3.7 The Distributive Property

Strand:
Number Sense and Algebra

Student Text Pages
160 to 169

Suggested Timing
80–160 min

Tools
- algebra tiles

Related Resources
BLM 3.7.1 Investigate Table
BLM 3.7.2 Practice: The Distributive Property
BLM 3.7.3 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☐ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.05 multiply a polynomial by a monomial involving the same variable [e.g., $2x(x + 4)$, $2x^2(3x^2 - 2x + 1)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil);

NA2.06 expand and simplify polynomial expressions involving one variable [e.g., $2x(4x + 1) - 3x(x + 2)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of the Get Ready questions prior to working on this section.

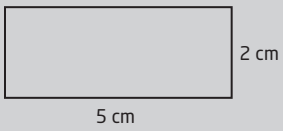
Warm-Up

1. Draw a rectangle having the width and length given.
a) 2 cm × 5 cm **b)** 3 cm × 4 cm

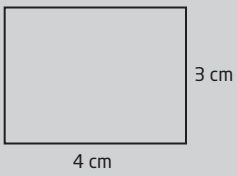
2. Find the area of each rectangle in #1.

Warm-Up Answers

1. a)



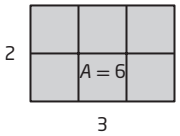
b)



2. a) 10 cm^2
b) 12 cm^2

Teaching Suggestions

- Assign the Warm-Up and have students work independently. (5 min)
- For the Investigate, have students work with a partner or in small groups. (10–15 min) Distribute copies of **BLM 3.7.1 Investigate Table.** Students should be familiar with rectangular area models as a means to illustrate multiplication statements, based on their learning experiences in elementary school, e.g.,



  In this example, the two factors of the product, 2 and 3, represent the dimensions of a rectangle, and the resultant, 6, represents the area of the rectangle. Algebra tiles are used in the Investigate to extend this geometric concept to the multiplication of algebraic expressions. The distributive property is discovered by examining the initial factors and the algebraic resultant.
- After having students work through the Investigate, debrief the results as a class. Then, use the nickel and dime example following the Investigate to consolidate the idea.

Common Errors

- Some students may distribute incompletely, for example, $2(x + 3) = 2x + 3$.

R_x Have students work with algebra tiles using simple examples. Have students build a rectangular area model and examine the area.

- Some students may distribute negative terms improperly, particularly in the middle of a long expression, for example, $2(x + y) - 3(x - y) = 2x + 2y - 3x - 3y$.

R_x Remind students that the sign to the left of the coefficient gets distributed with the coefficient. Review integer operations, as needed.

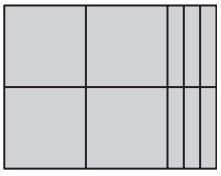
Ongoing Assessment

- Use Achievement Check question 17 to monitor student success. See the Achievement Check Answers and **BLM 3.7.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Use Examples 1 and 2 for a class discussion. (10–15 min)
- These examples illustrate the distributive property. Algebra tiles may be helpful in providing a visual/geometric connection, but use them with caution: Tiles are not recommended when negative coefficients are involved, or when distributing a variable. Use the tiles in the Investigate, and perhaps for Example 1a), and then move directly into symbolic reasoning.
- Assign Communicate Your Understanding questions C1 and C2 and follow up with a class discussion. (5 min)
- Assign questions 1 to 4 and 6 (omit 2c), 6d) and f)). (10 min)
- Debrief answers with a class discussion. (5 min)
- Have students work on Examples 3 and 4, and consolidate with a class discussion. (10–15 min)
- In Example 3, the distributive property is applied in conjunction with other algebraic skills, for example, collecting like terms. The distribution of monomials that consist of both a constant and variable part is also introduced. In part c), students must apply operations involving rational numbers. The ability to recall and apply a disparate array of operational skills, as needed, is critically important for the academic student.
- Nested brackets are introduced in Example 4. Caution students to use careful placement and removal of brackets. When simplifying expressions of this type, encourage students to work systematically from the inside (round brackets) out [square brackets]. Remind students that as round brackets are removed, they can replace the square brackets with round brackets.
- Assign Communicate Your Understanding questions C3 and C4. Follow up with a class discussion. (5 min)
- Assign the remaining questions as independent work. (balance of period)
- Depending on the progress of the class, you may wish to extend this section to 1.5 or 2 periods.
- You may wish to use **BLM 3.7.2 Practice: The Distributive Property** as a remediation tool.

Investigate Answers (page 160)

1.

Rectangle	Width	Length	Area	Equation $w \times \ell = A$
	2	$x + 1$	$2x + 2$	$2 \times (x + 1) = 2x + 2$
	x	$x + 2$	$x^2 + 2x$	$x \times (x + 2) = x^2 + 2x$
	x	$2x + 5$	$2x^2 + 5x$	$x \times (2x + 5) = 2x^2 + 5x$
	$2x$	$2x + 3$	$4x^2 + 6x$	$2x \times (2x + 3) = 4x^2 + 6x$
	x	$x + 3$	$x^2 + 3x$	$x \times (x + 3) = x^2 + 3x$

Accommodations

Memory—Provide students with visual or verbal clues to remember the steps involved when using the distributive property.

2. Take each term in the width and multiply it with each term in the length. Then collect all the like terms and add/subtract them.

