5.2 Partial Variation

Strand:
Linear Relations

Strand:
Analytic Geometry

Student Text Pages
246 to 253

Suggested Timing
80 min

Tools
- grid paper

Related Resources
BLM G10 Grid Paper
BLM 5.2.1 Practice: Partial Variation
BLM 5.2.2 Achievement Check Rubric
BLM A7 Thinking General Scoring Rubric

Mathematical Process Expectations Emphasis

- [x] Problem Solving
- [ ] Reasoning and Proving
- [ ] Reflecting
- [ ] Selecting Tools and Computational Strategies
- [x] Connecting
- [x] Representing
- [x] Communicating

Specific Expectations

Understanding the Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.04 compare the properties of direct variation and partial variation in applications, and identify the initial value (e.g., for a relation described in words, or represented as a graph or an equation);

Connecting Various Representations of Linear Relations

RE3.02 describe a situation that would explain the events illustrated by a given graph of a relationship between two variables;

RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

RE3.04 describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied (e.g., given a partial variation graph and an equation representing the cost of producing a yearbook, describe how the graph changes if the cost per book is altered, describe how the graph changes if the fixed costs are altered, and make the corresponding changes to the equation).

Investigating the Properties of Slope

AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation);

Using the Properties of Linear Relations to Solve Problems

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550).

Common Errors

- Some students may have difficulty seeing the constant of proportionality, since it is not simply multiplied by the independent variable.

R_x Have students graph the data in the problem. Draw right triangles between successive points illustrating the steps while going through the relation. Alternatively, add another column to their data chart and ask them to subtract successive values of the dependent variable.

Ongoing Assessment

- Use Achievement Check question 12 to monitor student success. See Achievement Check Answers and **BLM 5.2.2 Achievement Check Rubric**.
- Chapter Problem question 8 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Link to Get Ready

The Get Ready segment Percents provides the needed skills for this section. You may wish to have students complete or review Get Ready questions 7 and 8 before starting this section.

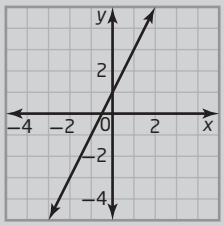
Warm-Up

1. Mental Math: Continue each of the following number patterns, stating the next two terms.

a) 2, 5, 8, 11 **b)** 15, 10, 5, 0 **c)** 6, 8, 10, 12 **d)** 1, 11, 21, 31

2. Which of the following represent direct variation?

a)



b) The page numbering in the student's textbook versus the number of pieces of paper.

c) (0, 0), (2, 4), (4, 8), (6, 12) **d)** $y = 8x$ **e)** $C = 2n + 5$

Warm-Up Answers

1. a) 14, 17 **b)** −5, −10 **c)** 14, 16 **d)** 41, 51

2. a) No **b)** Yes **c)** Yes **d)** Yes

e) No; The graph does not pass through the origin.

Teaching Suggestions

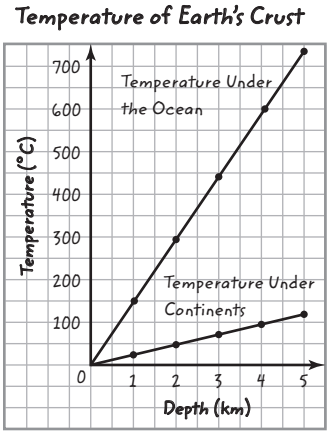
- Assign the Warm-Up. (2 min)
- Discuss the opening paragraph and what the students need to do in the Investigate.
- After students complete the Investigate, relate the multiple representations of partial variation numerically, verbally, graphically, and algebraically. Discuss how the equation is developed, for example, by looking at the fixed, initial, or starting value, and adding the variable term, which is formed by multiplying the constant of variation by the independent variable. You may wish to use **BLM G10 Grid Paper** to support this activity. (25 min)
- Present Examples 1 and 2, or similar examples that illustrate the multiple representations of relations. It is equally important that students experience abstract examples, such as Example 1 and applications such as Example 2. When looking at data, such as in Example 1, students may need assistance in finding the constant of variation. Some students divide each difference between successive y-values by the corresponding difference between successive x-values. Others may try to see a pattern in the terms. Both methods are valid. (20 min)
- Discuss the questions from Communicate Your Understanding. (10 min)
- Assign and discuss Practise questions 1 and 2. (10 min)
- You may wish to use **BLM 5.2.1 Practice: Partial Variation** for remediation or extra practice.

Investigate Answers (page 246)

1.

Depth (km)	Temperature Under Oceans (°C)	Temperature Under Continents (°C)
0	10	10.00
1	155	31.75
2	300	53.50
3	445	75.25
4	590	97.00
5	735	118.75

2. a) and b)



3. Both graphs start at 10 on the temperature axis and go in a straight line upward, from left to right. The "Temperature Under Oceans" graph rises more steeply than the other graph does.

4. They differ in that they do not start at (0, 0) on the vertical axis; they start at (0, 10).

5. a) 155, 300, 445, 590, 735; they are the same.

b) This equation works in a similar way to how an equation for direct variation works. There is a constant of variation that represents the constant average increase in temperature. The difference is that a value is added to the equation.

c) $T = 21.75d + 10$, where T represents the temperature, in degrees Celsius, and d represents the depth, in kilometres, under the continents.

