5.6 Connecting Variation, Slope, and First Differences

Strand:
Linear Relations

Strand:
Analytic Geometry

Student Text Pages
279 to 287

Suggested Timing
160 min

Related Resources
BLM 5.6.1 Practice: Connecting Variation, Slope, and First Difference
BLM5.6.2 Rule of Four
BLM G10 Grid Paper
BLM 5.6.3 Achievement Check Rubric

Tools
- grid paper
- pencils
- paper

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Understanding Characteristics of Linear Relations

RE2.01 construct tables of values, graphs, and equations, using a variety of tools (e.g., graphing calculators, spreadsheets, graphing software, paper and pencil), to represent linear relations derived from descriptions of realistic situations;

RE2.02 construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools (e.g., spreadsheets, graphing software, graphing calculators, paper and pencil), for linearly related and non-linearly related data collected from a variety of sources (e.g., experiments, electronic secondary sources, patterning with concrete materials);

RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Connecting Various Representations of Linear Relations

RE3.03 determine other representations of a linear relation, given one representation (e.g., given a numeric model, determine a graphical model and an algebraic model; given a graph, determine some points on the graph and determine an algebraic model);

Investigating the Properties of Slope

AG2.01 determine, through investigation, various formulas for the slope of a line segment or to determine the slope of a line segment or a line;

AG2.03 determine, through investigation, connections among the representations of a constant rate of change of a linear relation (e.g., the cost of producing a book of photographs is \$50, plus \$5 per book, so, an equation is $C = 50 + 5p$; a table of values provides the first difference of 5; the rate of change has a value of 5, which is also the slope of the corresponding line; and 5 is the coefficient of the independent variable, p, in this equation).

Link to Get Ready

You may wish to have students review their skills with rational numbers by completing Get Ready questions 1 and 2.

Teaching Suggestions

- You may wish to teach this concept over two days, leaving Examples 2 and 3 to the second day. Concentrate on the comparisons on the first day, and add some abstractions on the second day.
- You may wish to use **BLM 5.6.1 Practice: Connecting Variation, Slope, and First Differences** for remediation or extra practice.

Day 1

- Have students work through the Investigate independently or with a partner. Follow up with a class discussion of their findings. Allow students to state their findings in their own words, using formal language to summarize at the end. (20 min)
- Describe the symbolic language used for slope:

$$m = \frac{\text{rise}}{\text{run}} \text{ or } \frac{\text{change in } y}{\text{change in } x} \text{ or } \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{\Delta y}{\Delta x}$$

Explain that each is acceptable to use in a given situation. (10 min)

Common Errors

- Some students may not be able to make the connection between variation, slope, and rate of change.

R_x As many students are visual learners, have them graph each data relationship before considering the connections. Ask them to calculate the slope of the line formed by the data and to interpret it in words, relative to the problem at hand. Prompt them with terms such as *per* or *for each*.

Ongoing Assessment

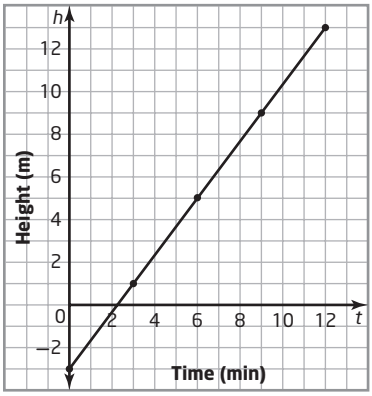
- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 5.6.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Make the connection between an initial, or constant, value of a relation and the *y*-intercept of a graph, *b*, and how this then forms the linear relation $y = mx + b$.
- Discuss Example 1 to consolidate the connection between variation, slope and rate of change. Stress the Rule of Four, which formalizes the multiple representations of relations. You may wish to use **BLM 5.6.2 Student Success: The Rule of Four** to help students consolidate the learning. (15 min)
- You may wish to use **BLM G10 Grid Paper**.
- Discuss Communicate Your Understanding questions C1 and C2 before assigning and taking up Practise questions 1 and 2. (10 min)

