4.3 Solve Equations Involving Fractions

Strand:
Number Sense and Algebra

Student Text Pages
204 to 210

Suggested Timing
80 min

Technology Tools
- Computer Algebra System
- *The Geometer's Sketchpad®*
- TI-89 calculators
- calculators
- graphing calculators
- computers

Related Resources
BLM 4.3.1 Practice: Solve Equations Involving Fractions
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM 4.3.2 Cross-Multiplication Method
BLM A8 Application General Scoring Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.3.3 Achievement Check Rubric
BLM A17 Teamwork Self Assessment
BLM 4.3.4 Student Success: Decision Tree

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.07 solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies).

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Students should have completed all of these prior to working on this section.

Warm-Up

Have students work alone or in pairs.

1. a) What is a trapezoid?
b) Draw a trapezoid to support your explanation.

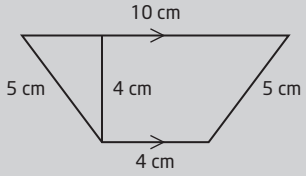
2. a) What is the formula for the area of a trapezoid?
b) Take measurements of the trapezoid you drew in 1b) and calculate its area.

3. Trade your work with another classmate or group and check each other's work for accuracy and correctness.

Warm-Up Answers

1. a) A trapezoid is a four-sided figure with two parallel sides. The other two sides may or may not have the same length.

b) Answers may vary. Sample solution:



2. a) $A = \frac{1}{2}h(a + b)$

b) Answers may vary. Sample solution based on diagram in 1 b).

$A = \frac{1}{2}h(a + b)$
$= \frac{4}{2}(4 + 10)$
$= 2(14)$
$A = 28$
The area is 28 cm^2.

Teaching Suggestions

- Have students complete the Warm-Up independently. (5–10 min)
- After the Warm-Up, discuss the problem posed in the Investigate. Discuss which variables are given, and which is unknown. Then, let students try to solve the equation for the unknown with a partner or in small groups. (5–10 min)
- There are different ways that this equation can be solved, without clearing fractions: guess and check, convert to decimals, divide both sides by $\frac{1}{2}$,

Common Errors

- Some students may multiply the side with the fraction twice when multiplying to clear fractions, for example,

$$2 \times \frac{(x+3)}{2} = 2 \times 5$$
$$2x + 6 = 10$$

R_x Have students clearly show the dividing out of the factor and what is left after dividing (in this case, 1s on the left hand side where the 2s are).

- Some students may forget to multiply the side without the fraction when multiplying to clear fractions, for example,

$$3 \times \frac{(y+1)}{3} = 7$$

R_x Remind students that any operation performed on one side of an equation must be performed identically on the other side in order for the equation to hold true.

Ongoing Assessment

- Use Achievement Check question 10 to monitor student success. See Achievement Check Answers and **BLM 4.3.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills (see Achievement Chart for levels)

Accommodations

Gifted and Enrichment—Encourage students to solve other simple, higher degree equations including quadratic, cubic, and quadratic equations.

Visual—Let students use a graphing calculator to graph linear relations related to the linear equations to be solved and relate the *x*-intercept to the root of an equation.

Motor/Memory—Give a multi-step equation. Then, provide each step to the solution on randomly ordered separate slips of paper. The student's task is to arrange the slips of paper in the correct order to present a coherent solution. Have students work with a partner and take turns explaining each step of the solution. You may wish to use **BLM A17 Teamwork Self Assessment** to assist you in assessing your students.

Language—Encourage students to determine how equations are formed in order to help them use a CAS (Computer Algebra System) when solving equations.

apply the distributive property with the fraction, etc. Ask students to share which approach or approaches they tried. Each of these methods can work, however the calculations can become difficult. After having wrestled with one or more of these approaches, students should be in a position to appreciate the techniques of clearing fractions, as presented in the Examples. You may wish to use **BLM 4.3.1 Practice: Solve Equations Involving Fractions** as remediation or extra practice.

