3.3 Discover the Exponent Laws

Strand:
Number Sense and Algebra

Student Text Pages
119 to 129

Suggested Timing
160 min

Tools
- calculators

Related Resources
BLM 3.3.1 Practice: Discover the Exponent Laws
BLM A8 Application General Scoring Rubric
BLM 3.3.2 Achievement Check Rubric
BLM 3.3.3 Student Success: Exponent Law for Multiplication of Powers
BLM 3.3.4 Student Success: Exponent Law for Division of Powers
BLM 3.3.5 Student Success: Exponent Law for Powers of Powers

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Operating with Exponents

NA1.03 derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;

Manipulating Expressions and Solving Equations

NA1.04 extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.

Link to Get Ready

Most of the concepts in the Get Ready section are needed here. Ensure that students have completed all of the Get Ready questions prior to working on this section.

Warm-Up

1. Identify the base and exponent of each power.
a) 5^3 **b)** 10^5 **c)** $(-2)^4$ **d)** $(-1)^7$

2. Write each power in question 1 in expanded form.

3. Evaluate each power in question 1.

Warm-Up Answers

1. a) base: 5, exponent: 3
b) base: 10, exponent: 5
c) base: −2, exponent: 4
d) base: −1, exponent: 7

2. a) $5 \times 5 \times 5$
b) $10 \times 10 \times 10 \times 10 \times 10$
c) $-2 \times -2 \times -2 \times -2$
d) $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1$

3. a) 125 **b)** 100 000 **c)** 16 **d)** −1

Teaching Suggestions

- Consider teaching this lesson over two periods.

Day 1: Product and Quotient Rule

- Assign the Warm-Up. (5 min)
- Have students complete Investigate A Parts A, B, and C in pairs or small groups. (30–35 min)
- Follow Examples 1 and 2 with a discussion on the product and quotient rules. (10–15 min)
- Assign question C1 of Communicate Your Understanding. Support this activity with a class discussion. (5 min)
- Assign questions 1 to 4 and 11 to 13 as independent work. (balance of period)

Common Errors

- Some students may change the base when applying exponent laws, for example, $2^3 \times 2^4 = 4^7$.

Rx Use expanded form (as in the Investigates) to illustrate to students why the base does not change. Have students verify with a calculator or by hand that the two expressions above are not equal, but that $2^3 \times 2^4 = 2^7$.

- Some students may confuse the product rule and the power of a power rule, for example, $3^2 \times 3^4 = 3^8$ and $(4^2)^3 = 4^5$.

Rx Use expanded form (as in the Investigates) to justify and reinforce the proper exponent laws to apply for each type of situation. Have students verify equality/non-equality of results with a calculator or by hand by evaluating initial and final expressions.

Ongoing Assessment

- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 3.3.2 Achievement Check Rubric** to assist you in assessing your students.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills (see **BLM 3.3.2 Achievement Check Rubric** for levels.)

- Depending on how the class is progressing on the first day, you may wish to move Quotient of Powers to the second day.

Day 2: Power of a Power and Simplifying Algebraic Expressions

- Have students work on Investigate B with partners or in small groups. (5–10 min)
- Work through the remaining examples as a class. (15–20 min)
- Assign Communicate Your Understanding questions C2 and C3, and follow up with a class discussion. (5 min)
- Assign questions 6 to 13 as independent work. (balance of period)
- Use **BLM 3.3.1 Practice: Discover the Exponent Laws** if extra practice is required.

Investigate Answers (page 119)

A.

Part A

1. 1 km = 1000 m; 10^3 m

2.

Unit	Number of these in 1 m	Power of 10
decimetre	10	10^1
centimetre	100	10^2
millimetre	1000	10^3

3. a) 100 000 cm in 1 km **b)** $10^2 \times 10^3 = 10^5$

4. a) 1 000 000 mm = 1 km **b)** $10^1 \times 10^2 \times 10^3 = 10^6$

5. Answers will vary. The exponents increase by one.

Part B

6.

Product	Expanded Form	Single Power
$3^2 \times 3^4$	$(3 \times 3) \times (3 \times 3 \times 3 \times 3)$	3^6
$4^3 \times 4^3$	$(4 \times 4 \times 4) \times (4 \times 4 \times 4)$	4^6
$6^4 \times 6^1$	$(6 \times 6 \times 6 \times 6) \times (6)$	6^5
$2^4 \times 2^2 \times 2^3$	$(2 \times 2 \times 2 \times 2) \times (2 \times 2) \times (2 \times 2 \times 2)$	2^9
$k^3 \times k^5$	$(k \times k \times k) \times (k \times k \times k \times k \times k)$	k^8

7. The bases of the powers in each product are the same.

8. The sum of the exponents in the first column is equal to the exponent in the last column.

9. You can write a product of powers with the same base by showing a sum in the exponent.

10. $x^a \times x^b = x^{a+b}$

Part C

11.

Quotient	Expanded Form	Single Power
$5^5 \div 5^3$	$\frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5}}$	5^2
$7^4 \div 7^1$	$\frac{\cancel{7} \times 7 \times 7 \times 7}{\cancel{7}}$	7^3
$10^6 \div 10^4$	$\frac{10 \times 10 \times \cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10} \times \cancel{10} \times \cancel{10}}$	10^2
$2^7 \div 2^6$	$\frac{2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}$	2^1
$p^8 \div p^5$	$\frac{p \times p \times p \times \cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p}}{\cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p} \times \cancel{p}}$	p^3
$A^4 \div A^2$	$\frac{\cancel{A} \times \cancel{A} \times A \times A}{\cancel{A} \times \cancel{A}}$	A^2

Accommodations

Gifted and Enrichment—Challenge students to investigate patterns in an exponent table and the Binomial Expansion (Pascal's Triangle).

