Mathematical Processes

Vocabulary

natural number
prime number
perfect square
vertex
pentomino
rational number
conjecture
mean
counter-example

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Overall Expectations

By the end of this course, students will:

NAV.01 demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions;

NAV.02 manipulate numerical and polynomial expressions, and solve first-degree equations

Specific Expectations

Manipulating Expressions and Solving Equations

By the end of this chapter, students will:

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Mathematical Processes

The mathematical processes have been included at the beginning of the textbook to reflect the spirit of the 2005 Curriculum Revisions. These processes are interwoven throughout the other chapters, but they are introduced here to set the tone.

The focus of all the problems in Chapter 1 is on numeracy skills. Students can review these skills through a process approach, instead of through pages of drill. This way, students will become engaged in mathematical Problem Solving, Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing, and Communicating.

Chapter 1 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 1 Opener • 15 min	2–3			
Get Ready • 80 min	4–5	• BLM G1 Integer Number Lines • BLM 1.GR.1 Practice: Get Ready	• BLM 1.GR.2 Get Ready Self-Assessment Checklist	
1.1 Focus on Problem Solving • 80 min	6–9	• BLM 1.1.1 Investigate A Table • BLM 1.1.2 Investigate B Table • BLM 1.1.3 Practice: Focus on Problem Solving • BLM 1.1.4 Sudoku	• BLM A18 My Progress as a Problem Solver	
1.2 Focus on Communicating • 80 min	10–13	• BLM 1.2.1 Practice: Focus on Communicating • BLM 1.2.2 Sudoku	• BLM A9 Communication General Scoring Rubric • BLM A4 Presentation Checklist	
1.3 Focus on Connecting • 80 min	14–18	• BLM 1.3.1 Practice: Focus on Connecting • BLM 1.3.2 Sudoku	• BLM A5 Problem Solving Checklist • BLM A10 Observation General Scoring Rubric	
1.4 Focus on Representing • 80 min	19–22	• BLM 1.4.1 Practice: Focus on Representing • BLM G9 Centimetre Grid Paper	• BLM A4 Presentation Checklist BLM A11 Group Work Assessment Recording Sheet • BLM A12 Group Work Assessment General Scoring Rubric	**Tools** • centimetre grid paper
1.5 Focus on Selecting Tools and Computational Strategies • 160 min	23–28	• BLM 1.5.1 Practice: Focus on Selecting Tools and Computational Strategies • BLM G10 Grid Paper • BLM G7 Square Dot Paper • BLM 1.5.2 Tangram	• BLM A5 Problem Solving Checklist	**Tools** • square tiles • linking squares • grid paper • geoboards • square dot paper • tangrams
1.6 Focus on Reasoning and Proving • 80 min	29–33	• BLM 1.6.1 Practice: Focus on Reasoning and Proving • BLM 1.6.2 Chess Board • BLM 1.6.3 Sudoku	• BLM A11 Group Work Assessment Recording Sheet • BLM A12 Group Work Assessment General Scoring Rubric • BLM A10 Observation General Scoring Rubric	
1.7 Focus on Reflecting • 80 min	34–36	• BLM 1.7.1 Practice: Focus on Reflecting	• BLM A11 Group Work Assessment Recording Sheet • BLM A12 Group Work Assessment General Scoring Rubric • BLM A9 Communication General Scoring Rubric	**Tools** • paper strips • tape
Chapter 1 Review • 80 min	37	• BLM 1.CR.1 Chapter 1 Review	• BLM A14 Self-Assessment Recording Sheet • BLM A15 Self-Assessment Checklist • BLM A20 Learning Skills Checklist • BLM A16 My Progress as a Mathematician • BLM A18 My Progress as a Problem Solver	

Chapter 1 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM G1	Integer Number Lines	Teacher Support
	BLM 1.GR.1	Practice: Get Ready	Practice
	BLM 1.GR.2	Get Ready Self-Assessment Checklist	Self-Assessment
1.1: Focus on Problem Solving			
	BLM 1.1.1	Investigate A Table	Student Support
	BLM 1.1.2	Investigate B Table	Student Support
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM 1.1.3	Practice: Focus on Problem Solving	Practice
	BLM 1.1.4	Sudoku	Student Support
1.2: Focus on Communicating			
	BLM A9	Communication General Scoring Rubric	Assessment
	BLM A4	Presentation Checklist	Assessment
	BLM 1.2.1	Practice: Focus on Communicating	Practice
	BLM 1.2.2	Sudoku	Student Support
1.3: Focus on Connecting			
	BLM A5	Problem Solving Checklist	Assessment
	BLM 1.3.1	Practice: Focus on Connecting	Practice
	BLM 1.3.2	Sudoku	Student Support
	BLM A10	Observation General Scoring Rubric	Assessment
1.4: Focus on Representing			
	BLM A4	Presentation Checklist	Assessment
	BLM 1.4.1	Practice: Focus on Representing	Practice
	BLM G9	Centimetre Grid Paper	Student Support
	BLM A11	Group Work Assessment Recording Sheet	Assessment
	BLM A12	Group Work Assessment General Scoring Rubric	Assessment
1.5: Focus on Selecting Tools and Computational Strategies			
	BLM A5	Problem Solving Checklist	Assessment
	BLM 1.5.1	Practice: Focus on Selecting Tools and Computational Strategies	Practice
	BLM G10	Grid Paper	Student Support
	BLM G7	Square Dot Paper	Student Support
	BLM 1.5.2	Tangram	Student Support

