4.4 Modelling With Formulas

Strand:
Number Sense and Algebra

Student Text Pages
211 to 219

Suggested Timing
80 min

Related Resources
BLM 4.4.1 Practice: Modelling With Formulas
BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 4.4.2 Use a Computer Algebra System Directly
BLM 4.4.3 Student Success: Terrific Triangles
BLM A18 My Progress as a Problem Solver
BLM G10 Grid Paper

Tools
- grid paper
- rulers or metre sticks

Technology Tools
- calculators
- graphing calculators
- Computer Algebra System
- TI-89 calculators
- *The Geometer's Sketchpad®*
- computers

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations

NA2.03 relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify and solve equations.

NA2.08 rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement).

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of these prior to working on this section.

Warm-Up

1. Use a calculator to evaluate. Round to one decimal place, if necessary.

a) $\sqrt{400}$ **b)** $\sqrt{225}$ **c)** $\sqrt{200}$ **d)** $\sqrt{1250}$

2. Substitute, then solve for the unknown variable.

a) $A = P + I$ $P = 500; I = 75$

b) $y = mx + b$ $m = \frac{1}{2}; x = 4; b = -9$

Warm-Up Answers

1. a) 20 **b)** 15 **c)** 14.1 d) 35.4

2. a) 575 **b)** −7

Teaching Suggestions

- Students may struggle with this section initially because the level of abstract symbolic reasoning is taken to a new level. Students may be used to finding a numeric "answer" to a math question, and now they are starting and finishing with an algebraic expression. It is important for students to gain some rationale and comfort level for learning such skills.
- They have some initial exposure in Chapter 3 where they were required to simplify an expression, however, here the "answer" often looks no simpler than the "question." They increasingly will need to apply this kind of reasoning in the future study of mathematics (e.g., factoring, etc.).
- Students may feel that they are simply performing arbitrary manipulations that serve no useful purpose, and thus lose interest. Try to connect the activities to real-world examples as often as possible. You may wish to use **BLM 4.4.1 Practice: Modelling With Formulas** for remediation or extra practice.
- Have struggling students create cue cards for various mathematical processes that list the steps involved in the process. Encourage students to keep their cue cards at hand and use them when needed.
- Have students complete the Warm-Up as independent work. (5 min)
- For the Investigate, have students work with a partner or in small groups. (5–10 min)

Common Errors

- Some students may apply the wrong operation to isolate the variable. For example, in Practise question 1a), they might subtract 4 from both sides, instead of divide by 4.

R_x Have students ask themselves, *What operation is being performed between the variable of interest and the other variables or constants? What is the opposite operation required to "undo" this?*

- Some students may perform operations in the incorrect order. For example, in Practise question 2a) some students may divide by t before first subtracting b.

R_x Remind students that they must isolate the term containing the variable of interest first, before dividing or multiplying to isolate the variable itself. This is similar to the approach used earlier in the chapter when actually solving for the root of an equation in one variable. Review a simple example of this, if necessary, such as: $3x + 4 = 10$ (you must subtract 4 first, before dividing by 3).

Ongoing Assessment

- Chapter Problem question 7 also can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Use the photo of the view from the CN Tower to generate a class discussion. If your school is far from the CN Tower, you may wish to use an interesting local alternative (e.g., Skylon Tower in Niagara Falls, any nearby rural fire towers, etc.). You may need to research some height data to adjust the Investigate.
- The purpose of the Investigate is to pose a problem that will prepare students to see the desirability of rearranging a formula into a different form. Students should have little difficulty in solving Investigate question 1, except possibly for how to handle the square root. Suggest substituting and simplifying the expression inside the square root symbol first.
- Have graphing calculators or software available for students to use for the Investigate activity. There are different ways that the students may try to solve 2b) of the Investigate, such as: guess and check, substitute and rearrange to solve the equation, and using a graphing calculator to enter a function and reading its graph. The latter may require some support, as students thus far have had limited or no experience in solving equations involving square roots. Remind students of the Pythagorean theorem, and the opposite relationship of squaring a number and taking its square root. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator**, **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Assign the Examples, and follow up with a class discussion. (15–20 min)
- After debriefing the results of the Investigate, use Example 1 to illustrate the various techniques for rearranging simple formulas. These examples require only one step, to apply the opposite operation in order to isolate the variable of interest. Students should be able to transfer skills learned earlier in the chapter to working with and possibly shifting more than one variable.
- Example 2 involves rearranging formulas in more than one step. The context chosen is an important one, as it connects to future work in Chapter 6. Operations typically are done in the opposite order, according to BEDMAS. The main idea is to isolate the term containing the variable of interest first, and then solve for the variable.
- A Computer Algebra System (CAS) can be useful for rearranging formulas, and could be presented as an alternate approach to Example 2. In Method 2, students enter each step, and CAS produces the result of that step. CAS is also capable of rearranging a formula directly, without the need for entering intermediate steps. You may wish to have students use this method for checking their work, however, it is important for the academic student to be able to master the basic skills involved here and not use the CAS as a substitute for pencil and paper. Use **BLM 4.4.2 Use a Computer Algebra System Directly** for this activity. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity.
- Communicate Your Understanding: class discussion (5 min)
- Assign the exercises as independent work. (balance of period)
- You may wish to use **BLM A18 My Progress as a Problem Solver** at any point in this lesson to assist you in assessing your students.

