4.5 Modelling With Algebra

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Manipulating Expressions and Solving Equations
NA2.09 solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods.

Link to Get Ready

Understanding of all of the concepts in the Get Ready section is required by this point. Ensure that students have completed all of these prior to working on this section.

Warm-Up

1. Solve for x.
$$x + 2x + 3x - 100 = 20$$

2. The formula for the area of a trapezoid is:
$$A = \frac{1}{2}h(a + b)$$

Find the area of a trapezoid whose parallel sides are 8 cm and 12 cm, and whose height is 10 cm.

3. Rearrange the following formula in terms of p.
$$a = 5p + q$$

Warm-Up Answers

1. 20   **2.** 100 cm^2 **3.** $\dfrac{a - q}{5} = p$

Teaching Suggestions

- Assign the Warm-Up as independent work. (5 min)
- Have students work on the Investigate as a class or in small groups. (10–15 min)
- The purpose of the Investigate is to provide students with an insight into the power of algebraic reasoning, in this case to debunk a magic trick. Conduct this as a class activity. The secret number can be posted on the board out of view of the magician. You may wish to be the magician, or choose a student volunteer. You may wish to have the student dress up as a magician for this class. Instruct students to leave books closed, except as needed. Use **BLM 4.5.1 Investigate: Magic With Algebra** for the designated magician.
- After groups have discussed question 5 of the Investigate for a few minutes, you may wish to give the class a hint: Simplify this expression: $[3(2x + 50) - 100] \div 2$
- Compare the original expression to the audience's steps in question 2. Compare the simplified expression to the magician's secret steps in question 3.
- Assign the Examples, and follow up with a class discussion. (25–35 min)
- Example 1 illustrates how an algebraic equation can be built to model and solve a payroll problem, using one variable. It is important for students to be able to build the expressions for the different workers' pay using a single variable. To do this, they must translate the given information into

- Some students may have trouble setting up an algebraic equation, based on given information.

R_x Have students look for key words and what operations they imply, for example, total, difference, times, etc. Provide practice with just translating phrases before delving into full problems.

- Some students may have incomplete solutions. For example in question 9, the student may solve for x, which represents one of the integers, and stop there, without stating the other integers.

R_x Typically, these problems require a number of steps. Have students look back to the question and verify that they have answered it completely.

Ongoing Assessment

- Use Achievement Check question 15 to monitor student success. See Achievement Check Answers and **BLM 4.5.3 Achievement Check Rubric**.
- Chapter Problem question 8 also can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

algebraic expressions. Ensure that students practise this skill alone before assigning full problems.
- In Example 2, a formula model is built involving several variables. The formula is then rearranged in terms of one of the other variables. This is done in two different ways. Method 1 is more straightforward conceptually, but results in a rather complicated formula, involving a fraction and a decimal. Method 2 produces a more elegant result but requires an insightful manipulation that clears the need for any fractions or decimals. In a class discussion, have students debate the merits of each approach.
- Example 3 requires synthesizing equation-solving concepts with area relationships and the Pythagorean theorem. Two approaches are presented and compared. Method 1 is algebraic, although it requires some geometric reasoning. Method 2 is a dynamic geometry approach that is far more visual. You may wish to demonstrate Method 2 to the class. In a class discussion, have students compare the different approaches to solving the problem. You may wish to use **BLM T4** *The Geometer's Sketchpad® 3* or **BLM T5** *The Geometer's Sketchpad® 4* to support this activity.
- Have students work with a partner or in small groups for the Communicate Your Understanding questions. Have students share their ideas with the class in a class discussion. (5 min)
- Assign the balance of the exercises and have students work independently. (balance of period)
- Encourage diverse approaches to problem solving and encourage students to share their approaches as they work independently or in small groups to tackle the exercise problems.
- Alternatively, break up the lesson:
  - Assign Example 1, and assign Practise questions 1 to 5.
  - Debrief answers to Practise questions.
  - Have students do Examples 2 and 3, and assign the remaining exercise questions.
- Based on the progress of your students, you may wish to split the lesson over two days and provide more practice. You may wish to use **BLM 4.5.2 Practice: Modelling With Algebra**.