	BLM	Title	Purpose
1.6: Focus on Reasoning and Proving			
	BLM A11	Group Work Assessment Recording Sheet	Assessment
	BLM A12	Group Work Assessment General Scoring Rubric	Assessment
	BLM 1.6.1	Practice: Focus on Reasoning and Proving	Practice
	BLM 1.6.2	Chess Board	Student Support
	BLM A10	Observation General Scoring Rubric	Assessment
	BLM 1.6.3	Sudoku	Student Support
1.7: Focus on Reflecting			
	BLM A11	Group Work Assessment Recording Sheet	Assessment
	BLM A12	Group Work Assessment General Scoring Rubric	Assessment
	BLM 1.7.1	Practice: Focus on Reflecting	Practice
	BLM A9	Communication General Scoring Rubric	Assessment
Chapter 1 Review			
	BLM A14	Self-Assessment Recording Sheet	Student Self-Assessment
	BLM A15	Self-Assessment Checklist	Student Self-Assessment
	BLM 1.CR.1	Chapter 1 Review	Review
	BLM A20	Learning Skills Checklist	Assessment
	BLM A16	My Progress as a Mathematician	Student Self-Assessment
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment

Get Ready

Common Errors

- Some students may find the common denominator when multiplying or dividing fractions.

R_x Have students think about the reasons behind finding the common denominator when adding and subtracting—in order to combine parts of a whole, fractions must relate to each other with a common denominator. When multiplying and dividing, there is no need to do that.

- Some students may mix up the rules for adding two negatives and give a positive answer.

R_x Use integer tiles to illustrate adding 2 red tiles with 3 red tiles equals 5 red tiles, rather than 5 green tiles.

Accommodations

Perceptual—Encourage students to use a number line when adding and subtracting fractions.

Memory—Allow students to use a scientific calculator to evaluate order of operations questions.

Teaching Suggestions

- As this chapter is meant to highlight mathematical processes, the Get Ready should be used for quick warm-ups, rather than as a full period exercise.
- Refer to the Link to Get Ready in the chapter sections of the Teacher's Resource.
- Have students work with each part when it is relevant to the section in the chapter. You may wish to use **BLM G1 Integer Number Lines** for questions 4 to 6.
- You may wish to use **BLM 1.GR.1 Practice: Get Ready** for remediation or extra practice.
- All BLMs referred to throughout this chapter can be found on the Principles of Mathematics 9 Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; have students place a checkmark beside topics in the Get Ready in which they feel confident with the necessary skills, or have students use **BLM 1.GR.2 Get Ready Self-Assessment Checklist**. Remedial action can be taken in small groups or with a whole class skill review.

Focus on Problem Solving

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Warm-Up

Mental Math:

1. Describe each pattern.
   a) 2, 4, 6, 8   **b)** 1, 2, 2, 3, 3, 3, 4, 4, 4, 4
   c) 5, 10, 15, 20

2. Multiply.
   a) 7×7 **b)** 8×8
   c) 9×9 **d)** 10×10
   e) 11×11 **f)** $2 \times 2 \times 2 \times 2 \times 2 \times 2$

3. Express as a decimal.
   a) $\frac{1}{2}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{4}$ **d)** $\frac{1}{5}$ **e)** $\frac{2}{3}$ **f)** $\frac{4}{5}$

4. What is missing from each list?
   a) A, I, U, O   **b)** J, F, M, A, M, J, J, A, O, N, D
   c) 1, 4, 9, 16, 36, 49

Warm-Up Answers

1. a) Increasing by 2s
   b) Each number written that number of times.
   c) Increasing by 5s

2. a) 49   **b)** 64   **c)** 81   **d)** 100   **e)** 121   **f)** 64

3. a) 0.5   **b)** $0.\overline{3}$ **c)** 0.25   **d)** 0.2   **e)** $0.\overline{6}$ **f)** 0.8

4. a) E   **b)** S (September)   **c)** 25

Teaching Suggestions

- This section focuses on problem-solving processes. Draw students' attention to the four steps:
  - Understand the problem
  - Choose a strategy
  - Carry out the strategy
  - Reflect
- When considering and choosing a strategy, ensure students consider one or all of the strategies listed. They may also suggest other strategies.
- Investigate A instructs students to make an organized list or table. Encourage students to consider this method in any situation for organizing their thoughts. As mathematics is generally the study of patterns, have students concentrate on looking for, describing, and extending patterns. Have students use **BLM 1.1.1 Investigate A Table** to produce their tables. (15 min)

- Some students may not know where to start.

R_x Have students start with a smaller problem. Have them look at the relationship between two terms, or what happens at the beginning, rather than focus on the "final answer" from the start. An organized list in chart form usually helps the most. Have the students put things into words or a diagram, depending on the type of learner.

Accommodations

Gifted and Enrichment—Encourage students to use different strategies to solve mathematical problems and to extend their knowledge of patterns in mathematics.

