6.3 Graph a Line Using Intercepts

Strand:
Analytic Geometry

Student Text Pages
315 to 322

Suggested Timing
80 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- graphing calculators
- CBR™ motion sensor

Related Resources
BLM G16 Investigate Graph
BLM A2 Attitudes Assessment Checklist
BLM G10 Grid Paper
BLM 6.3.1 Practice: Graphing a Line Using Intercepts
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.04 identify and explain any restrictions on the variables in a linear relation arising from a realistic situation (e.g., in the relation $C = 50 + 25n$, C is the cost of holding a party in a hall and n is the number of guests; n is restricted to whole numbers of 100 or less, because of the size of the hall, and C is consequently restricted to \$50 to \$2550).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

1. Solve for x.
a) $3x + 2(0) = 6$ **b)** $5x - 2(0) = -10$

2. Solve for x.
a) $0 + 2x = 8$ **b)** $5(0) - 3x = -12$

Warm-Up Answers

1. a) 2 **b)** -2
2. a) 4 **b)** 4

Teaching Suggestions

- Assign the Warm-Up as individual work. (5 min)
- Assign the Investigate and have students work individually or with a partner. Use **BLM G16 Investigate Graph** to support this activity. (5–10 min)
- The Investigate provides a scenario in which the intercepts of a linear relation have a significant meaning and can be used to build a linear model. In this case, however, some points on the line do not have contextual meaning. This provides a good opportunity to discuss the relationship between a mathematical model and the situation that it represents. Although the linear model is useful for solving problems, students must have sufficient understanding to be able to identify its limitations.

Common Errors

- Some students may misunderstand the language. [e.g., when asked for a point, they may say −4 when they should say (−4, 0).]

R_x Reinforce the difference between intercept points and intercept.

- Some students may neglect to include brackets around a point (e.g., 4, 0).

R_x Reinforce the appropriate form.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- After debriefing the Investigate, present the third common form of the equation of a line, and discuss its usefulness in finding intercepts. Example 1 presents another method for graphing lines, by identifying the *x*- and *y*-intercepts. As students progress through this chapter, they should be able to judiciously apply an efficient graphing technique (e.g., table of values, slope *y*-intercept, intercepts), appropriate to the form of equation that is given to them.
- Assign the Examples, and follow up with a class discussion. (15–20 min)
- In Example 2, explain and then demonstrate this strategy: to quickly (i.e., mentally) solve for the *x*-intercept, cover the *y*-term with your hand (if at the chalkboard), and mentally solve the resulting equation. Use the same technique for the *y*-intercept.
- Example 3 shows how you can use intercepts to quickly find the slope of a line. Point out the triangle that is formed by both intercepts and the origin. Explain that finding the slope using intercepts is easy because the rise and run is simply the values of the intercepts, and if you use the formula, two of the values are zero. Two lines of reasoning are presented here: application of the graph and the formula. Students should see both methods and appreciate the connection between the geometry and algebra of the slope *y*-intercept, intercepts relationship.
- Assign the exercises as individual work or with partners. (balance of period)
- You may wish to use **BLM A2 Attitudes Assessment Checklist** to assist you in assessing your students.
- You may wish to use **BLM G10 Grid Paper** to support the activities.
- You may wish to use **BLM 6.3.1 Practice: Graph a Line Using Intercepts** for remediation or extra practice.

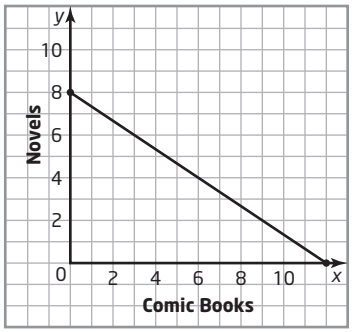
Investigate Answers (page 315)

1. 12 comic books

2. 8 novels

3. (12, 0); (0, 8)

4. **Joanne's Book Store Purchases**



5. a) 2 novels and 9 comic books; 4 novels and 6 comic books; 6 novels and 3 comic books; any other combination of novels and comic books that uses fewer of one or both types of books.

b) Answers will vary. The students could use trial and error. Alternatively, a graph can be used to discover combinations that work by looking at the line and seeing which ordered pairs of whole numbers lie on it.

6. a) Answers will vary. The graph can be used to discover combinations that work by looking at the line and seeing which ordered pairs of whole numbers lie on it. For example, the pair (9, 2) lies on the line, so Joanne can buy 9 comic books and 2 novels as these cost $9 \times 4 + 2 \times 6 = 48$ dollars.

b) Answers will vary. The point $\left(\frac{3}{2}, 7\right)$ lies on the line but $\frac{3}{2}$ in the *x* position represents the number of comic books that Joanne could buy. She can't buy $\frac{3}{2}$ comic books. Only whole numbers in the two positions of the ordered pair (x, y) will make sense as *x* and *y* represent the number of comic books and the number of novels that Joanne could buy.

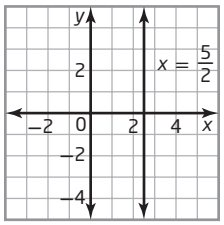
c) Answers will vary. Possible answers: $\left(11, \frac{2}{3}\right)$; $\left(\frac{15}{2}, 3\right)$; These points have no meaning because one coordinate in each point is an improper fraction and these fractions cannot represent numbers of novels or comic books that Joanne can buy.

