6.4 Parallel and Perpendicular Lines

Strand:
Analytic Geometry

Student Text Pages
326 to 329

Suggested Timing
80 min

Tools
- grid paper
- protractors

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G16 Investigate Graph
BLM G10 Grid Paper
BLM G4 Protractor
BLM 6.4.1 Practice: Parallel and Perpendicular Lines
BLM A7 Thinking General Scoring Rubric
BLM 6.4.2 Achievement Check Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☐ Problem Solving
- ☑ Reasoning and Proving
- ☐ Reflecting
- ☐ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☐ Communicating

Specific Expectations

Investigating the Properties of Slope

AG2.04 identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using graphing technology to facilitate investigations, where appropriate.

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

Review the processes and keystrokes for graphing lines on a graphing calculator, including the Zoom and Window options. You may wish to refer students to the Technology Appendix.

Teaching Suggestions

- Do the Warm-Up. (5 min)
- For the Investigate, have students work individually or with a partner. Use **BLM G16 Investigate Graph** to support this activity. (15–20 min)
- The purpose of this Investigate is to consolidate and confirm the relationships between the slopes of parallel and perpendicular lines.
- The Investigate activity can be done either on grid paper, or using graphing calculators. Use **BLM G10 Grid Paper** or refer students to the Technology Appendix, depending on which method you choose.
- If you choose to have students use a graphing calculator, setting the Zoom to square will ensure that perpendicular lines actually appear to cross at 90°. If students are graphing lines on grid paper by hand, encourage them to be as accurate as possible.
- Assign the Example, and follow up with a class discussion. (5–10 min)
- The Example illustrates that parallel lines have equal slopes and that perpendicular lines have slopes that are negative reciprocals. After

Common Errors

- Some students may reverse the relationships between parallel lines and perpendicular lines.

R_x Have students picture parallel lines and remember that they have the same direction, and hence the same slope.

- Some students may forget either the negative part or the reciprocal part of negative reciprocals.

R_x Have students check that the product of the negative reciprocals equals -1.

Ongoing Assessment

- Use Achievement Check question 11 to monitor student success. See Achievement Check Answers and **BLM 6.4.2 Achievement Check Rubric.**
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to determine the reason that perpendicular lines do not always appear to be perpendicular when graphing using a graphing calculator.

Perceptual—Encourage students to create tables of values to interpret distance-time graphs.

Spatial—Let students use graphing calculators to graph the linear relations in this section.

Memory—Allow students to work with a partner or in small groups to review the sequential steps to rewrite equations from standard form ($Ax + By + C = 0$) to slope y-intercept form ($y = mx + b$).

Student Success

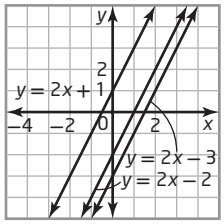
Use **concept attainment** to have students identify the relationships between parallel and perpendicular lines and their slopes.

working through part b), take some time to consolidate the concept of negative reciprocal. A negative reciprocal is easiest to identify if the number is expressed as a proper or improper fraction, so, some numeric manipulation is sometimes required.

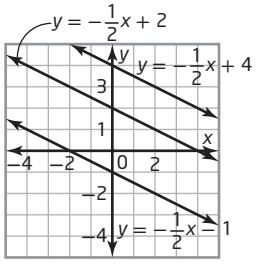
- Assign the questions as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM G4 Protractor** to support the activities.
- You may wish to use **BLM 6.4.1 Practice: Parallel and Perpendicular Lines** for remediation or extra practice.

Investigate Answers (page 326)

1. a)

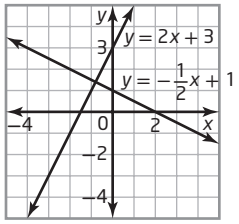


b)

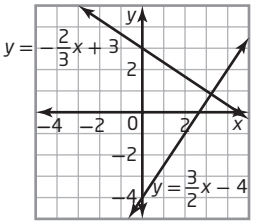


2. Answers will vary. The lines are parallel. The equations differ by a constant but have the same slope.

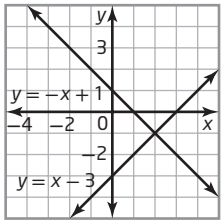
3. a)



b)



c)



4. Answers will vary. The lines are perpendicular. The slope constants of the two equations in each set are negative reciprocals of each other.

5. a) The lines' slopes are the same.
b) The lines' slopes are negative reciprocals.

Communicate Your Understanding Responses (page 327)

C1. Lines **A** and **C**; They are parallel because their slopes are the same.

C2. Lines **A** and **B** and lines **C** and **D**; They are perpendicular because their slopes are negative reciprocals.

Practise

For questions 1 and 2, remind students that it is not always obvious when two lines have equal slopes, or slopes that are negative reciprocals. Encourage students to apply their number sense when comparing numbers in various forms, for example, proper and improper fractions, integers, mixed numbers, and decimals. Some students may find it easier to convert all slopes to proper or improper fractions.

Connect and Apply

Question 3 is similar to questions 1 and 2 from Practise.

Question 6 provides an opportunity to draw connections between parallel lines and direct and partial variation. Remind students that when you change the fixed part (or vertical intercept), you produce a line parallel to the original.

For questions 8 to 11, remind students that right triangles have two sides that meet at 90°, and this infers that the slopes of the segments forming the

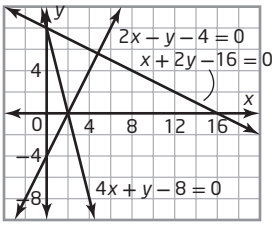
right angle must be negative reciprocals.

For question 10, there are two possible solutions in the Cartesian plane. For an interesting extension to question 10, explain to students that there are infinite solutions in three-dimensional space. Have students consider the segment KL as the axis of rotation about which they can find an infinite number of points M that satisfy the condition. This type of reasoning and visualization will be of benefit to any students who may study Geometry and Algebra in grade 12. You may wish to use **BLM A7 Thinking General Scoring Rubric** for questions 8 and/or 10 to assist you in assessing your students.

Question 11 is an Achievement Check. You may wish to use **BLM 6.4.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 329)

11. a)



b) Yes, the triangle appears to be a right triangle.

c) The slopes of the three lines are -4, 2, and -0.5, respectively. The last two slopes are negative reciprocals (and have a product of -1). Therefore, these two lines are perpendicular and the triangle is a right triangle.

d) Answers will vary. $y = 3x - 5$, $y = -\frac{1}{3}x + 4$, $y = x$. Two equations must have slopes that are negative reciprocals and the third line must have a different slope from the other two lines.

Extend

For questions 12 and 13, consider providing access to graphing software or a graphing calculator. These are good exercises to help students continue to develop perceptual connections between the algebraic representation of a line (i.e., its equation) and its geometric representation (i.e., its graph). You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6, 8
Typical	1–10
Extension	12–14