Visual—Allow students to work with Sudoku puzzles where the numbers are written on extra pieces of paper and can be moved around, rather than written and erased.

Perceptual—Encourage students to work together in groups or with a partner when solving problems.

- Investigate B encourages students to use logic or reasoning. Ask students, *What fits together? How can the descriptions be extended further?* Use **BLM 1.1.2 Investigate B Table** to produce their tables. (15 min)
- Students will find that, based on the clues, the values of A, B, D, E, F, H, and I can be determined. C and G could be either 3 or 5, so two answers are possible.
- In all problem-solving situations, encourage students to reflect on their answers. You may wish to use **BLM A18 My Progress as a Problem Solver** to assist you in assessing your students.
- Have a class discussion on the students' results from the Investigates before assigning and discussing Communicate Your Understanding questions C1 and C2. (20 min)
- Assign and take up Practise question 1. (5 min)
- Use **BLM 1.1.3 Practice: Focus on Problem Solving** for remediation or extra practice.

Investigate Answers (page 6)

A.

1. Use pennies to form a triangle. Form a pattern by increasing the number of pennies used for the base starting with 2. A triangle with 10 pennies in its base contains how many pennies altogether?

3.

Diagram Number	Number of Pennies
1	3
2	6
3	10
4	15
5	21
6	28
7	36
8	45
9	55

4. The number of pennies can be calculated by adding the diagram number plus 1 to the number of pennies in the previous triangle.

B.

1. Each letter in the grid represents a different natural number from 1 to 9. Consider these clues and find the value of each letter.

4.

7	1	3
9	6	2
5	4	8

or

7	1	5
9	6	2
3	4	8

Communicate Your Understanding Responses (page 8)

C1. a) Other strategies that were used were patterning and making diagrams.
b) Answers will vary.

C2. Answers will vary. A sample answer: Other strategies I used were logic or reasoning, systematic trial, and find needed information.

Practise

Encourage students to work without a calculator, as these exercises are meant to also review numeracy skills.

For question 2, encourage students to make an organized list.

Connect and Apply

For question 6, you may wish to have the students use the **BLM 1.1.4 Sudoku**. Question 6 may lead into giving students easy Sudoku puzzles to try on their own. More challenging Sudokus will be provided in future sections.

Question 7 is often of interest to students. Stress the importance of using a systematic method.

In question 8, some students may have difficulty visualizing the chess tournament structure. Encourage them to draw a diagram to illustrate the pairings.

Extend

Question 10 can be solved using logic and considering sums of available digits. It is accessible to level 3 and 4 students.

Question 11 is much more challenging and requires logic and reasoning.

Literacy Connections

Make Your Own Mnemonics

Explain to students that making up their own mnemonic can help them to remember how to approach problems. For example, **MOM** could remind them to **M**ake your **O**wn **M**odel, or **DAD** could remind them to **D**raw **A** **D**iagram. Have students think of a mnemonic to help them remember some of the other ways to approach a problem shown in this section.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1, 2, 5, 8
Typical	1–9
Extension	10, 11

Focus on Communicating

Mathematical Process Expectations Emphasis

☐ Problem Solving

☐ Reasoning and Proving

☑ Reflecting

☑ Selecting Tools and Computational Strategies

☑ Connecting

☐ Representing

☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Link to Get Ready

The Get Ready segment Operations With Fractions provides the needed skills for this section. You may wish to have students complete Get Ready questions 1, 2, and 3 before starting this section.

Teaching Suggestions

- Communication in mathematics comes in many forms, including drawing diagrams, interpreting diagrams and graphs, communicating one's understanding of mathematics, and communicating the results of solutions. This section will focus on the forms of communicating. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.
- The Investigate shows students how other historical cultures have developed communication systems in mathematics, and has the students communicate and extend their own understanding. As a follow up, have students research ancient number systems on the Internet or in the library and prepare a poster on their findings. You may wish to use **BLM A4 Presentation Checklist** to assist you in assessing your students. (15 min)
- The Example illustrates the value of a good diagram. As many students are visual learners, explain the importance of accurately drawn diagrams that are well labelled. Also, discuss the difference between a picture and a diagram. The former may look attractive, but the latter gives needed information to be able to solve a problem. (15 min)
- Assign and discuss the Communicate Your Understanding questions. (20 min)
- Assign and take up Practise question 1. (5 min)
- You may wish to use **BLM 1.2.1 Practice: Focus on Communicating** for remediation or extra practice.

Investigate Answers (page 10)

1. 13: ∩ ||| 126: ⊚∩∩ ||||||

1291: ⊚ ⊚ ⚱ ∩∩∩∩∩∩∩∩∩ |

2. a) �   **b)** ⬭ or ‖ / |||‖ / ⬭ / |||
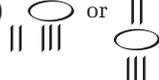
3. Answers will vary. The ancient Chinese used horizontal lines to represent one and the Egyptians used vertical lines.

4. Answers will vary and may include: Our number system is more like the Egyptian system in that we place our number symbols side-by-side across the page instead of stacking them.