3. a) $4x + 12$ **b)** $2x^2 + 7x$
c) $3x^2 + 6x$

4. Answers will vary. For example, $7(2y - 5)$. First multiply $7 \times 2y = 14y$. Then, multiply $7 \times (-5) = -35$. The result is $7(2y - 5) = 14y - 35$.

Communicate Your Understanding Responses (page 165)

C1. The terms in the brackets are unlike terms and therefore cannot be added. The 3 in front of the bracket is understood to multiply the bracket. The distributive property does this.

C2. Distributive property is used to multiply a polynomial by a monomial. To apply the distributive property, multiply each term in the polynomial by the monomial. For example:

$$\begin{aligned} & 4a(3b + 4c + 1) \\ &= 4a(3b) + 4a(4c) + 4a(1) \\ &= 12ab + 16ac + 4a \end{aligned}$$

C3. a) In the second line, Dmitri did not multiply the term outside of the brackets ($3x$) by every term in the brackets ($x + 2$). The second line should read: $3x^2 + 6x$, which is the answer.

b) Verify by substituting a value for x into the original expression and into Dmitri's answer. When simplified, the value of the original expression and the simplified expression will not be the same.

C4. The distributive property, which should be used to simplify this expression, has not been used correctly. -5 should be multiplied by x^2, $-3x$, and 1, and then each product should be simplified. The correct answer is $-5x^2 + 15x - 5$.

C5.

$3(x + 2(x - 1))$	Apply the distributive property to the innermost set of brackets.
$= 3(x + 2x - 2)$	Simplify the expression inside the brackets.
$= 3(3x - 2)$	Apply the distributive property again.
$= 9x - 6$	

Practise

Students may struggle with distributing terms with negative coefficients, for example, in question 3c) and d). Review and consolidate integer operations with students.

Connect and Apply

For question 10, make a connection with Chapter 5, Section 5.2 Partial Variation.

Question 12 makes a connection between measurement and algebra. This question also justifies the need to apply the distributive property to rational terms.

Algebra tiles may be helpful for question 13.

In question 15, have students distribute into all sets of brackets simultaneously. Remind students to take care as to which terms get distributed into which brackets. Ensure they also indicate the signs of the results when distributing terms with negative coefficients.

For question 16, distribution of rational numbers requires careful work operating with both fractions and integers. Remind students to take extra care to clearly show all steps to their solutions, to more easily troubleshoot any problem areas.

Connections to perimeter are made in question 17. To solve and part c), set length = width and solve.

Achievement Check Answers (page 169)

17. a) Perimeter $= x + (2x - 1) + (2x + 3)$
$= 5x + 2$

Perimeter $= 2(2x + 1) + 2x$
$= 6x + 2$

Perimeter $= (2x + 1 + 2x - 1 + 3x + 1 + 20 - 4x)$
$= 3x + 21$

Perimeter $= 3(2w + 3) + 3(3w - 2)$
$= 6w + 9 + 9w - 6$
$= 15w + 3$

b) Perimeter $= 2(2x - 1) + 2(8 - 2x)$
$= 4x - 2 + 16 - 4x$
$= 14.$

So the perimeter is always 14, BUT the smallest value for x is $\frac{1}{2}$, otherwise the length is negative and the largest value is 4, otherwise the width is negative. In both of these cases, the rectangle degenerates into a line.

c) If the rectangle is also a square, then the length = the width, so $2x - 1 = 8 - 2x$ or $4x = 9$ and $x = 2.25$.

Extend

The type of reasoning in question 19 will be applied in grade 10 when students will learn how to multiply two binomials.

Question 20 is a natural extension.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1, 3a), c), e), 4a) c), 5a), c), 6a), c), e), 7a), c), e), 8a), c), 10, 12, 15a), c), e)
Typical	1, 3–16a), c)
Extension	18–21

Use Technology

Computer Algebra Systems

Strand:
Number Sense and Algebra

Student Text Pages
170 to 173

Suggested Timing
80 min

Technology Tools
• TI-89 calculators

Related Resources
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.04 add and subtract polynomials with up to two variables [e.g., $(2x - 5) + (3x + 1)$, $\left(3x^2y + 2xy^2\right) + \left(4x^2y - 6xy^2\right)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil);

Operating with Exponents

NA1.03 derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents;

NA1.04 extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents.

Warm-Up

- Hand out the TI-89 calculators, and go over review some basics with the students, such as how to:
  - turn the calculator on and off
  - get to the Home Screen
  - find certain important keys (operation keys, ENTER, etc.)
  - perform basic operations
  - clean up the home screen

Teaching Suggestions

- Some of the TI-89 functions may not be intuitive to students. Have them work with a partner, but encourage them to take turns entering commands. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this section.
- Some of the layout features and functions are similar to the TI-83+/84+; students with prior experience with them may be able to transfer some working knowledge to the new TI-89 environment. For example, leading negatives have a special key, the keyboard layouts are similar, the second ENTER command brings you backward through previously entered commands, etc.
- Most students will have had some experience working in a Windows-like computer environment. The TI-89 has similar Cut-Copy-Paste commands, which are very useful when entering lengthy repeated expressions.
- For Investigate A, selecting NewProb also clears the variables to which a previous user may have assigned values.
- Investigate C could be assigned as a performance task.
- Investigate D, question 5 would be a very good activity for gifted students. Have students give demonstrations to the class.
- Use discretion in assigning question 5 if you are concerned about students getting lost, or changing modal settings on the calculator.