5

Modelling With Graphs

Vocabulary

direct variation
constant of variation
partial variation
slope
rise
run
rate of change
first differences

Curriculum Expectations
Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING
MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING
MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING
MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING
MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Overall Expectations

By the end of this course, students will:

LRV.01 apply data-management techniques to investigate relationships between two variables;

LRV.02 demonstrate an understanding of the characteristics of a linear relation;

LRV.03 connect various representations of a linear relation.

Specific Expectations

Understanding Characteristics of Linear Relations

By the end of this chapter, students will:

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

RE2.03 identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;

RE2.04 compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Connecting Various Representations of Linear Relations

By the end of this chapter, students will:

RE3.01 determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;

RE3.02 describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;

RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

RE3.04 describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).

Overall Expectations

By the end of this course, students will:

AGV.01 determine the relationship between the form of an equation and the shape of its graph with respect to linearity and non-linearity;

AGV.02 determine, through investigation, the properties of the slope and *y*-intercept of a linear relation;

AGV.03 solve problems involving linear relations.

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

By the end of this chapter, students will:

AG1.01 determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation).

Investigating the Properties of Slope

By the end of this chapter, students will:

AG2.01 determine, through investigation, various formulas for the slope of a line segment or a line;

AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is $50, plus $5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation).

Using the Properties of Linear Relations to Solve Problems

By the end of this chapter, students will:

AG3.03 describe the meaning of the slope and *y*-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to $50 to $2550).

Chapter Problem

The Chapter Problem is introduced in the Chapter Opener. Have students discuss their understanding of the toothpick patterns. There are many patterns that can be drawn out of the diagram in the Chapter Problem. Allow students to explore them. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 291. Alternatively, you may wish to assign the Chapter Problem when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment.

Chapter 5 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 5 Opener • 15 min	234–235			
Get Ready • 80 min	236–237	• BLM 5.GR.1 Practice: Get Ready	• BLM 5.GR.2 Get Ready Self-Assessment Checklist	
5.1 Direct Variation • 80 min	238–245	• BLM G10 Grid Paper • BLM T6 *Fathom*^{TM} • BLM 5.1.1 Practice: Direct Variation	• BLM A9 Communication General Scoring Rubric	**Tools** • grid paper **Technology Tools** • *Fathom*^{TM} • computers • graphing calculators
5.2 Partial Variation • 80 min	246–253	• BLM G10 Grid Paper • BLM 5.2.1 Practice: Partial Variation	• BLM 5.2.2 Achievement Check Rubric • BLM A7 Thinking General Scoring Rubric	**Tools** • grid paper
5.3 Slope • 80 min	254–263	• BLM G10 Grid Paper • BLM 5.3.1 Practice: Slope	• BLM A16 My Progress as a Mathematician • BLM A23 News Report Checklist	**Tools** • grid paper
5.4 Slope as a Rate of Change • 80 min	264–271	• BLM T6 *Fathom*^{TM} • BLM 5.4.1 Practice: Slope as a Rate of Change • BLM G10 Grid Paper	• BLM A12 Group Work Assessment General Scoring Rubric • BLM 5.4.2 Achievement Check Rubric	**Tools** • grid paper **Technology Tools** • *Fathom*^{TM} • computers • graphing calculators
5.5 First Differences • 80 min	272–278	• BLM G10 Grid Paper • BLM 5.5.1 Practice: First Differences	• BLM A20 Learning Skills Checklist	**Tools** • grid paper **Technology Tools** • graphing calculators
5.6 Connecting Variation, Slope, and First Differences • 160 min	279–287	• BLM 5.6.1 Practice: Connecting Variation, Slope, and First Differences • BLM 5.6.2 Student Success: The Rule of Four • BLM G10 Grid Paper	• BLM 5.6.3 Achievement Check Rubric	**Tools** • grid paper
Chapter 5 Review • 80 min	288–289	• BLM G10 Grid Paper • BLM 5.CR.1 Chapter 5 Review	• BLM A4 Presentation Checklist	**Tools** • grid paper
Chapter 5 Practice Test • 80 min	290–291	• BLM G10 Grid Paper	• BLM 5.PT.1 Chapter 5 Practice Test • BLM 5.CT.1 Chapter 5 Test	**Tools** • grid paper **Technology Tools** • graphing calculators
Chapter 5 Problem Wrap-Up • 40–80 min	291	• BLM G10 Grid Paper	• BLM 5.CP.1 Chapter 5 Problem Wrap-Up Rubric	**Tools** • grid paper • toothpicks and/or algebra tiles **Technology Tools** • graphing calculators