Day 2

- Review the Rule of Four with students, and discuss Example 2 as a class, or a similar example that begins with a word description and proceeds to develop a table of values, a graph, and an equation. You may wish use an enlarged, completed copy of **BLM 5.6.2 Student Success: The Rule of Four** and use as a poster, or have students produce their own poster. Follow up with Example 3, or similar abstract example, that begins with an equation of a line in $y = mx + b$ form, and proceeds to complete the Rule of Four. (30 min)
- Discuss Communicate Your Understanding question C3. (10 min)
- Before assigning the questions, summarize the different representations of slope, the meaning of the *y*-intercept, and the Rule of Four. (5 min)

Investigate Answers (page 279)

1. 

It is a partial variation.

2. *t*-values increase by 3; since first differences are the same, 4, the relation is linear.

3. $\frac{4}{3}$

4. The slope is the value of the first differences divided by the amount that the *t*-values increase.

5. −3 m

6. $h = \frac{4}{3}t - 3$

7. In a linear relation, the slope is equal to the value of the first differences of the dependent variable divided by the amount that the independent variable increases. The slope is the constant of variation. The initial value of the dependent variable is the same as the fixed value of a partial variation.

Communicate Your Understanding Responses (page 284)

C1. They both tell how one quantity changes as another quantity changes.

C2. For example, calculate the rise over the run, the first differences divided by the increment in *x*-values in a table of values, or $\frac{y_2 - y_1}{x_2 - x_1}$ for two points (x_1, y_1) and (x_2, y_2) on the graph.

C3. The line has a rise of 3 and a run of 2, so, the slope is $\frac{3}{2}$ or 1.5.

Accommodations

Perceptual—Encourage students to create visual representations using either pencils and paper or technology to relate the constant of variation to slope to first differences.

Memory—Allow students to use words instead of variables when working with rates of change.

Student Success

Use the **timed retell** strategy to ensure that all students understand the relationship between slope, direct variation, and first diiferences.

Practise

The Practise questions are straightforward and consolidate the Examples. In questions 4, 5, and 6, ensure the students complete the Rule of Four fully and in clear language. If you have not used **BLM 5.6.2 Student Success: The Rule of Four** previously, you may wish to have students complete it now.

Connect and Apply

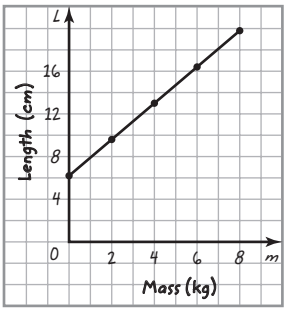
Most questions are straightforward and will be accessible for most students. Level 1 and 2 students may find the abstractions in questions 9 to 12 challenging. In all questions, the key is the multiple representations of the relation.

Students also need to demonstrate skills in interpolation, such as in question 13e) and with extrapolation in question 14e). You may wish to use **BLM G10 Grid Paper** to support these activities.

For question 14, you may wish students to use technology for graphing. Refer students to the Technology Appendix at the back of their textbook.

Achievement Check Answers (page 287)

14. a)



b) The 6.2 cm represents the starting length of the elastic band when no mass is attached.

c) slope $= \frac{9.6 - 6.2}{2 - 0}$ or slope $= \frac{19.8 - 9.6}{8 - 2}$

$= 1.7$ $= 1.7$

The various calculations show that the slope is constant.
The slope represents the additional stretch of 1.7 cm that occurs for each 1 kg of mass that is added.

d) $L = 6.2 + 1.7m$ or $y = 1.7x + 6.2$

e) $L = 6.2 + 1.7(10) = 23.2$
The elastic band will be 23.2 cm long.

f) The graph would no longer be linear. This might indicate that the elastic band is near its maximum stretching capacity. Therefore, the length for 10 kg might be 19.2 cm or else the band might break.

Extend

Question 15 requires students to multiply by percents greater than 100 and then make a comparison to a given relationship. This question is accessible to level 3 and 4 students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–8, 11, 12
Typical	1–13
Extension	15, 16