5.1 Direct Variation

Strand:
Linear Relations

Strand:
Analytic Geometry

Student Text Pages
238 to 245

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- *Fathom*™
- computers
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM T6 *Fathom*™
BLM 5.1.1 Practice: Direct Variation
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Understanding the Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.04 compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

Investigating the Properties of Slope

AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation);

Connecting Various Representations of Linear Relations

RE3.02 describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;

RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

RE3.04 describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).

Using the Properties of Linear Relations to Solve Problems

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550);

Link to Get Ready

The Get Ready segments, Rational Numbers, and Ratio and Proportion, provide the needed skills for this section. You may wish to have students complete Get Ready questions 1 to 6 before starting this section.

Common Errors

- Some students may have difficulties deciding which is the independent variable.

R_x Have students consider not only cause and effect, but which variable must be known before the other can be calculated. Refer students to Chapter 2, Section 2.3.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Warm-Up

1. Mental Math: Multiply each of the following.

a) 10×12 **b)** 6×9 **c)** 11×7
d) 100×72 **e)** 15×3 **f)** 30×5

2. Divide each of the following.

a) $72 \div 8$ **b)** $100 \div 5$ **c)** $24 \div 6$
d) $60 \div 3$ **e)** $1000 \div 10$ **f)** $99 \div 11$

Warm-Up Answers

1. a) 120 **b)** 54 **c)** 77 **d)** 7200 **e)** 45 **f)** 150

2. a) 9 **b)** 20 **c)** 4 **d)** 20 **e)** 100 **f)** 9

Teaching Suggestions

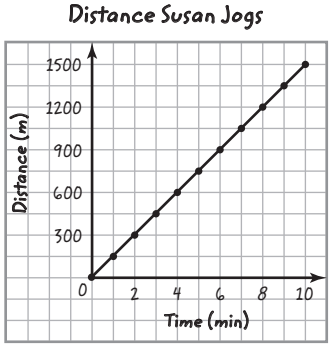
- Discuss the opening photo and what is meant by speed. Ask how speed is related to distance and time and how to write the relationship as an equation. (5 min)
- Have students work on the Investigate. At the conclusion of the Investigate, discuss the multiple representations (words, numbers, graph, equations) of the relationship, which is the key to this activity. You may wish to use **BLM G10 Grid Paper** to support this activity. (15 min)
- Discuss Examples 1 and 2 as a class, or examples similar to them, showing the multiple representations of direct variation. (20 min)
- Students will have worked with proportions in grade 7 and 8, but will not have calculated a constant of variation. Stress the importance of this constant, which is what makes the graph a straight line. Ensure that students understand that the graphs must pass through the origin. Although included in the examples, technology is not as important a tool for understanding direct variation as the multiple representations. You may wish to use **BLM T6 *Fathom*™** to support this activity.
- Review the vocabulary in this section (independent variable, dependent variable, direct variation, constant of variation) before moving on to Communicate Your Understanding. (10 min)
- Discuss the Communicate Your Understanding questions C1 and C2. (10 min)
- You may wish to use **BLM 5.1.1 Practice: Direct Variation** for remediation or extra practice.

Investigate Answers (page 238)

1. a)

Time (min)	Distance (m)
0	0
1	150
2	300
3	450
4	600
5	750
6	900
7	1050
8	1200
9	1350
10	1500

b) Independent variable: time; dependent variable: distance



Accommodations

Visual—Encourage students to work together with a reading buddy to receive verbal instructions for the questions in this section.

Perceptual—Encourage students to use visual and verbal cues to identify the independent variable and the dependent variable in linear relations where the variables are letters other than x and y.

Spatial—Provide students with opportunities to use technology to relate direct variation ($y = mx$) to a linear relation with a y-intercept of zero.

Motor—Give students extra time to complete the questions in this section.

ESL—Allow students to work together when working through the questions in this section, and encourage them to use their dictionaries to understand the meanings of the new words.

c) The graph is a straight line that slopes upward, from left to right, and intersects the vertical axis at the origin, (0, 0).

d) $d = 150t$

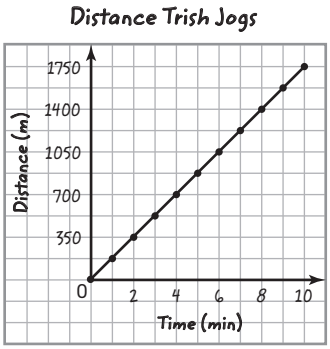
e) 6000 m or 6 km

f) The distance is doubled. The distance is tripled.

2. a)

Time (min)	Distance (m)
0	0
1	175
2	350
3	525
4	700
5	875
6	1050
7	1225
8	1400
9	1575
10	1750

b) time, distance



c) The graph is a straight line that slopes upward, from left to right, and intersects the vertical axis at the origin (0, 0).

d) $d = 175t$

e) 7000 m or 7 km

f) The distance is doubled. The distance is tripled.

3. The distance jogged is a product of the time and the average speed. For example, if the average speed is 120 m/min, then an equation for the distance, d, in metres, travelled in t minutes would be $d = 120t$.

Communicate Your Understanding Responses (page 242)

C1. $A = 2C$ is an example of direct variation, because it has the same form as $y = kx$. The variable C varies directly with the variable A.

C2. a) Both graphs are straight lines that start at the origin and go upward, from left to right.

b) The difference between the graphs is their level of steepness. The graph of $d = 2t$ has a slope of 2 and the graph of $d = 3t$ has a slope of 3, so, the graph of $d = 3t$ is steeper.

Practise

Students should not have difficulties with the Practise questions; they are straightforward and reinforce the Examples. You may wish to use **BLM G10 Grid Paper** to support questions 1 to 5.

Connect and Apply

Questions 6 and 7 are straightforward. In part c) of both questions, students use the formula for extrapolation.

Questions 8c) and 9c) provide a good opportunity for students to use their reasoning and communication skills.

For question 10, you may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Questions 11, 12, and 13 provide interesting contexts for the students. You may wish to use **BLM G10 Grid Paper** to support questions 7, 9, and 12.

Extend

Question 14 involves re-arranging an equation involving two variables and decimal coefficients. It is accessible to most level 3 and 4 students.

Question 15 discusses an interesting scientific relationship among four variables.

Questions 16 and 17 are Math Contest questions.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6, 8
Typical	1–10, 11, 13
Extension	14—17