Memory—Encourage students to use cue cards to memorize the exponent laws.

Student Success

Have students start off in "home triads" (groups of 3). Assign an exponent law to each home triad, and have them work on the Investigates in these triads. Then, split up the home triads and have one expert for each exponent law per new group. Have the new groups work on the Examples. Finally, have students return to their home triads, and teach the new exponent law they learned to the others in the triad. You may wish to use **BLM 3.3.3 Student Success: Exponent Law for Multiplication of Powers, BLM 3.3.4 Student Success: Exponent Law for Division of Powers, and BLM 3.3.5 Student Success: Exponent Law for Powers of Powers** to support this activity.

12. The bases of the powers in each quotient are the same.

13. If you subtract the exponents in the first column you get the exponent of the single power in the last column.

14. You can write a quotient of two powers with the same base using one power by showing a subtraction symbol between the exponents. For example, $4^7 \div 4^5 = 4^{7-5} = 4^2$.

15. $x^a \div x^b = x^{a-b}$

B.

1.

Power of a Power	Expanded Form	Single Power
$(2^2)^3$	$(2^2) \times (2^2) \times (2^2)$ $= (2 \times 2) \times (2 \times 2) \times (2 \times 2)$	2^6
$(5^3)^4$	$(5^3) \times (5^3) \times (5^3) \times (5^3)$ $= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times$ $(5 \times 5 \times 5)$	5^{12}
$(10^4)^2$	$(10^4) \times (10^4)$ $= (10 \times 10 \times 10 \times 10) \times (10 \times 10 \times 10 \times 10)$	10^8
$(3^3)^3$	$(3^3) \times (3^3) \times (3^3)$ $= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \times (3 \times 3 \times 3)$	3^9

2. The exponent in the last column is equal to the product of the exponents in the first column.

3. You can write the power of a power as a single power by keeping the same base with exponent equal to the product of the two exponents. For example,
$(4^3)^2 = 4^{3 \times 2}$
$= 4^6$

4. $(x^a)^b = x^{ab}$

Communicate Your Understanding Responses (page 126)

C1. a) quotient rule **b)** power rule
c) no law applies, the bases are different **d)** product rule
e) quotient rule **f)** no law applies, the bases are different

C2. Answers will vary. For example:
a) $y^5 \times y^4 = y^9$ **b)** $(y^5)^4 = y^{20}$ **c)** $\frac{y^5}{y^2} = y^3$

C3. a) Substitute into the simplified expression because it is easier to solve. It requires less arithmetic.
b) 6

Practise

When applying exponent laws, it is important for students to realize that:
- the base of each power must be identical
- the base does not change

Watch for these key things as students begin to apply exponent laws in the early practise questions. Ensure that students put brackets around bases containing negative signs and/or fractions. Omitting the brackets changes the meaning of the expression, for example,

$$\left(\frac{2}{5}\right)^6 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$
$$= \frac{64}{15625}$$
$$\text{but } \frac{2}{5}^6 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{5}$$
$$= \frac{64}{5}$$

$$
\begin{aligned}
(-3)^6 &= (-3)(-3)(-3)(-3)(-3)(-3) \\
&= 729 \\
\text{but } -3^6 &= -3 \times 3 \times 3 \times 3 \times 3 \times 3 \\
&= -729
\end{aligned}
$$

Students may be confused by the use of nested brackets (for example, question 6b), c), and d)). Discuss the need for these: the round brackets are used to simply identify the base of the power, and the square brackets denote a power of that power.

Connect and Apply

Questions 10 and 14 are useful in illustrating how application of exponent laws can reduce the complexity level of an expression, and the calculations involved when substituting. Question 11 draws connections between measurement and application of exponent laws. Students will need to recall the formula for the volume of a prism. You may wish to use **BLM A8 Application General Scoring Rubric** for questions 11 to assist you in assessing your students.

Questions 12 and 13 draw connections between probability and exponents. Have students draw a tree diagram to illustrate why the probability of flipping heads three times in a row is $\left(\frac{1}{2}\right)^3$.

Achievement Check Answers (page 128)

14. a) Substitute $m = 4$, $n = -3$, which gives $\frac{(-3)(16)(-27)(64)(9)}{(4)(16)(81)(3)(4)(-3)} = \frac{-16 \times 9}{9}$
$= -16$

b) $\frac{-3m^2n^3 \times 4m^3n^2}{(2mn^2)^2 \times 3mn} = \frac{(-12)(m^5)(n^5)}{4(m^2)(n^4)(3)(m)(n)}$
$= \frac{-12(m^5)(n^5)}{12(m^3)(n^5)}$
$= -m^2$

c) Substitute $m = 4$, value is $(-4 \times 4) = -16$

d) Method b) is the better solution as it involves substituting in a simpler expression.

e) Josie forgot to add the exponent 3 on the m in the numerator of the expression and she forgot to add the exponent 2 on the m inside brackets in the denominator. Since Josie's expression reduces to the same value as the original, i.e., $-m^2$, the solutions will be the same.

Extend

For questions 15 to 18, scientific notation is explicitly mentioned in the science curriculum only, however some exposure to application of exponent laws to numbers in scientific notation may be beneficial. Assign these questions only if students had a chance to do question 15 on introduction to scientific notation in Section 3.2.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–7, 8, 10, 11
Typical	1, 14
Extension	15–20