4.2 Solve Multi-Step Equations

Strand:
Number Sense and Algebra

Student Text Pages
196 to 203

Suggested Timing
80–160 min

Tools
- grid paper

Technology Tools
- Computer Algebra System
- computers
- *The Geometer's Sketchpad®*
- TI-89 calculators
- calculators

Related Resources
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM 4.2.1 Alternate Solution Method
BLM 4.2.2 Construct a Geometric Model With *The Geometer's Sketchpad®*
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.2.3 Practice: Solve Multi-Step Equations
BLM A7 Thinking General Scoring Rubric
BLM G10 Grid Paper

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies).

Link to Get Ready

Students will need to recall how to collect like terms, the distributive property, and geometric relationships for this section. Ensure that students have completed questions 1 to 6 of the Get Ready prior to starting this section.

Warm-Up

1. Solve.

a) $2x - 5 = 7$ **b)** $3 - 4x = -9$

2. Check your answers to question 1 using the left side/right side method.

Warm-Up Answers

1. a) $2x - 5 = 7$
$2x = 7 + 5$
$2x = 12$
$x = 6$

b) $3 - 4x = -9$
$3 + 9 = 4x$
$12 = 4x$
$3 = x$

2. a) LS $= 2(6) - 5$ RS $= 7$
$= 7$

b) LS $= 3 - 4(3)$ RS $= -9$
$= -9$

Teaching Suggestions

- Assign the Warm-Up as independent work. (5 min)
- Discuss the photo of the bridge in preparation for the Investigate, which provides an opportunity to draw connections between measurement and algebra. Triangles will be revisited from a geometric perspective in Example 3. It is important for students to appreciate that algebra is closely connected and applied to other branches of mathematics and used in the real world, and not simply a collection of irrelevant mechanical skills.
- Have students work on the Investigate with a partner or in small groups. (5–10 min)
- Assign the Examples, and follow up with a discussion. (20–25 min)
- Example 1a) introduces the method of collecting variable terms on the left side and constant terms on the right side.
- Example 1b) reinforces this technique with a slightly more complicated equation. Although movement of both terms is performed simultaneously, some students may prefer to perform these one step at a time, i.e., add $5k$ to both sides in the first step, and subtract 7 from both sides in the second step. A CAS method of solution is presented that still requires students to invoke each algebraic step. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.

Common Errors

- Some students may bring terms to the other side of an equation without changing their sign.

R_x Remind students that they must perform the opposite operation in order to remove a term from one side of the equation, and that identical operations must be performed on both sides of an equation for it to hold true.

- Some students may mix up signs when simplifying equations.

R_x This may be due to a lingering weakness in integer operations. Remediate as needed.

- Some students may handle fractions incorrectly, or not reduce fractional answers to lowest terms.

R_x This may be due to a lingering weakness in rational number operations. Remediate as needed.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Example 1c) shows that it does not matter which side of the equation the variables and constants are collected on. Show both methods to illustrate this.
- Example 2 introduces equations involving brackets. You may wish to use **BLM 4.2.1 Alternate Solution Method** to show students an alternate solution strategy.
- For Example 2b), consider using the following strategy to spark interest:
  - Tell students to close their textbooks.
  - Instruct the class to think about each step for a moment.
  - Then, select a volunteer to write the next step of the solution at the board and explain the step.
  - Allow for class discussion of each step. The steps may look slightly different from the solution presented in the textbook.
  - Have all students transcribe the complete solution and write their own explanation of each step.
- Example 3 applies equation solving to a geometric context. You may wish to present an alternate solution to students using an overhead projector. Use **BLM 4.2.2 Construct a Geometric Model With *The Geometer's Sketchpad*®**, and **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Alternatively, break up the lesson:
  - Assign Example 1, and Practise questions 1 to 3.
  - Debrief answers to Practise questions.
  - Have students do Examples 2 and 3, and assign the remaining exercises.
- Based on the progress of your students, you may wish to split the lesson over two days and provide more practice. You may wish to use **BLM 4.2.3 Practice: Solve Multi-Step Equations** for remediation or extra practice.
- Assign the Communicate Your Understanding questions, and consolidate with a class discussion. (5 min)
- Have students complete the balance of the exercises as independent work. (balance of period)

Investigate Answers (page 196)

1.