Communicate Your Understanding Responses (page 11)

C1. a) A dot represents 1 unit and a line represents 5 units.

b) ══ ●● / ══ / ═══
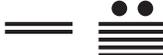
C2. Starting with 13, add 2 to each term: 24, 26

C3. a) Greg should rewrite his work using three lines that begin with the equal sign and write 8% as 0.08.

b) Answers will vary and may include: Using correct mathematical form reduces the chance of errors and makes it easier for others to read and understand the solutions.

Practise

Question 1 may take level 1 and 2 students quite a bit of time, since there are many numeracy skills involved here. You may wish to use **BLM 1.2.1 Practice: Focus on Communicating** for remediation or extra practice.

Connect and Apply

Although these are not difficult problems, each question contains a valuable communication component. Students need to practise explaining their reasoning, justifying their solution, and supporting their solutions with diagrams.

Extend

Question 9 is a relatively easy Sudoku puzzle. It is accessible to level 2, 3, and 4 students. The challenge lies in the description of the problem-solving process. Use **BLM 1.2.2 Sudoku** for this activity.

Question 10 is a very challenging problem that requires a well-labelled diagram.

Additional Extend Question: Why do computers use a binary number system instead of a base-10 system?
Answer: Computers are made of switches (originally mechanical, now electronic), which could be in only one of two states: on or off (1 or 0). Memory, numbers, and mathematical operations consist of combinations of these switches. Hence, everything, including numbers, is still represented in binary.

Literacy Connections

Explanations

Explain to students that when a question asks them to explain their thinking, they need to write in full sentences and really "tell" the reader how they found the answer. Have students consider what they would say to a friend or classmate if they had to explain their work, and then write out what they would say.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	C1, C2, 1, 3, 5, 6, 7
Typical	C1, C2, 1, 2–6, 8
Extension	9, 10

Focus on Connecting

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations
NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;
NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Teaching Suggestions

- Making Connections means applying math to the world around us. It can also mean making connections among different areas of math, or using a variety of math skills to solve a problem. This section makes those connections and encourages students to make multiple representations.
- In the Investigate, ensure that students draw a well-labelled diagram, illustrating the movement of a bouncing ball. This could be extended for level 4 students to include the question, *What if the ball continued bouncing indefinitely?* (10 min)
- Example 1 makes the connection with sports and the problem is solved by making an organized list in table form. Students may have other strategies that they would use. (10 min)
- Fermi problems are problems involving large numbers and requiring estimation skills. Students need to ask "big" questions when solving them and make connections to the appropriate area of mathematics. You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students. (20 min)
- Discuss the questions in Communicate Your Understanding before assigning the exercise. (10 min)
- You may wish to use **BLM 1.3.1 Practice: Focus on Connecting** for remediation or extra practice.

Investigate Answers (page 14)

1. A Superball is dropped from a height of 160 m. Each time it bounces, the Superball reaches 75% of its previous height. After bouncing 5 times, what distance has the Superball travelled?

2. The diagram must show the height from which the ball was dropped as well as four of the rebounds. The heights of the rebounds will be labelled. These will be calculated by multiplying the previous height by 75% (0.75):

$$160 \text{ m} \times 0.75 = 120 \text{ m}$$
$$120 \text{ m} \times 0.75 = 90 \text{ m}$$

3. Only the first four bounces must be shown since the distance is only being calculated until the moment when the ball bounces for the fifth time but not including that rebound.

160 cm   120 cm

90 cm

67.5 cm
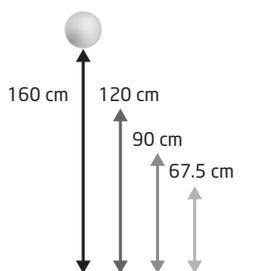
4. The Superball travelled a distance of $160 + 120 + 120 + 90 + 90 + 67.5 + 67.5 = 715$ cm. This seems reasonable considering the original height of 160 cm and the four rebounds at 75%.

Communicate Your Understanding Responses (page 17)

C1. Answers may vary. A sample answer: A diagram is used to illustrate a situation. It is clearly drawn with labels, measurements, and possibly calculations to show all the important information. A picture is an image of a situation; any information is assumed from the picture—there is no written information.

C2. Answers will vary. You will need to find or estimate the number of students per class, the number of classes, the number of teachers and staff within the school, and the percentage of the school population that are sitting.

Practise

Help for question 1 can be found in the Investigate. A good diagram will also help.

The key to question 3 is to ask the right questions regarding the shape and size of a puck and of the classroom. Students will need to know the formulas for the volume of a box and a cylinder.

Connect and Apply

Question 5 requires logic and reasoning skills.

Question 6 is best done by investigating the number of triangles of each size. A good extension is to have the students look for a pattern in these different-sized triangles and determine the number of triangles in a much larger triangle.

Question 8 seems simple in its approach. Students need to consider what happens on the final day.

Extend

The Sudoku in question 10 is moderately more difficult than the one that appears in Section 1.2. Students need to develop logic and reasoning strategies. Use **BLM 1.3.2 Sudoku** for this activity.

Question 12 is best solved by solving a smaller problem, and then extending it to larger polygons. Students need to look for a pattern. You may wish to use **BLM A10 Observation General Scoring Rubric** to assist you in assessing your students.

