3.2 Work With Exponents

Strand:
Number Sense and Algebra

Student Text Pages
110 to 118

Suggested Timing
80 min

Tools
- grid paper

Related Resources
BLM A1 Assessment Recording Sheet
BLM 3.2.1 Practice: Work With Exponents
BLM G15 Placemat
BLM G10 Grid Paper

Technology
- graphing calculator
- scientific calculator

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Operating with Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8^3 by using a calculator]);

NA1.02 describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by $(x)(x)$ or x^2; volume, which is three dimensional, can be represented by $(x)(x)(x)$, $(x^2)(x)$, or x^3];

Manipulating Expressions and Solving Equations

NA2.03 relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

Link to Get Ready

Students will multiply integers and rational numbers in this section. Have students complete questions 3, 5, and 6 of the Get Ready before beginning this section.

Warm-Up

1. Evaluate.

a) $(-3)(-3)$ **b)** $(-2)(-2)(-2)$

2. Write in expanded form, then evaluate.

a) 5^2 **b)** 2^4 **c)** 3^3 **d)** 10^5

3. Evaluate.

a) $\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$ **b)** $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

Warm-Up Answers

1. a) 9 **b)** -8

2. a) $5 \times 5 = 25$ **b)** $2 \times 2 \times 2 \times 2 = 16$

c) $3 \times 3 \times 3 = 27$ **d)** $10 \times 10 \times 10 \times 10 \times 10 = 10\,000$

3. a) $\frac{9}{16}$ **b)** $\frac{1}{8}$

Teaching Suggestions

- Assign the Warm-Up as independent work. (5 min)
- The cartoon and the Investigate that follows are designed to illustrate exponential growth and how it can occur. Have students complete the activity in pairs or small groups. (10–15 min)
- Then, discuss other areas in which such growth can occur, such as, population growth, investments, contagious diseases, rumours, and e-mail chain letters.

Common Errors

- Some students may think that the base is multiplied by the exponent, for example, $2^3 = 6$.

R_x Students associate powers with multiplication, which is why this error occurs. Have students practise writing out powers in expanded form, for example, $2^3 = 2 \times 2 \times 2$, before evaluating them. Once students have demonstrated clear understanding, allow them to skip this step.

- Some students may improperly interpret and evaluate powers having negative bases.

R_x Use several comparative examples, such as those in Example 1b) and c) to reinforce proper form for identifying whether a negative is part of the base of a power. Have students practise writing out such questions in expanded form before evaluating so that they can clearly see the number of negative factors involved.

Ongoing Assessment

The Chapter Problem question 13 can be used as an assessment tool. You may wish to use **BLM A1 Assessment Recording Sheet**.

- Discuss limitations of the model in the cartoon, for example, some friends may hear about the restaurant more than once, some may tell more or less than two friends, not everyone will try the restaurant, etc.
- Discuss the Examples. (10–15 min)
- Example 1 illustrates most of the mechanical skills that students need in order to evaluate powers with rational bases. Remind students to pay special attention when negative bases are present and in the placement of brackets (see parts b, c, and d).
- Example 2 uses a contextual problem to draw connections between powers, algebra, and measurement. Prior knowledge required includes the formulas for finding the volume of a cube and the volume of a cylinder.
- When the exponents are small, encourage students to use mental math skills to evaluate the expression. When the exponents are large or the base contains a decimal number, encourage students to use a calculator.
- Discuss the Communicate Your Understanding questions. (5 min)
- Have students work independently to complete the questions. (balance of period)
- Use **BLM 3.2.1 Practice: Work With Exponents** for extra practice if needed.

Investigate Answers (page 111)

1.

Day	New Customers	Expanded Form	Power
1	2	2	2^1
2	4	2×2	2^2
3	8	$2 \times 2 \times 2$	2^3
4	16	$2 \times 2 \times 2 \times 2$	2^4

2. $2^7 = 128$. Each day, the number of new customers increases by a factor of 2. Therefore, based upon this model, on day 7, Barney should expect 2^7 new customers.

3. $2^{14} = 16\,384$. Using the model, Barney should expect 16 384 new customers on day 14. This is not realistic. The model has limitations; the same pattern cannot continue indefinitely because there is no guarantee that each customer that visits Barney's will also bring two new customers the next day, each of whom in turn will bring another two customers the day after, and so on. The pattern will slow down.