Investigate Answers (page 220)

2. Answers will vary. Sample solution: Choose the number 2.

    a) 4   **b)** 54   **c)** 162   **d)** 62   **e)** 31   **f)** 31

3. Abracadabra

    a) 6   **b)** 2   **c)** 2

4. Answers will vary. Sample Solution: Choose the number 9. Results are: 18, 68, 204, 104, 52, 52. Magician: 27, 9.

6. a) No; Pick a number x and follow the magician's instructions to the audience.
Step 1: $2x$
Step 2: $2x + 50$
Step 3: $6x + 150$
Step 4: $6x + 50$
Step 5: $3x + 25$
Step 6: Final Result is $3x + 25$
Now, consider the magician's secret steps to find the number.
Step 1: $3x$
Step 2: x
So, after doing a series of calculations, one arrives at the number itself. The initial changes in the number introduced by addition and multiplication are countered by subtraction and division. In simple terms, these operations represent a circle. One starts at a certain point and performs some calculations (travels along the circle) and finds the number (arrives at the starting point).

b) Yes, this trick will work for both numbers greater than 10 and for negative numbers. As above, the calculations are independent of the number someone chooses. You can substitute any value for x and the steps, described above, still hold.

c) Answers will vary. You can use algebra to work backwards from the desired answer. For example, if you want x then, you can use algebraic equations to develop a series of steps that will result in x.

Communicate Your Understanding Responses (page 226)

C1. D. Let Hanna's age be h. Rufio is 5 years older than Hanna, so, Rufio's age is $(h + 5)$. Their ages add up to 37, so $h + (h + 5) = 37$.

C2. B. p is the amount earned delivering papers; Brittany earned twice as much babysitting ($2p$) for a total of $800: $800 = p + 2p$

C3. C. $12h$ represents the $12 Asraf earns per hour plus the 10% (0.1) commission on sales (s): $12h + 0.1s$

Practise

Use the Practise questions as a means to develop students' skills at translating information given in prose form into algebraic expressions. Often, this is one of the most challenging aspects of algebraic modelling for students. If students struggle with this step, consider assigning additional questions in this area prior to assigning the Practise, or have students work on the Practise questions with a partner. Use **BLM 4.5.2 Practice: Modelling With Algebra** for additional practice.

Connect and Apply

Have students work with a partner on some of these problems to help build confidence.

Students may solve questions 9 and 10 using guess and check. This is a perfectly valid strategy as long as students can demonstrate and communicate how their method works.

For questions 11 to 13, encourage students to start with a diagram and to label any information that they are given.

You may wish to have students use **BLM G10 Grid Paper** for Achievement Check question 15.

Achievement Check Answers (page 228)

15. a) An algebraic expression for Paloma's total earnings is $9h + 12m$, where h is the time worked, in hours, and m is the number of memberships sold.

b) $9(8) + 12(7) = 156$ Paloma earned $156.

c) Substituting and solving,

$$9(24) + 12m = 600$$
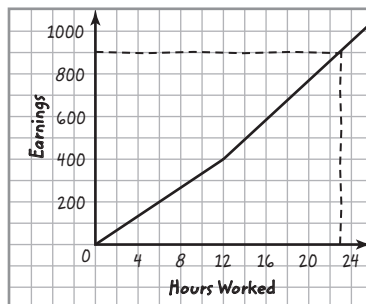$$12m = 600 - 216$$
$$12m = 384$$
$$m = 32$$

Paloma must sell 32 memberships in order to earn $600.

d) In the first 12 h, Paloma earns $9 + 2(12) = 33$, or $33/h.
In the next 12 h, Paloma earns $9 + 3(12) = 45$, or $45/h.
In order to graph this, students need to draw a line that rises at $33/h for the first 12 h. From that point, they must draw a line that rises at $45/h for the next 12 h. the graph will intersect the $900 line at the 23 h mark.

Extend

For question 16, the Pythagorean theorem is required to set up the equation for the unknown side. Then, the area of a square relationship is applied. The Pythagorean theorem is also required for question 17.

Literacy Connections

Wages

Ask students, *Have you ever been paid for shovelling someone's driveway? If so, were you paid an hourly wage for the time you spent, or were you given*

a flat fee for completing the driveway? Explain that there are many different ways to be paid for work done. Sometimes people are paid an hourly wage and sometimes the amount is based on how much salespeople sell. Often in retail, the more people sell the more they make. Have students check on the Internet and with their friends and family about the variety of ways to earn money. Instruct students to write a brief report comparing and contrasting two methods of earning money at a job. You may wish to use **BLM A22 Earning Money Report Checklist** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–7
Typical	1–7, 9–14
Extension	16–20