Chapter 5 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 5.GR.1	Practice: Get Ready	Practice
	BLM 5.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
5.1: Direct Variation:			
	BLM G10	Grid Paper	Student Support
	BLM T6	*Fathom*™	Technology
	BLM 5.1.1	Practice: Direct Variation	Practice
	BLM A9	Communication General Scoring Rubric	Assessment
5.2: Partial Variation			
	BLM G10	Grid Paper	Student Support
	BLM 5.2.1	Practice: Partial Variation	Practice
	BLM 5.2.2	Achievement Check Rubric	Assessment
	BLM A7	Thinking General Scoring Rubric	Assessment
5.3: Slope			
	BLM G10	Grid Paper	Student Success
	BLM A16	My Progress as a Mathematician	Student Self-Assessment
	BLM 5.3.1	Practice: Slope	Practice
	BLM A23	News Report Checklist	Assessment Literacy
5.4: Slope as a Rate of Change			
	BLM A12	Group Work Assessment General Scoring Rubric	Assessment Group Work
	BLM T6	*Fathom*™	Technology
	BLM 5.4.1	Practice: Slope as a Rate of Change	Practice
	BLM G10	Grid Paper	Student Support
	BLM 5.4.2	Achievement Check Rubric	Assessment
5.5: First Differences			
	BLM G10	Grid Paper	Student Support
	BLM A20	Learning Skills Checklist	Assessment
	BLM 5.5.1	Practice: First Differences	Practice
5.6: Connecting Variation, Slope, and First Differences			
	BLM 5.6.1	Practice: Connecting Variation, Slope, and First Differences	Practice
	BLM 5.6.2	Student Success: The Rule of Four	Student Success
	BLM G10	Grid Paper	Student Support
	BLM 5.6.3	Achievement Check Rubric	Assessment

	BLM	Title	Purpose
Chapter 5 Review			
	BLM 5.CR.1	Chapter 5 Review	Review
	BLM G10	Grid Paper	Student Support
	BLM A4	Presentation Checklist	Assessment
Chapter 5 Practice Test			
	BLM 5.PT.1	Chapter 5 Practice Test	Diagnostic Assessment
	BLM 5.CT.1	Chapter 5 Test	Student Assessment
	BLM G10	Grid Paper	Student Support
Chapter 5 Problem Wrap-Up			
	BLM G10	Grid Paper	Student Support
	BLM 5.CP.1	Chapter 5 Problem Wrap-Up Rubric	Student Assessment

Get Ready

Student Text Pages
236 to 237

Suggested Timing
80 min

Related Resources
BLM 5.GR.1 Practice: Get Ready

BLM 5.GR. 2 Get Ready Self-Assessment Checklist

Common Errors

• Some students may treat negative mixed fractions as the sum of a negative and a positive, for example $-2\frac{1}{3} = -2 + \frac{1}{3}$.

R_x Explain that the negative sign belongs with the entire number. Then, show them on a number line that $-2\frac{1}{3}$ is between -2 and -3, just as $2\frac{1}{3}$ is between 2 and 3.

Accommodations

Gifted and Enrichment—Encourage students to learn more about mathematical puzzles, to solve them, and to create their own.

Memory—Encourage students to review the rules for equivalent fractions and expression ratios in simplest form.

Teaching Suggestions

• Students need a sound understanding of rational numbers, as slopes and rates of change will involve rationals. For a more complete development of operations with rational numbers, see Chapter 1, Section 1.5.
• If students need remedial work on rationals, you may wish to use **BLM 5.GR.1 Practice: Get Ready** for remediation or extra practice.
• Refer to the Link to Get Ready in the chapter sections of the Teacher's Resource.
• All **BLMs** referred to throughout this chapter can be found on the *Principles of Mathematics 9* Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation as students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; students can place a checkmark beside topics in the Get Ready in which they feel confident. You may wish to use **BLM 5.GR.2 Get Ready Self-Assessment Checklist** as a self-assessment for students. Remedial action can be taken in small groups or with a whole class skill review.