4. Answers will vary. Possible answer: In a school of 500 students, all students will know about Barney's in 9 days. $2^9 = 512$

5. a) 3^2, or 9 **b)** 3^4, or 81
c) For the school with 500 students, all students will know in 6 days. $3^6 = 729$

6. Answers will vary. Exponents can be used to show some growth patterns, when the pattern is changing by a multiple of the same factor.

Communicate Your Understanding Responses (page 113)

C1. a) base: 3, exponent: 4
b) base: $\frac{1}{2}$, exponent: 4
c) base: -2, exponent: 6
d) base: 2, exponent: 6
e) base: 1.2, exponent: 2

C2. a) 81, $\frac{1}{16}$, 64, -64, 1.44
b) The answers in part c) and d) are different due to the presence of brackets, which changes the order in which the operations are performed.

C3. The next step is to square the value of r.

C4. Answers may vary. I would use a calculator to evaluate the expressions in parts c) and e) because they involve higher powers and, in part c) a decimal base.

Accommodations

Visual—Provide students with photocopies of the charts to be completed in this section instead of copying the information in their notebooks.

Perceptual—Allow students to use patterning to understand the exponents of negative numbers.

Language—Encourage students to use their dictionaries and research the new words in this section on the Internet.

ESL—Encourage students to use their dictionaries or translators to understand the meanings of the new words in this section.

Student Success

Working in groups of four, have students complete a placemat activity. Instruct each student to list examples of powers or exponents used in formulas or real-life situations in their area of the placemat. Then, have each group place a summary list of examples in the centre of the placemat. You may wish to use **BLM G15 Placemat** to support this activity.

Practise

Use questions 3 to 5 to emphasize proper form when writing powers. In particular, negative bases should be surrounded by brackets in both power form and expanded form. A single negative sign in front of a power (see question 5c), for example), means that the base of the power is positive, and that the entire expression is negative. Use of a calculator is recommended for evaluating powers involving decimals (see question 3b), for example).

Review BEDMAS for question 6.

For question 7, remind students to use brackets when substituting into expressions.

Connect and Apply

Questions 9 to 10 are good examples of real non-linear relations. Discuss the implications of such growth patterns as they compare to linear growth.

Students interested in music may appreciate the context in question 11.

For question 14, note to students that exponential decay is the counterpart to exponential growth. Have students explore this type of curve on a graphing calculator and see what happens as the amount approaches (but never reaches) zero. This type of thinking will be expanded upon in future mathematics courses when students explore asymptotes and limits. You may wish to use **BLM G10 Grid Paper** to support questions 9 and 14.

Extend

Question 15 mentions **scientific notation**, a concept only explicitly mentioned in the science curriculum. However, students may benefit from some exposure to it in this section to consolidate the skills and appreciate how the two disciplines of study are related. Special attention to entering and interpreting numbers in scientific notation using a scientific calculator can be quite helpful, particularly if a surprising result appears in this form. Students should have some basic awareness of how to work with such values.

Literacy Connections

Literal and Numerical Coefficients

Explain to students that a *literal coefficient* is a letter, and a *numerical coefficient* is a number. Remind students that the terms used in mathematics make a lot of sense when they examine what they actually mean and where the words come from. Use this discussion to lead into a discussion on prefixes.

Prefixes

Explain that many of the prefixes used in mathematics also make a lot of sense. For example, a *monomial* has only one term. Ask students to think of other examples in real life where the *mono-* prefix is used. (For example, monocle, monologue, monolingual, monochromatic, monochrome, monogamy, monolith, etc.) Tell students that binomials and trinomials are special polynomials and that they are used so often that they are referred to by their specific names. The word *polynomial* refers to the whole group of expressions with two or more terms. Ask students to think of other examples where we use the prefix *bi* and *tri*. (For example, bicycle, bicarbonate, biceps, bicultural, bicuspid, etc., and tricycle, triangle, triad, triangulate, trilateral, triceps, etc.)

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 6a), c), e), 7a), b), c), 8
Typical	1–12, 14
Extension	15–20