Accommodations

Gifted and Enrichment—Challenge students to research geostationary satellites and share their new knowledge with their classmates.

Visual—Encourage students to highlight the variables in the formulas in different colours.

Perceptual—Allow students to use visual cues, such as the distance, speed, time triangle, to help them when they are manipulating formulas.

Memory—Encourage students to review how to use the functions on their scientific and graphing calculators.

Student Success

After teaching this section, introduce students to **formula triangles**. Emphasize the mathematical operations behind why the triangles work. Use **BLM 4.4.3 Student Success: Terrific Triangles** to support this activity.

Investigate Answers (page 211)

1. a) 67 km **b)** 76 km

2. a) 781 m

b) Substitute $d = 100$ into the formula and solve for h.

Step 1: Substitute $d = 100$: $100 = 2 \times \sqrt{3.2 \times h}$

Step 2: Divide both sides by 2: $50 = \sqrt{3.2 \times h}$

Step 3: Square both sides: $50^2 = 3.2 \times h$

Step 4: Divide both sides by 3.2: $h = \frac{50^2}{3.2}$

Step 5: Simplify: $h = 781.25$

You would have to go up 781 m, to the nearest metre.

3. Formulas can be rearranged to isolate one of the variables. Then, the known values can be substituted in the equation to calculate the isolated variable.
For example: $N = 7x + 12$, evaluate x when $N = 82$.

$$N = 7x + 12$$
$$N - 12 = 7x$$
$$\frac{N - 12}{7} = x$$

Substitute $N = 82$ in the rearranged equation to evaluate x.

$$\frac{82 - 12}{7} = x$$
$$10 = x$$

Communicate Your Understanding Responses (page 214)

C1. a) Multiply both sides by t.

b) Subtract b from both sides.

c) Divide both sides by π.

C2. The probability of errors is reduced. Rearranging the formula helps to simplify the expression and make the substitution easier.

C3. Answers may vary. Sample solution:

Step 1: Divide both sides by 2.

$$\frac{p}{2} = \frac{2(\ell + w)}{2}$$

Step 2: Divide the 2s on the right side.

$$\frac{p}{2} = \frac{\cancel{2}(\ell + w)}{\cancel{2}}$$

Step 3: Subtract w from both sides.

$$\frac{p}{2} - w = \ell + w - w$$

Step 4: Simplify.

$$\frac{p}{2} - w = \ell$$

Yes, there is more than one way to isolate ℓ. You could expand the expression on the right side and then subtract $2w$ from the both sides. Next, you would have to divide both sides by 2 to isolate ℓ. However, the results of both expressions will be equivalent.

Practise

Students may have difficulty getting started because they are uncertain what to do. Reinforce that they first need to identify the variable of interest. Then, have them consider the operation that will allow them to isolate that variable. This will usually require applying the opposite operation to numbers and/or other variables.

Connect and Apply

Have graphing calculators or **BLM G10 Grid Paper** available as a number of the questions ask students to compare algebraic and graphic models. For questions 3 and 4, use a ruler or metre stick (that also shows inches) to show students where this relationship comes from.

Question 4 connects to linear relations, a key theme of the chapter, as well as section 5.2 Direct Variation.

Question 6 connects solving equations and future work in section 5.3 Partial Variation, as well as linear and non-linear relations.

Question 8 connects measurement relationships and equations. Some students may benefit from exploring this problem using *The Geometer's Sketchpad*® or a graphing calculator. You may wish to use **BLM T7 The Computer Algebra System (CAS) on the TI-89 Calculator**, **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Questions 10 to 12 connect to future work in senior chemistry and physics. Although a thorough understanding of the scientific principles is not necessary here, students will benefit from seeing how mathematics is required for manipulating these formulas.

Extend

Questions 13 and 14 extend the idea of inverse operations applied to formulas into the idea of inverse functions, which students will study in depth in senior mathematics. At this level however, students should be able to simply identify some similar and distinguishing features between the curves, without any formal analysis. They should also realize that you could use either curve (e.g., A versus ℓ, or ℓ versus A) to find specific information about the relationship between the two variables.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–7
Typical	1–8, 10–12
Extension	13–19