Literacy Connections

Life Experiences

Nearly every experience can add to your knowledge of mathematics. Ask students, *Can you think of three things you have done in the past week that connect math to your life?* Have student discuss them with their classmates and see how many they can find.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–9
Extension	10–12

1.4 Focus on Representing

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☐ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Warm-Up

1. Plot each of the following points. What shape do the points form?
  a) $(3, 0)$, $(3, 5)$, $(0, 5)$, $(0, 0)$
  b) $(-2, 5)$, $(-2, 2)$, $(6, 2)$
  c) $(7, -1)$, $(5, -4)$, $(-3, -4)$, $(-1, -1)$

2. What is $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$?

Warm-Up Answers

1. a) rectangle   **b)** right triangle   **c)** parallelogram

2. $\frac{1}{16}$

Teaching Suggestions

- Both mathematical and real-life situations can be expressed using many different representations, for example, verbally, with a diagram, numerically, and algebraically.
- In the Investigate, students are asked to represent the situation in words, numerically, and with a diagram. The strategy of working backward is effective here because the end result is known and the steps can be reversed. (10 min)
- As a follow up, have students research shipwreck diving in Ontario on the Internet or in the library and prepare a presentation on their findings. Information on Fathom Five National Marine Park can be found at http://canadianparks.com/ontario/ffivemp/index.htm. You may wish to use **BLM A4 Presentation Checklist** to assess students' presentation skills.
- For the Example, have students act it out to help them in the problem-solving process. (10 min)
- Alternate solution to the Example: Each of the 8 people could shake 7 hands. We must then divide by 2 because each person would shake another person's hand twice: $8 \times 7 \div 2 = 28$ handshakes.
- Assign and discuss the Communicate Your Understanding questions. (10 min)
- Assign and take up Practise questions 1 to 3. (20 min)
- Use **BLM 1.4.1 Practice: Focus on Representing** as extra practice or remediation.

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

**Spatial**—Allow students to use a graphing calculator when plotting points to determine patterns.

Language—Encourage students to work in groups to complete the questions in this section.

ESL—Encourage students to use a translator to understand the meaning of the new words in this section.

Investigate Answers (page 19)

1. Kevin is diving in a shipwreck. He dives down 6 m, swims 5 m up, dives down 9 m, and then another 2 m. He is now at 32 m below the surface. Where was he when he was at the beginning.

2.

5 m
6 m
9 m
2 m or 32 m below sea level
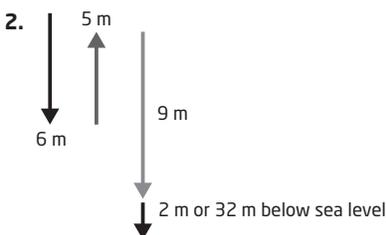
3. $-32\ m + 2\ m + 9\ m - 5\ m + 6\ m = -27\ m$

4. $-32\ m + 2\ m + 9\ m - 5\ m + 6\ m = -27\ m$

Communicate Your Understanding Responses (page 20)

C1. Answers will vary. A sample answer: Use algebra to represent the situation. Then, solve the equation:
$x - 6 + 5 - 9 - 2 = -32; x = -27$

C2. The first girl, Sarah, is 12 m ahead of the last girl, Barbara. Students' answers on how to solve this problem will vary. A sample answer: Draw a labelled diagram and add the distances between the first and last of the girls.

Practise

A diagram will help in each of questions 1 to 4. Working backward will also help in question 1.

Connect and Apply

Questions 5 and 8 help the student represent patterns and shapes graphically and will benefit them when discussing relationships in Chapter 2.

You may wish to use **BLM G9 Centimetre Grid Paper** for questions 5 and 8.

In question 6, students need to convert to a common denominator in diagram form. In part 6a), students would shade in sixths.

In question 7, it would help to represent the gears in ratio form.

Extend

In question 9, students need to determine the height needed to give an area of 15 square units, and apply the method used in question 8. You may wish to use **BLM G9 Centimetre Grid Paper**.

In question 10, students need to determine what effect, if any, another cog has on the number of turns. In this situation, a middle gear has no effect on the number of turns by the first gear because the middle gear simply transfers the force to the third gear.

Literacy Connections

Strategies

Have students list all the strategies they have devised this chapter. Ask students, *How can you remind yourself of the different ways to represent information? Can you think of any more mnemonics? Is there one type of representation you like more than the others? Explain your reasons.* With a partner, have students compare what strategies they like. You may wish to use **BLM A12 Group Work Assessment Recording Sheet** and/or **BLM A13**

Group Work Assessment General Scoring Rubric to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–8
Extension	9, 10

Focus on Selecting Tools and Computational Strategies

Mathematical Process Expectations Emphasis

☐ Problem Solving

☐ Reasoning and Proving

☑ Reflecting

☑ Selecting Tools and Computational Strategies

☑ Connecting

☑ Representing

☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations
NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;
NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Link to Get Ready

The Get Ready segments Operations With Fractions, Operations With Integers, and Order of Operations provide the needed skills for this section. Students should complete all the Get Ready questions before starting this section.

> **Warm-Up**
> Have students work through the Get Ready sections mentioned above, or similar examples.

