3.1 Build Algebraic Models Using Concrete Materials

Strand:
Number Sense and Algebra

Student Text Pages
104 to 109

Suggested Timing
80 min

Tools
- algebra tiles
- linking cubes

Technology Tools
- *The Geometer's Sketchpad*®
- computers

Related Resources
BLM 3.1.1 Practice: Build Algebraic Models Using Concrete Materials
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM A7 Thinking General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Operating with Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8^3 by using a calculator]);

NA1.02 describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by $(x)(x)$ or x^2; volume, which is three dimensional, can be represented by $(x)(x)(x)$; $(x^2)(x)$, or x^3];

Manipulating Expressions and Solving Equations

NA2.03 relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

Warm-Up

1. How can you find the area of a square?
2. Find the area, including proper units, of a square having a side length of:
   a) 3 cm **b)** 5 mm
3. How can you find the volume of a cube?
4. Find the volume, including proper units, of a cube having a side length of:
   a) 2 m **b)** 4 cm

Warm-Up Answers

1. Multiply the length by the width, or square one dimension.
2. **a)** 9 cm^2 **b)** 25 cm^2
3. Multiply the height, width, and length, or cube one dimension.
4. **a)** 8 m^3 **b)** 64 cm^3

Teaching Suggestions

- Have students complete the Warm-Up, working independently. (5–10 min)
- Have students work in pairs or small groups for the Investigate. (30–40 min)
- To prepare for the Investigate, hand out the algebra tiles, and let students examine them briefly before beginning the Investigate, especially if this is their first encounter with algebra tiles.
- Conduct a class discussion. Some prompting questions to get students familiar with the tiles include:
  - *What shapes do the tiles come in?*
  - *Are there any relationships between the dimensions of different tiles? What are they?*
  - *How many colours do the tiles come in? Why might this be so?*

Common Errors

- Some students may interpret the x-tile (or x^2-tile) as having a fixed length of five units.

R_x Remind students that the x-tile represents a variable length. Use virtual tiles in a software program, such as *The Geometer's Sketchpad*®, to demonstrate. Virtual tiles have an advantage in that you can click and drag their lengths to change them (you can not click and drag the length of a virtual unit tile, however). This can provide a good opportunity to discuss the distinction between a variable quantity and a constant quantity. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3 or BLM T5 *The Geometer's Sketchpad*® 4** if you are using virtual tiles.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

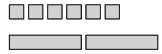
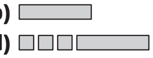
- Have students work in pairs or small groups on Part A of the investigation. Then, have the class discuss how unit tiles and x-tiles can be used to connect visual and algebraic representations of length.
- Note that most tile sets are such that the length of the x-tile is equal to the length of five unit tiles. It is important to realize that the x-tile represents a *variable* length, and generally is not equal to five. This is simply a limitation of the physical model.
- Have students move on to Part B, and discuss how x^2-tiles can be used to relate visual and algebraic representations of area. Clarify that these tiles are intended to represent variable area quantities (not 5 × 5 areas). However, the relationship between the length of an x-tile and the side length of an x^2-tile is important.
- To extend into dimensional relationships of volume, have students continue with Part C, using linking cubes. It is important for students to recognize through these investigations why units of measure depend on the dimensional factors that comprise them, for example,

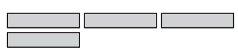
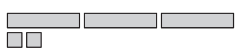
$$\begin{aligned}
\text{length} &= 5\text{ cm}\\
\text{area} &= 5\text{ cm} \times 5\text{ cm}\\
&= 5 \times 5\text{ cm} \times \text{cm}\\
&= 25\text{ cm}^2\\
\text{volume} &= 5\text{ cm} \times 5\text{ cm} \times 5\text{ cm}\\
&= 5 \times 5 \times 5\text{ cm} \times \text{cm} \times \text{cm}\\
&= 125\text{ cm}^3
\end{aligned}$$

- Ensure that students understand the Key Concepts before assigning the Communicate Your Understanding questions. (5–10 min)
- Assign the Practise as independent work. (balance of period)
- Use **BLM 3.1.1 Practice: Build Algebraic Models Using Concrete Materials** if additional practice or remediation is required.

