5.5 First Differences

Strand:
Linear Relations

Strand:
Analytic Geometry

Student Text Pages
272 to 278

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM A20 Learning Skills Checklist
BLM 5.5.1 Practice: First Differences

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

RE2.03 identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear;

Connecting Various Representations of Linear Relations

RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

Investigating the Relationship Between the Equation of a Relation and the Shape of its Graph

AG1.01 determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of nonlinear relations (e.g., use a graphing calculator or graphing software to graph a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; connect an equation of degree one to a linear relation).

Teaching Suggestions

- First differences are best learned through an investigative approach. Have students investigate the first differences of data in charts and compare the results to their graphs. You may wish to use **BLM G10 Grid Paper** to support this activity.
- Graphing calculators are not essential, since students will quickly pick up on the concept while graphing by hand. However, the graphing calculator will make it easier to quickly graph quadratic and exponential relations. The key is for students to recognize first differences and how they apply to linear and non-linear relations, and to be able to recognize the equations of linear and non-linear relations. (20 min)
- Assign and take up Communicate Your Understanding C1 and C2 and Practise questions 1 to 3. (30 min)
- You may wish to use **BLM A20 Learning Skills Checklist** at any time during this section to assist you in assessing your students.
- You may wish to use **BLM 5.5.1 Practice: First Differences** for remediation or extra practice.

Common Errors

- Some students may subtract the smaller value from the greater value, always getting a positive first difference.

R$_x$ Have students draw small arrows pointing upward between successive data values as a reminder to subtract "upward."

Ongoing Assessment

- Chapter Problem question 6 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to create and investigate tables of values with third differences or fourth differences.

Perceptual—Encourage students to use scientific calculators to calculate first differences.

Spatial—Let students use graphing calculators to determine if functions are linear or non-linear.

Motor—Provide students with photocopied exercise pages to complete the columns when determining missing values in tables of values.

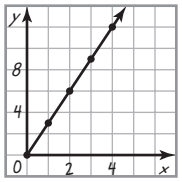
Investigate Answers (page 272)

A.

1. a)

x	y
0	0
1	3
2	6
3	9
4	12

b)



c) The relation is linear.

2. a) They increase by 1 each time.

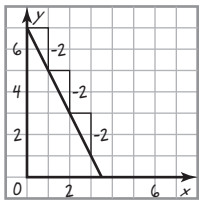
b)

x	y	First Differences
0	0	
1	3	3
2	6	3
3	9	3
4	12	3

c) Both values are 3.

3. a)

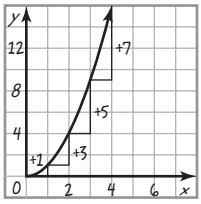
x	y	First Differences
0	7	
1	5	−2
2	3	−2
3	1	−2
4	−1	−2



linear; x-values increase by 1; first differences are the same, −2

b)

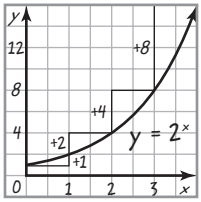
x	y	First Differences
0	0	
1	1	1
2	4	3
3	9	5
4	16	7



non-linear; x-values increase by 1; first differences are not the same

c)

x	y	First Differences
0	1	
1	2	1
2	4	2
3	8	4
4	16	8



non-linear; x-values increase by 1; first differences are not the same

4. The first differences of linear relations are the same.

5. a) linear **b)** non-linear **c)** linear

6. If the first differences of a relation are the same, then the relation is linear. If the first differences are not the same, then the relation is non-linear.

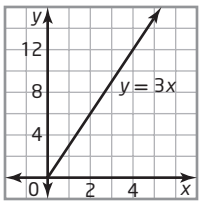
7. If an equation is of the form of a direct variation $(y = kx)$ or a partial variation $(y = mx + b)$, then the relation is linear.

B.

1.

$y = 3x$	
x	y
0	0
1	3
2	6
3	9
4	12
5	15

2.

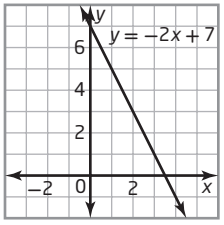


The graph is linear.

3. a) They increase by 1. **b)** 3
c) They are all the same, 3.

4. a)

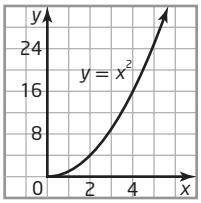
$y = -2x + 7$		First Differences
x	y	
0	7	
1	5	−2
2	3	−2
3	1	−2
4	−1	−2
5	−3	−2



linear; x-values increase by 1; first differences are the same

b)

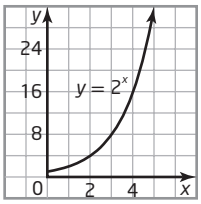
$y = x^2$		First Differences
x	y	
0	0	
1	1	1
2	4	3
3	9	5
4	16	7
5	25	9



non-linear; x-values increase by 1; first differences are not the same

c)

$y = 2^x$		First Differences
x	y	
0	1	
1	2	1
2	4	2
3	8	4
4	16	8
5	32	16



non-linear; x-values increase by 1; first differences are not the same

5. The first differences of linear relations are the same.

6. a) linear **b)** non-linear **c)** linear

7. If the first differences of a relation are the same, then the relation is linear. If the first differences are not the same, then the relation is not linear.

8. If an equation is of the form of a direct variation $(y = kx)$ or a partial variation $(y = mx + b)$, then the relation is linear.

Communicate Your Understanding Responses (page 275)

C1. a) It is possible, because x-values are evenly spaced.
b) It is not possible, because x-values are not evenly spaced.

C2. Jacob's hourly wage is the same as the first differences.

Practise

The Practise questions are straightforward and consolidate the Investigate.

Connect and Apply

Students will enjoy the toothpick patterns in question 6, the Chapter Problem.

Extend

In question 8, students need to generate their own data from the situation. You may have to remind students of the formula for the area of a triangle.

Question 10 requires proportional reasoning to generate the data.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3
Typical	1–5, 7
Extension	8–10