Teaching Suggestions

• This section focuses on the process of selecting the right tools and computational strategies for the problem at hand. You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students. The use of graphing calculators and dynamic software, such as *The Geometer's Sketchpad®* and *Fathom™*, are left to the appropriate sections in future chapters.
• This section also focuses on operations with rational numbers, as this expectation is barely touched upon in grade 8 and is to be embedded as needed into problems in grade 9.

Day 1:
• Have students use pentominos to act out the problem in Investigate A. Have students work with a partner for this investigation. (15 min)
• Investigate B allows the student to select the appropriate tool. Most students will select a calculator. Others will do the calculations in their head or on paper. Allow the students to develop their own strategies for solving this problem. Students will end up with different descriptions of their patterns. (15 min)
• Have a class discussion on how and when to select the appropriate tools. Explain to students that not all problems require a model or diagram in order to visualize it. Calculators need not be the tool of choice for calculations. It is recommended that grade 9 students have calculators, graph paper, a ruler, and a variety of writing implements available. (10 min)
• Discuss question C1 in Communicate Your Understanding with the class before assigning exercises. (10 min)
• Assign and take up Practise questions 1a) to d). (10 min)

- The Example develops computational strategies that can be used with operations involving rational numbers. You may wish to supplement with more examples of a similar type. (15 min)
- Discuss question C2 in Communicate Your Understanding before assigning exercise questions. (10 min)
- Assign and take up Practise questions 2, 7, 9, and 10. (20 min)
- You may wish to use **BLM 1.5.1 Practice: Focus on Selecting Tools and Computational Strategies** for remediation or extra practice.

Investigate Answers (page 23)

A. 1. a)
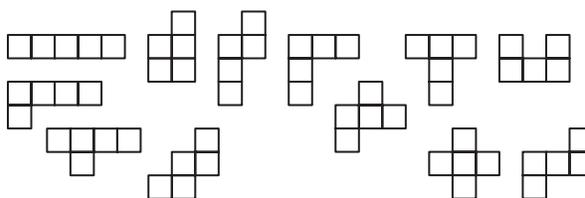
    c) There are 12 possible pentominos.

  2. Answers will vary. A sample answer: Square tiles, linking squares, and grid paper are all effective tools.

B. 1. a) $1^3 + 2^3 = 9$
    b) $1^3 + 2^3 + 3^3 = 36$
    c) $1^3 + 2^3 = 9 = 3^2 = (1 + 2)^2$
      $1^3 + 2^3 + 3^3 = 36 = 6^2 = (1 + 2 + 3)^2$
      $1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2 = (1 + 2 + 3 + 4)^2$
      The sum of the cubes of the natural numbers is the sum of the natural numbers squared.

  2. a) Answers will vary.
    b) The sum of the cubes of the first 15 natural numbers is 125 600.

  3. Answers will vary. A sample answer: Once the first few sums were solved and the equations written out, I used logic and reasoning to look for new ways to write the numbers $\left(\text{e.g., } 36 = 6^2, 3 = 1 + 2\right)$, and I started to see patterns.

Communicate Your Understanding Responses (page 25)

C1. Answers will vary. A sample answer: Calculator, speedometer, odometer of the truck, and metre stick.

C2. Ted's error was rounding off the repeated decimals. He should use the $\boxed{a^{b/c}}$ function on his calculator.

Practise

You may wish to use **BLM G10 Grid Paper** for questions 1b) and c). Most questions are fairly straightforward, as the tools are suggested.

    Question 3 requires the student to visualize in three dimensions. A good tool is to build a model.

    In questions 7 and 8, students will use varying strategies. Ask students who did the questions in their heads to explain how.

Connect and Apply

Assign and discuss question 11 before assigning question 12.

    For question 13, have students try the folding activity with different sizes and thicknesses of paper to show that the size does not matter. Allow them to come up with their own hypotheses as to why they can't fold it more than eight times. Scientifically, it is because 256 sheets of paper are being stretched as they are folded, with the outer ones being stretched much farther than the inner ones.

A geoboard with elastics is a valuable tool to have available to answer question 15. Alternatively, you may wish to use **BLM G7 Square Dot Paper**.

In questions 16 and 17, students should be familiar with a tangram. It is a good visual tool for students to use to understand operations with rational numbers. Use **BLM 1.5.1 Tangram** for questions 16 and 17.

Extend

Question 21a) is a good indicator of level 4 reasoning, especially with justification.

In question 22, students could build a model by rolling up a piece of paper for a tube and using string. Alternatively, students could draw an opened and flattened version of the tube. Students will need to know the formula for the circumference of a circle and the Pythagorean theorem.