Communicate Your Understanding Responses (page 319)

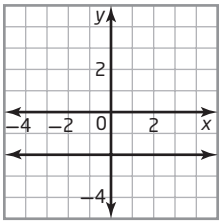
C1. (3, 0); (0, −4)

C2. a) Yes; a vertical line not lying on the *y*-axis has no *y*-intercept

b) Answers will vary. $x = \frac{5}{2}$



C3. This is a horizontal line that intersects the *y*-axis at the point (0, −2);



Accommodations

Gifted and Enrichment—Challenge students to use a graphing calculator to investigate the number of x-intercepts in other simple higher degree equations including quadratic, cubic, and quartic equations.

Perceptual—Encourage students to create a two-point mini table of values to record the x-intercept as the point $(a, 0)$ and the y-intercept as the point $(0, b)$ to show that they represent two different points on the line before graphing the line.

Spatial—Let students use a table of values with five sets of coordinates, where all of the x-values are the same and the y-values are different to graph a vertical line [e.g., $(3, -5)$, $(3, -1)$, $(3, 2)$, $(3, 3)$, $(3, 5)$], and a table of values with five sets of coordinates, where all of the y-values are the same and the x-values are different, to graph a horizontal line [e.g., $(-2, -2)$, $(2, -2)$, $(3, -2)$, $(1, -2)$, $(5, -2)$].

ESL—Have ESL students consider the words in their language that relate to the ones in the margin items, and see if there are similarities or differences in the sense of the words. Have students work with a partner to discuss the words they find. Non-ESL students may wish to learn some of the terms, such as perpendicular, parallel, slope, and point, in a second language.

Student Success

Have students **use technology** (graphing calculators or software) to construct graphs and equations of lines from their intercepts. Students can then investigate how the equations change as the intercepts are varied.

Practise

For question 2, caution students that the list does not present ordered pairs, but rather provides two points (i.e., the intercepts) in each case.

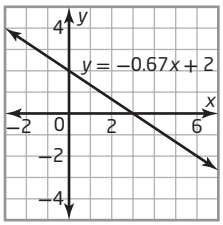
Connect and Apply

Question 6 provides another opportunity to attach physical meaning to the intercepts of a linear graph. A CBR™ demonstration at the front of the classroom may be helpful.

For questions 8 to 11, students may struggle with some of these problems. Consider having them work with a partner to solve these.

You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support question 10. Question 10 can be done using Cabri Jr. Alternatively, if you have access to TI-Navigator then you can have this image pre-constructed and send it to all student calculators. The students can then begin processing the question.

Sample image:



Extend

Question 13 presents a good opportunity for students to explore non-linear relations, which they will do more of beyond grade 9. Have graphing calculators or graphing software available for this exercise. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Question 15 is a good exercise in algebraic reasoning, a skill that some students find challenging. Have students work with numerical values first, and then try to generalize to construct the formulas. Note it is not critical that students be able to memorize and apply these formulas.

Literacy Connections

It Just Makes Sense II

To intercept something is to cross it or touch it. Is this another situation where "it just makes sense"? Did you have this picked out already as one of the places in this chapter where things "just make sense"?

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 7
Typical	1–9, 11
Extension	12–15

Use Technology

Use *The Geometer's Sketchpad*® to Explore Parallel and Perpendicular Lines

Strand:
Analytic Geometry

Student Text Pages
323 to 325

Suggested Timing
80 min

Tools
- protractors

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- graphing calculators

Related Resources
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 6.UT.1 Use Technology: Using a Slider
BLM 6.UT.2 Use Technology: Use the TI-83 Plus or TI-84 to Explore Parallel and Perpendicular Lines
BLM A9 Communication General Scoring Rubric
BLM G4 Protractor

Common Errors

- Some students may find that the desired options do not appear to be available in various pull-down menus.

R_x Have students check that they do not have either too many or too few things selected before searching the menu. Remind students to deselect before selecting new objects to work with.

Accommodations

Gifted and Enrichment—Challenge students to extend their knowledge of parallel and perpendicular lines by creating different families of parallel lines with positive, negative, and zero slopes.

Visual—Provide verbal instructions for students to complete the Investigate.

Specific Expectations

Investigating the Properties of Slope

AG2.02 identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

Review the basic commands of *The Geometer's Sketchpad*®.

Teaching Suggestions

- Point out to students that *The Geometer's Sketchpad*® displays perpendicular lines accurately, that is, they really do meet at 90°, unlike some graphing calculator displays.
- You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Although function terminology and function notation are not expectations at the grade 9 level, *The Geometer's Sketchpad*® uses them. You may wish to give a basic introduction to these concepts. Students will learn these concepts in more depth in later grades, and should not be assessed on their understanding at this level.
- Another way to dynamically change the slope of a line is to use a slider. You may wish to use **BLM 6.UT.1 Use Technology: Using a Slider** for instructions.
- This Investigate can also be performed using graphing calculators or graphing software, however these may not have the same dynamic capabilities (e.g., motion controllers, etc.). You may wish to use **BLM 6.UT.2 Use Technology: Use the TI-83 Plus or TI-84 to Explore Parallel and Perpendicular Lines** to support this activity.
- For question 5, you may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.
- For question 8, you may wish to use **BLM G4 Protractor** to support this activity.