal check of the solution to the equation
- Conclusion is clearly stated with relationship to equation solving process