Exercise Guide

Category	Question Number
Day 1:	
Minimum (essential questions for all students to cover the expectations)	1, 3–6, 13
Typical	1, 3–6, 13, 15
Extension	18–21
Day 2:	
Minimum (essential questions for all students to cover the expectations)	2, 7–12
Typical	2, 7–12, 14, 16, 17
Extension	22

1.6 Focus on Reasoning and Proving

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☐ Representing
- ☐ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Warm-Up

Review the meaning of powers $\left(\text{e.g., } 3^4 = 3 \times 3 \times 3 \times 3\right)$ and the order of operations (BEDMAS). Evaluate:

a) 5^2 **b)** $(-5)^2$ **c)** 5^3
d) $(-5)^3$ **e)** $7^2 - 8^2$ **f)** $(7 - 8)^2$
g) $9 + 3 \times 2$ **h)** $5 - 9^2 \div 3$

Warm-Up Answers

a) 25 **b)** 25 **c)** 125
d) -125 **e)** -15 **f)** 1
g) 15 **h)** -22

Teaching Suggestions

- This section concentrates on using reasoning skills. Students need to develop a sense of what works well and when. For example, a Guess and Check method involves an educated guess, based on their skills and knowledge. Sudoku puzzles also help develop reasoning skills. Students need to be able to determine whether an answer is reasonable, for example, whether it is too big or whether a negative value makes sense. Students need to develop methods of proving or verifying whether an answer or conjecture is true or false. Providing a counter-example is a useful tool in proving a conjecture false; algebra is often used to prove a conjecture true.
- The Investigate suggests the use of Guess and Check to solve the problem. Discuss with the students what makes a good educated guess. It may help to have students work with a partner. At the conclusion of the investigation, discuss as a class what other strategies would be useful. You may wish to use **BLM A11 Group Work Assessment Recording Sheet** and/or **BLM A12 Group Work Assessment General Scoring Rubric** to assist you in assessing your students. (15 min)
- Discuss the Example, or a similar one, that involves proving a conjecture. Algebra is used here as a reasoning tool. Although algebra is used, much of the reasoning comes in the form of verbal communication. (10 min)
- Review the meaning of the word *conjecture* and the importance of developing a well-structured proof when asked to prove a conjecture. Verify that it is true with an example, or false with a counter-example. Explain to students that if the first step is true, then proceed to the proof, which involves communicating with opening statements, proceeding through clear steps that prove the conjecture for all examples, and stating a final conclusion that refers back to the original conjecture. (5 min)
- Assign and discuss the questions in Communicate Your Understanding

- Level 1 students may become discouraged, not due to any common errors, but rather due to lack of success in their reasoning abilities. They may have difficulties seeing the big picture or even the results of a particular guess in the Guess-and-Check method.

R$_x$ Offer plenty of teacher support or support from their peers. Many students are visual or kinesthetic learners. Encourage them to draw a diagram or to act out the problem. Scaffolding the steps may also help in solving problems.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Perceptual—Encourage students to work with a reading buddy to understand the questions in this section.

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before assigning the exercise questions. (5 min)
- Assign and take up Practise questions 1 and 3. (10 min)
- Use **BLM 1.6.1 Practice: Focus on Reasoning and Proving** for remediation or extra practice.

Investigate Answers (page 29)

1. A vending machine containing 90 coins has 3 times as many quarters as dimes, and twice as many nickels as dimes. How much money is in the machine?

2. The number of nickels must be even and the number of quarters must be divisible by 3.

3. There are 15 dimes, 45 quarters, and 30 nickels.

4. There is only one possible solution that fits all the clues.

5. The quickest strategy is to write equations to fit all the clues and to solve for all of the variables.

Communicate Your Understanding Responses (page 31)

C1. a) Jay used Guess and Check, and Patterning.
   b) Answers will vary.

C2. a) 48°; 37°; 50°; 81°   **b)** Each angle is less than 90°.
   c) No. These triangles are examples of isosceles triangles that support the statement but the statement is not proved for all isosceles triangles.

Practise

Questions 1 and 2 are generally best completed through Guess and Check. Students may use other methods, though. Most students will answer question 3 using algebra. Question 4 is a good chance for students to provide a verbal argument supporting the conjecture.

Connect and Apply

All of these questions are accessible to students working at levels 3 and 4. Students working at level 2 will reach some level of success. Students working at level 1 or below will probably experience great difficulties without teacher support. Many questions, in particular, questions 6, 7, and 11, provide good practice of numeracy skills. Question 8 is an important exercise in finding counter-examples. You may wish to supplement with additional similar examples. Use **BLM 1.6.2 Chess Board** for question 9.

Extend

The Sudoku puzzle in question 14 is fairly challenging for grade 9 students. Use **BLM 1.6.3 Sudoku** for question 14. Question 16 provides a good exercise in interpreting patterns, and where simple extension of the pattern takes too long. Good reasoning skills are key for all three Extend questions. You may wish to use **BLM A10 Observation General Scoring Rubric** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 8
Typical	1–13
Extension	14–16

1.7 Focus on Reflecting

Mathematical Process Expectations Emphasis

☑ Problem Solving

☑ Reasoning and Proving

☑ Reflecting

☑ Selecting Tools and Computational Strategies

☑ Connecting

☑ Representing

☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Warm-Up

1. One quarter of 8 is increased by 5 then doubled. What is the result?

2. Multiply 6 by -3, then square it. What is the result?

3. What is the sum of the squares of 4 and 7?

4. What are all the divisors of 24? 60? 17?

5. Calculate the area of a rectangle with width of 10 cm and length that is twice the width.

Warm-Up Answers

1. 14

2. 324

3. 65

4. 24: 1, 2, 3, 4, 6, 8, 12, 24
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
17: 1, 17

5. 200 cm^2

Teaching Suggestions

• In this chapter, we have stressed mathematical processes and the importance of selecting the appropriate strategies. The final step in any solution is reflecting. Students need to be able to reflect on their final answer as well as on their strategy and make adjustments if necessary.

