1.6 Focus on Reasoning and Proving

Strand:
Number Sense and Algebra

Student Text Pages
29 to 33

Suggested Timing
80 min

Related Resources
BLM A11 Group Work Assessment Recording Sheet
BLM A12 Group Work Assessment General Scoring Rubric
BLM 1.6.1 Practice: Focus on Reasoning and Proving
BLM 1.6.2 Chess Board
BLM A10 Observation General Scoring Rubric
BLM 1.6.3 Sudoku

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☐ Representing
- ☐ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.01 simplify numerical expressions involving integers and rational numbers, with and without the use of technology;

NA2.02 solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion.

Warm-Up

Review the meaning of powers (e.g., $3^4 = 3 \times 3 \times 3 \times 3$) and the order of operations (BEDMAS). Evaluate:

a) 5^2 **b)** $(-5)^2$ **c)** 5^3
d) $(-5)^3$ **e)** $7^2 - 8^2$ **f)** $(7 - 8)^2$
g) $9 + 3 \times 2$ **h)** $5 - 9^2 \div 3$

Warm-Up Answers

a) 25 **b)** 25 **c)** 125
d) −125 **e)** −15 **f)** 1
g) 15 **h)** −22

Teaching Suggestions

- This section concentrates on using reasoning skills. Students need to develop a sense of what works well and when. For example, a Guess and Check method involves an educated guess, based on their skills and knowledge. Sudoku puzzles also help develop reasoning skills. Students need to be able to determine whether an answer is reasonable, for example, whether it is too big or whether a negative value makes sense. Students need to develop methods of proving or verifying whether an answer or conjecture is true or false. Providing a counter-example is a useful tool in proving a conjecture false; algebra is often used to prove a conjecture true.
- The Investigate suggests the use of Guess and Check to solve the problem. Discuss with the students what makes a good educated guess. It may help to have students work with a partner. At the conclusion of the investigation, discuss as a class what other strategies would be useful. You may wish to use **BLM A11 Group Work Assessment Recording Sheet** and/or **BLM A12 Group Work Assessment General Scoring Rubric** to assist you in assessing your students. (15 min)
- Discuss the Example, or a similar one, that involves proving a conjecture. Algebra is used here as a reasoning tool. Although algebra is used, much of the reasoning comes in the form of verbal communication. (10 min)
- Review the meaning of the word *conjecture* and the importance of developing a well-structured proof when asked to prove a conjecture. Verify that it is true with an example, or false with a counter-example. Explain to students that if the first step is true, then proceed to the proof, which involves communicating with opening statements, proceeding through clear steps that prove the conjecture for all examples, and stating a final conclusion that refers back to the original conjecture. (5 min)
- Assign and discuss the questions in Communicate Your Understanding

Common Errors

- Level 1 students may become discouraged, not due to any common errors, but rather due to lack of success in their reasoning abilities. They may have difficulties seeing the big picture or even the results of a particular guess in the Guess-and-Check method.

R_x Offer plenty of teacher support or support from their peers. Many students are visual or kinesthetic learners. Encourage them to draw a diagram or to act out the problem. Scaffolding the steps may also help in solving problems.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Perceptual—Encourage students to work with a reading buddy to understand the questions in this section.

before assigning the exercise questions. (5 min)
- Assign and take up Practise questions 1 and 3. (10 min)
- Use **BLM 1.6.1 Practice: Focus on Reasoning and Proving** for remediation or extra practice.

Investigate Answers (page 29)

1. A vending machine containing 90 coins has 3 times as many quarters as dimes, and twice as many nickels as dimes. How much money is in the machine?
2. The number of nickels must be even and the number of quarters must be divisible by 3.
3. There are 15 dimes, 45 quarters, and 30 nickels.
4. There is only one possible solution that fits all the clues.
5. The quickest strategy is to write equations to fit all the clues and to solve for all of the variables.

Communicate Your Understanding Responses (page 31)

C1. a) Jay used Guess and Check, and Patterning.
b) Answers will vary.

C2. a) 48°; 37°; 50°; 81° **b)** Each angle is less than 90°.
c) No. These triangles are examples of isosceles triangles that support the statement but the statement is not proved for all isosceles triangles.

Practise

Questions 1 and 2 are generally best completed through Guess and Check. Students may use other methods, though. Most students will answer question 3 using algebra. Question 4 is a good chance for students to provide a verbal argument supporting the conjecture.

Connect and Apply

All of these questions are accessible to students working at levels 3 and 4. Students working at level 2 will reach some level of success. Students working at level 1 or below will probably experience great difficulties without teacher support. Many questions, in particular, questions 6, 7, and 11, provide good practice of numeracy skills. Question 8 is an important exercise in finding counter-examples. You may wish to supplement with additional similar examples. Use **BLM 1.6.2 Chess Board** for question 9.

Extend

The Sudoku puzzle in question 14 is fairly challenging for grade 9 students. Use **BLM 1.6.3 Sudoku** for question 14. Question 16 provides a good exercise in interpreting patterns, and where simple extension of the pattern takes too long. Good reasoning skills are key for all three Extend questions. You may wish to use **BLM A10 Observation General Scoring Rubric** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 8
Typical	1–13
Extension	14–16