6. For $T = 145d + 10$, T represents the temperature, in degrees Celsius, at a depth of d kilometres under the oceans, 145 is the constant of variation, and 10 is the starting temperature. For $T = 21.75d + 10$, T represents the temperature, in degrees Celsius, at a depth of d kilometres under the continents, 21.75 is the constant of variation, and 10 is the starting temperature. The d is the "depth" column in the table and the T is the temperature column. Both equations are of the form of a direct variation equation $(y = kx)$, except there is a value added on to them $(y = kx + \text{value})$.

Communicate Your Understanding Responses (page 250)

C1. a) Partial variation; it is a straight line that does not pass through the origin.

b) Neither; it is not a straight line.

c) Direct variation; it is a straight line that passes through the origin.

C2. The initial value of the cost of repair occurs when the number of hours is 0. The initial value of the cost of repair is \$50. The constant of variation is 40. This relationship is a partial variation, so, its equation will be of the form $c = mh + b$, where c is the cost of repair, m is the constant of variation, h is the number of hours, and b is the constant value. Substitute $b = 50$ and $m = 40$ to get the equation $c = 40h + 50$. As the number of hours changes from 0 to 1, the cost of repair changes from \$50 to \$90. Therefore, the cost of repair increases by \$40 as the number of hours increase by 1.

C3. As x changes from 0 to 1, y changes from 10 to 14. Therefore, y increases by 4 as x increases by 1. The initial value of y occurs when x is 0, so, the initial value of y is 10. So, the constant of variation is 4. Substitute $b = 10$ and $m = 4$ into the equation $y = mx + b$ to get $y = 4x + 10$.

Practise

In questions 2 and 3, students may not see the pattern in the growth of the y-values. Suggest that they count by 5s for each question. You may wish to use **BLM G10 Grid Paper** to support questions 2 and 3.

Connect and Apply

Students may find question 7 challenging, since it is the first time the independent variable does not increase by 1. Ask them what the increase in cost is per 100 and to convert to a rate.

Question 8 refers to the Chapter Problem. Note to students that the first toothpick pattern has been coloured red. This will help students see the initial, or fixed, value of the relation, since this is the first example where the independent variable begins at 1 instead of 0. Encourage students to make a chart to help them see the pattern of numbers.

You may wish to use **BLM G10 Grid Paper** to support questions 6, 11, and 12.

Accommodations

Perceptual—Allow students to use words instead of variables to identify the independent variable and the dependent variable.

Spatial—Provide students with opportunities to use technology to relate direct variation ($y = mx + b$) to a linear relation with a fixed initial amount or y-intercept of b.

Memory—Encourage students to colour code the slope and y-intercept to relate the variable cost to the slope and the fixed cost to the slope.

ESL—Give students extra time to interpret and complete the questions in this section.

Student Success

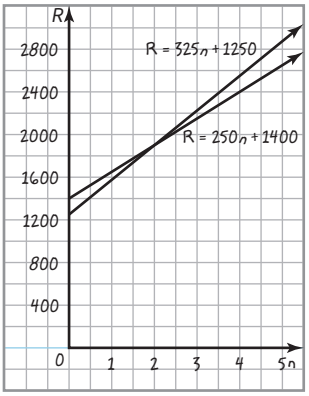
Give each student a different direct or partial variation problem to solve. Then, use an inside/outside circle to have stdents present their problem and solution to others.

Achievement Check Answers (page 253)

12. a) The equation for *Cats* is a partial variation:
$R = 325n + 1250$, where R is the royalty fee, in dollars, and n is the number of shows performed.
The equation for *Fame* is a partial variation:
$R = 250n + 1400$, where R is the royalty fee, in dollars, and n is the number of shows performed.

b)

Number of Shows	0	1	2	3	4
Royalty for *Cats* ($)	1250	1575	1900	2225	2550
Royalty for *Fame* ($)	1400	1650	1900	2150	2400



c) The company pays the same amount for each show, $1900, if there are two performances.

d) Answers will vary. Points to consider:
- If direct variation is used, the cost for theatre groups performing more than one or two shows will be high. For example, if the direct variation is based on royalties of $1900 after two shows then four shows would cost $3800 compared with the $2550 and $2400 for partial variation.
- The company needs a fixed price to ensure a reasonable income from groups that perform only one or two shows. In the direct variation example above one show would cost $950 compared to the $1575 and $1650 of the partial, variation model. Hence, partial variation is used to set a rate that is more equitable for performers and royalty rights holders.

Extend

Question 13 asks students to solve the equation for the dependent variable. It is accessible to level 3 and 4 students. You may wish to use **BLM A7 Thinking General Scoring Rubric** for question 13 to assist you in assessing your students.

Question 14 is a good example of a piecewise linear function. Students should graph each "piece" separately, but on the same grid. You may wish to use **BLM G10 Grid Paper** to support question 14.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6, 10
Typical	1–7, 9, 10, 11
Extension	13, 14