Investigate Answers (pages 104–106)

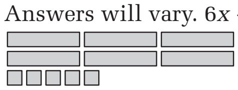
A.

1. a) 
b) 
c) 
d) 

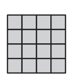
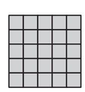
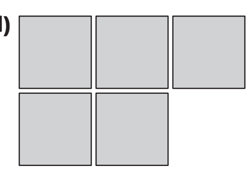
2. a) 
b) 
c) 
d) 

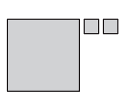
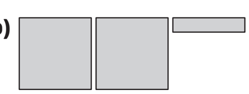
3. a) 4 **b)** $x + 1$ **c)** $3x$ **d)** $5x + 3$

4. Answers will vary. $6x + 5$



B.

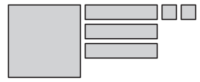
5. a) 
b) 
c) 
d) 

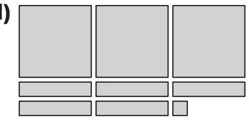
6. a) 
b) 

Accommodations

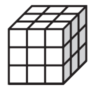
Gifted and Enrichment—Challenge students to research famous mathematicians in the library or on the Internet who have researched exponents. Have students present their findings to the class.

Spatial—Encourage students to use algebra tiles and linking cubes to give a visual representation of the questions in this section.

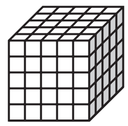
c) 

d) 

C.

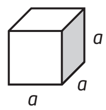
7. a) 

b) 27 cm^3
c) 3^3 cm^3

8. a) 

b) 125 cm^3
c) 5^3 cm^3

9. Answers will vary.

10. a) 

b) x^2 **c)** x^3

11. Answers will vary. For example, linking cubes and algebra tiles can be used to build algebra models. Unit tiles can be used to represent constants, x-tiles to represent unknown lengths, and cubes of various sizes can be used to represent volume.

Communicate Your Understanding Responses (page 107)

C1. a) length: 1 unit, width: 1 unit
b) length: x, width: 1 unit
c) length: x, width: x

C2. a) The width of an x tile is same as the length and width of a unit tile.
b) The length and width of an x^2 tile is same as the length of an x tile.

C3. a) Answers will vary.
b) Answers will vary.
c) Answers will vary.

C4. Answers will vary.
a) By lining up a series of tiles, students may better understand the concept of length.
b) By arranging tiles in a pattern and then counting them, students may better understand the concept of area.
c) By making a three-dimensional figure using linking cubes and counting them, students may better understand the concept of area.

Practise

These initial questions provide a good opportunity to lay the groundwork for future discussions of coefficients and variables. For example, to model $x^2 + 3x$, students need to use one x^2-tile and three x-tiles. This will be helpful later in understanding why the "invisible" coefficient of the squared term is one, and not zero.

Connect and Apply

For question 5, have some students bring their tile models to the front, and display them on the overhead. Ask the class to identify each expression.

Questions 6 to 8 require students to identify more clearly the relationship between volume and area. It may be counterintuitive to some that you can discuss area as it relates to one or more faces of a three-dimensional object. Cubes, three-dimensional models, and drawn or computer-generated nets are useful tools for demonstrating such relationships. You may wish to use **BLM A7 Thinking General Scoring Rubric** to assist you in assessing your students.

Extend

Students should be familiar building area models using multiplication from their work in elementary school, for example, modelling $2 \times 3 = 6$ using a rectangular array of tiles. The new skill here is applying variable length measures within the dimensions. This is a useful skill as area models using tiles are applied later in the chapter when investigating the distributive property, and in grade 10 when factoring polynomials.

Literacy Connections

Variables

Explain to students that, in mathematics, we often use the letters x, y, and z to represent variables, put in place of values that can change or vary. One common use of these variables is as the coordinates of points on a Cartesian plane. For example, the point (2, 3) is a particular point when x is 2 and y is 3. The point (x, y) represents many different points on a Cartesian plane, depending on the values chosen for x and y.

Expressions and Equations

Explain to students that when we use the word *expression* in English, we are referring to a short part of a sentence. In mathematics, however, an expression refers to a series of numbers and letters joined by addition and/or subtraction—a small part of an equation.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 6 a), b)
Typical	1–8
Extension	9–14