• Have students work through Investigate A with a partner. It is important that they be given a chance to reflect on their results and to explain them. (20 min)

• Have students work individually and then compare their strategies with a partner. You may wish to use **BLM A11 Group Work Assessment Recording Sheet** and/or **BLM A12 Group Work Assessment General Scoring Rubric** to assist you in assessing your students. (10 min)

• Encourage students to reflect on and evaluate their strategies.

• Assign and discuss the questions in Communicate Your Understanding. (20 min)

• Assign and take up Practise questions 1 and 2. Have students reflect on their solutions. (10 min)

• You may wish to use **BLM 1.7.1 Practice: Focus on Reflecting** for remediation or extra practice.

- Some students may give minimal reflection on their solutions.

R_x Have students with different strategies work in groups to discuss their solutions. Provide exemplars of student reflection.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to repeat the Möbius strip investigation using a full twist, then a twist and a half, etc., before taping the ends of the strip of paper, about 4 cm wide and 28 cm long, together.

Investigate Answers (page 34)

A. 2. Answers will vary. The line continued all the way around, dividing the strip in half and eventually meeting back at the starting point. It went on both sides of the original paper.

3. Answers will vary. A sample answer: I would end up with two thinner Möbius strips.

4. Answers will vary. A sample answer: When cut along the centre, the Möbius strip became one larger Möbius strip with the strip completely twisted twice.

6. After cutting along the two lines, the Möbius strip turned into one larger Möbius strip like before, but there was a smaller Möbius strip going through it. The smaller strip was the centre of the original Möbius strip.

B. 1. Answers will vary. Sample answers: Write out the multiples of 7 until you go past 100. Or divide 100 by 7 and that is how many times 7 goes into 100.

2. The multiples of 7 are:

7	14	21	28	35	42	49	56	63	70	77	84	91	98

There are 14 in all. Notice that $100 \div 7$ is 14 R2.

3. Answers will vary. A sample answer: Dividing 100 by 7 was a faster solution, and would be more effective for larger problems.

Communicate Your Understanding Responses (page 35)

C1. a) He might try $5 + 7 + 9 + 11 + 13$ because he needs more than 35.
b) No. There are many odd numbers and the question does not specify that they should be consecutive odd numbers.

C2. If it is a right triangle, then the Pythagorean theorem will work, and $3^2 + 5^2$ should equal 7^2. Since $3^2 + 5^2 = 34$ and not 49, it is not a right triangle.

C3. a) The attendant should have done some mental arithmetic to estimate the sum of the two prices.
b) The attendant might have entered the wrong item into the cash register, or entered too many items.

Practise

These questions are straightforward. Encourage students to reflect on their answers and strategies.

Connect and Apply

For all of these questions, encourage a class discussion on methods and final answers.

Question 6 allows students to use a variety of strategies. Have students compare strategies with their classmates.

Question 7 stresses verification of the student's rule.

Question 10 stresses error analysis, a skill students need when reflecting on solutions. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Extend

Question 11 is a Fermi problem that may require some research, or educated guesses, into pizza sales.

The solutions to the magic squares in question 12 are relatively straightforward. The discussion of strategies is the key.

Literacy Connections

Think It Through

Ask students, *What is your style for solving a problem?* Encourage students to discuss their style with a classmate. Explain that writing as if in a diary may help them reflect on the problem solving they have done.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 10
Typical	1–10
Extension	11, 12

Chapter 1 Review

Ongoing Assessment

Have students create a summary
sheet of the skills in the chapter. You
may wish to use **BLM A20 Learning
Skills Checklist** to assist you in
assessing your students.

 • Upon completing the Chapter
   Review, have students answer the
   following questions:

   ▪ *Did you work with yourself or
     with others?*

   ▪ *What questions did you find
     easy? Difficult? Why?*

   ▪ *How often did you have to check
     the related section in the text
     for Examples or Key Concepts?
     For which questions was this
     necessary?*

 • You may wish to use **BLM A16
   My Progress as a Mathematician**
   and/or **BLM A18 My Progress as
   a Problem Solver** to assist you in
   assessing your students.

Using the Chapter Review

Each question in the Chapter Review reviews a different skill and/or
concept. Have students work independently to complete the Chapter
Review, then with partners to compare solutions. Provide an opportunity for
the students to discuss any questions containing strategies or questions with
features they find difficult. You may wish to use **BLM A14 Self-Assessment
Recording Sheet** and/or **BLM A15 Self-Assessment Checklist** to assist you in
assessing your students. You may wish to use **BLM 1.CR.1 Chapter Review**.

   Although other Chapter Reviews later in the text refer to section
numbers, Chapter 1 Review does not. The Chapter Review questions
are meant to consolidate the mathematical process skills developed in
the chapter. After completing each question, have students identify the
mathematical processes they used and reflect on those strategies.
After they complete Chapter 1 Review, encourage students to make a list of
questions that caused them difficulty, and include the related sections and
teaching examples.