Solution	Explanation
$5x + 3 = 3x + 7$	Given equation.
$5x + 3 - 3 = 3x + 7 - 3$	Apply opposite operations: subtract 3 from both sides.
$5x = 3x + 4$	Simplify by adding integers.
$5x - 3x = 3x + 4 - 3x$	Apply opposite operations: subtract $3x$ from both sides.
$2x = 4$	Simplify by collecting like terms.
$\frac{2x}{2} = \frac{4}{2}$	Apply opposite operations: divide both sides by 2.
$x = 2$	Simplify.

2.

Solution	Explanation
$5x + 3 = 3x + 7$	Given equation.
$5x + 3 - 7 = 3x + 7 - 7$	Apply opposite operations: subtract 7 from both sides.
$5x - 4 = 3x$	Simplify by adding integers.
$5x - 4 - 5x = 3x - 5x$	Apply opposite operations: subtract $5x$ from both sides.
$-4 = -2x$	Simplify by collecting like terms.
$\frac{-4}{-2} = \frac{-2x}{-2}$	Apply opposite operations: divide both sides by -2.
$2 = x$	Simplify.

Accommodations

Gifted and Enrichment—Challenge students to show more than one solution for solving the equations in this section.

Spatial—Have students use grid paper to organize their solutions when solving equations. You may wish to use **BLM G10 Grid Paper**.

Motor/Memory—For some students, it may be beneficial to pose a multi-step equation and provide (in multiple choice format) a number of possible choices for the first step. Have students pick which is correct with justification. This requires less writing and the choices presented may provide cues for students with weak memory functions.

Memory—Encourage students to solve the equations in sequential steps reminding them to highlight the like terms.

Student Success

Provide **extrinsic rewards** for solving equations (e.g., secret code decoders, patterns in matching exercises, etc.).

3. a) The results from steps 1 and 2 are the same.

b) Answers will vary. Use opposite operations to collect the variable terms on one side. Do the same for constant terms to collect them on the other side. Then, simplify the equation.

4. Replace every x in the expressions for the side lengths with 2 and simplify to find the actual side lengths of each triangle. The lengths are 3, 4, and 6 for the first triangle, and 2, 5, and 6 for the second triangle. This checks as both have perimeter of 13.

Communicate Your Understanding Responses (page 200)

C1. a) Add 9 to both sides, to collect the like constant terms on the left side.

b) Add 8 to both sides, to collect the like constant terms on the left side.

c) Expand the brackets on both sides.

C2. C. If, after substituting $p = -2$ into both sides of the equation, the left side has the same value as the right side, then we know that $p = -2$ is the correct solution for that equation.

Practise

Remind students that it does not matter to which side of the equation they move the variables. For example, in 1c), it is easier to work with the variables on the right side.

For question 5, encourage students to use the left side/right side method of checking solutions, as opposed to simply substituting into the equation and simplifying. Working with LS/RS independently becomes particularly important in future mathematics chapters when non-linear equations can lead to extraneous roots. Mastering the technique now will benefit students later.

In question 6, students may struggle in setting up the initial equation. Encourage them to start with a diagram and to label it using the given information.

Connect and Apply

When working with CAS in questions 11 and 12, students may get error messages due to syntax problems. Watch in particular for missing parentheses when nested brackets are needed.

For question 14, students may struggle in setting up the initial equation. Encourage them to start with a diagram, and to label it using the given information.

Extend

Some students may have trouble understanding three-term ratio notation in questions 15 and 16. They also may struggle in setting up the equation. Provide support, as needed. You may wish to use **BLM A7 Thinking General Scoring Rubric** for question 15 to assist you in assessing your students.

Question 17 requires facility in fraction operations. Students will learn another method for solving such equations in the next section. Plodding through these questions first may help students appreciate the technique of clearing fractions.

Literacy Connections

Constant

Explain to students that a constant is something that does not change. The same is true in math. A constant term in an equation or expression is a value that will not change; it is a number.

BEDMAS

Tell students that the term *BEDMAS* is a mnemonic. A mnemonic is a device to aid the memory. Explain that the mnemonic BEDMAS helps us to remember the order of operations. There are many mnemonics in mathematics. Ask students, *Can you think of any others that you know already?*

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1a), c), 2a), c), 3a), c), 4a), c), 5a), c), 6, 9a), c), 10, 13
Typical	1–10, 13, 14
Extension	15–21