- Assign the Examples, and follow up with a discussion. (15–20 min)
- Example 1 introduces a technique for clearing a single fraction, which is to multiply both sides of the equation by the fraction's denominator. Notice the two forms that the equations can appear in: either with a rational coefficient multiplied by a polynomial, as in part a), or with the entire side of the equation appearing as a fraction, as in part b).
- A CAS solution method is provided for b). Note that while the initial fraction is cleared in the same way as the paper-and-pencil method, the distributive property can be avoided by dividing by the coefficient 3. Because the resulting fraction poses no difficulty for the CAS to handle, the result is a shorter, more elegant solution. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- Example 2 addresses equations involving more than one fraction. **BLM 4.3.2 Cross-Multiplication Method** provides an alternate technique, called cross-multiplication. Note that the technique of cross-multiplication can only be used if you have two rational expressions equal to each other. If you have more than two expressions, you must clear denominators using the lowest common denominator.
- Assign the Communicate Your Understanding questions and consolidate with a class discussion. (5 min)
- Assign the balance of the exercises as independent work. (balance of period)

Investigate Answers (page 204)

1. $50 = \frac{(8 + b)5}{2}$

2. $b = 12$; The garden is 12 m wide at the back.

3. Answers will vary. A sample answer: To work with the fraction $\frac{5}{2}$ was somewhat difficult. If the denominator, 2, could be eliminated, it would be easier to find the solution.

Communicate Your Understanding Responses (page 207)

C1. a) Multiply both sides by 4.
b) Multiply both sides by 3.

C2. A and C

C3. a) 30; The lowest common multiple of 5 and 6 is 30.
b) 9; The lowest common multiple of 9 and 3 is 9.

Practise

For question 1, remind students that the entire side of an equation is multiplied. For example in 1a), some students may be tempted to multiply the $\frac{1}{3}$ by 3, the x by 3, and the -2 by 3. Some will forget to also multiply the 5 by 3.

When working with CAS in question 2, especially when entering complicated equations, ensure that students watch for the correct number and placement of open and closed parentheses to avoid syntax errors or incorrect equations.

Student Success

Construct a **decision tree** for solving equations with the class (see **BLM 4.3.4 Student Success: Decision Tree** for an example). Use a large roll of chart paper and lots of colours. Or, have students construct their own copies on ledger paper.

Connect and Apply

Question 6 provides practice in trouble-shooting skills. Encourage students to try to diagnose their own errors, as much as possible. Remind students that this requires they clearly show all steps to their solutions.

An interesting extension to question 7 is to have students rearrange this equation to express F in terms of C. Talented students may explore this for enrichment. The skill of rearranging formulas is the focus of the next section. You may wish to use **BLM A8 Application General Scoring Rubric** for question 7 to assist you in assessing your students.

Question 8 draws another connection between measurement and algebra. Students will need to recall the formula for the area of a triangle.

Students need to synthesize various concepts for question 9: solving equations, area, perimeter, and the Pythagorean theorem.

For question 10, some students may benefit from exploring this problem using *The Geometer's Sketchpad*®. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Achievement Check Answers (page 209)

10. a) If a square has perimeter of 144 m, then each side is 36 m.

b) The perimeter is given by $w + 3w + w + 3w = 144$, or $8w = 144$.
So, $w = \frac{144}{8}$. The rectangle is 18 m wide and 54 m long.

c) The perimeter is $x + x + \frac{x - 2}{3} = 144$.
$\frac{7x}{3} = 144 + \frac{2}{3}$
$7x = 3(144) + 2$
$7x = 434$
$x = 62$
Therefore, the sides of the triangle are 62 m, 62 m, and $\frac{62 - 2}{3}$ m, or 20 m.

d) The areas of the three figures are:
Square: $36 \times 36 = 1296$
Rectangle: $18 \times 54 = 972$
Triangle: The isosceles triangle has a base of 20 m, and the height (altitude) bisects the base. Using the Pythagorean theorem to find the height gives
$h = \sqrt{62^2 - 10^2}$
$= \sqrt{3744}$
$\doteq 61.2$
So, the area is $\frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(20 \times 61.2)$ or 612.
Therefore, the square has the greatest area ($1296\ \text{m}^2$) and the triangle the least area ($612\ \text{m}^2$).

Extend

The same technique from Example 2 can be used for question 11. The extension is that you must find the lowest common denominator of three or more fractions.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1, 3, 5, 7
Typical	1, 3, 5–9
Extension	11–13