6.6

Find an Equation for a Line Given Two Points

Strand:
Analytic Geometry

Student Text Pages
338 to 343

Suggested Timing
80 min

Tools
- grid paper
- rulers

Technology Tools
- CBR™ motion sensors
- graphing calculators

Related Resources
BLM G16 Investigate Graph
BLM A18 My Progress as a Problem Solver
BLM G10 Grid Paper
BLM 6.6.1 Practice: Find an Equation for a Line Given Two Points
BLM 6.6.2 Student Success: Think Aloud

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.02 determine the equation of a line from information about the line (e.g., the slope and y-intercept; the slope and a point; two points);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is $40 per evening, plus $2 per person for equipment rental; the vertical intercept, 40, represents the $40 cost of renting the gym; the value of the rate of change, 2, represents the $2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package).

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

1. Find the equation of a line having a slope of -2, and passing through the point $(3, -1)$.
2. Find the slope of a line that passes through $(2, 5)$ and $(-2, 3)$.

Warm-Up Answers

1. $y = -2x + 5$
2. $y = \frac{1}{2}x + 4$

Teaching Suggestions

- Assign the Warm-Up as individual work. (5 min)
- For the Investigate, have students work individually or with a partner. You may wish to use **BLM G16 Investigate Graph** for this activity. (10–15 min)
- You may wish to have students use **BLM A18 My Progress as a Problem Solver** as a self-assessment.
- The Investigate problem is an example of a partial variation in which neither the variable cost nor the fixed cost is known. This is the same as knowing neither the slope nor y-intercept of the equation of the line. What is known are two points that satisfy the linear relationship.
- The purpose of the Investigate is to have students discover that you can still generate the equation of a line with this information. The first step is to use the two known points to calculate the slope of the line, and then proceed as in the previous section to find the y-intercept, and then the equation.

Common Errors

- Some students may struggle with operations involving fractions and integers when they arise, for example, in Practise question 1d).

R_x Review basics and provide remediation as needed. Students in the academic program must become proficient at mechanical manipulations of this sort. You may wish to use **BLM 6.6.1 Practice: Find an Equation for a Line Given Two Points**.

- Some students may become confused by the fact that they use both known points to find the slope, then one of the known points to find the *y*-intercept, and then neither of the points to find the equation of the line.

R_x Ensure that students understand the purpose of each step in the process, and are not just memorizing mechanical steps to an algorithm. Consider applying a literacy strategy to consolidate this understanding. For example, pair students and assign them roles A and B. Then, have them take turns explaining to each other each step in the process for finding the equation of a line, given two points. For example:

A tells B: "First we substitute both points into the slope formula in order to calculate the slope of the line."

B tells A: "Next we select one of the points and..."

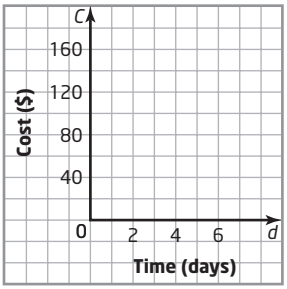
Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- After debriefing the Investigate, summarize the three steps to finding the equation of a line when given two points. Then, use the Example to illustrate and consolidate understanding of the process. Remind students to pay attention to detail when performing mechanical manipulations, particularly those involving integers and fractions.
- Assign the Example, and follow up with a class discussion. (10–15 min)
- Communicate Your Understanding question C5 can be useful as a study aid. As an extension, have students create examples to support their explanations.
- Assign the exercises as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM G10 Grid Paper** to support these activities.
- You may wish to use **BLM 6.6.1 Practice: Find an Equation for a Line Given Two Points** for remediation or extra practice.

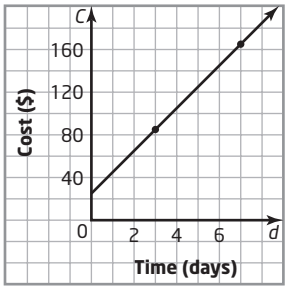
Investigate Answers (page 338)

1. a), b) **Renting a Snowboard**



2. a) The point (3, 85) represents Josh's 3-day snowboard rental at a cost of $85 and the point (7, 165) represents Kylie's 7-day snowboard rental at a cost of $165.

b) **Renting a Snowboard**



3. a) 20. It means that it costs $20 dollars a day to rent the board.

b) 25; The *d*-intercept represents the flat insurance cost of the snowboard rental.

4. $C = 20d + 25$

5. a) By using the graph we find that it costs $125 to rent a board for 5 days.

b) $C = md + b$
$C = 20(5) + 25$
$C = 125$

c) Yes. Both methods resulted in an answer of $125.

6. Answers will vary. Yes. The slope may be determined by subtracting the *C*-values and *d*-values of the two points, and then, dividing the subtracted *C*-values by the subtracted *d*-values. Then, the *C*-intercept of the equation may be determined by substituting the slope and *d*- and *C*-coordinates of one of the points into the slope *C*-intercept equation. The equation may be found by substituting the slope and *C*-intercept into the equation.

Accommodations

Visual—Let students use graphing calculators to understand coincident lines when graphing two lines.

Perceptual—Allow students to use visual cues such as highlighting for, colour coding, or bolding for the different forms used to calculate the slope when given two points.

Memory—Have students use graphing calculators when calculating the numerical values of slopes.

Student Success

Use the **think aloud** strategy to help students understand the thought processes involved in finding the equation of a line, given different information. (An example is included as **BLM 6.6.2 Student Success: Think Aloud**.)

Communicate Your Understanding Responses (page 341)

C1. a) Answers will vary. The equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

b) Answers will vary. Using $y = mx + b$, substitute slope for m, the values of the point for x and y and solve for b.

c) Answers will vary. Calculate the slope by using $m = \frac{y_2 - y_1}{x_2 \times x_1}$. Then, substitute using $y = mx + b$.

C2. Answers will vary.

a) If $m = 10$, $b = 7$
then $y = mx + b$
$y = 10x + 7$

b) If $m = 10$, P(1, 17)
then $y = mx + b$
$17 = 10(1) + b$
$b = 7$, Therefore, $y = 10x + 7$

c) $P_1(1, 17)$, $P_2(5, 57)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{57 - 17}{5 - 1}$$
$$m = \frac{40}{4}$$
$$m = 10$$
$$y = mx + b$$
$$17 = 10(1) + b$$
$$b = 7$$

Therefore, $y = 10x + 7$.

C3. a) It does not matter which point you substitute.

b) It is easier to use the point (1, 2) because the equation is easier to solve when there are smaller numbers in it but the final answer is the same.

C4. a) False. The walker started at a distance of 7 m from the sensor.

b) True. The point (3, 1) shows that the walker was 1 m from the sensor after 3 s.

c) True. The slope of the line is -2. This shows that the distance between the walker and the sensor decreases by 2 m each second. That is, the walker's speed is 2 m/s toward the sensor.

C5. a) Use the points to fine the slope. Substitute the slope and a point into the y-intercept equation and solve for b. Substitute the values for m and b into the y-intercept equation.

b) Use the points to find the slope. The y-intercept is given, so, substitute the values for b into the y-intercept equation and use the other point to solve for m.

c) Use the points to find the slope. The y-intercept is given, so, substitute the values for m and b into the y-intercept equation.

Practise

For questions 1 and 2, students will realize that whether points are given as ordered pairs or on a graph, the approach to finding the equation of a line is the same: start by using two known points to find the slope.

In question 3, the two points are implicitly given, for example, an x-intercept of 4 implies that (4, 0) is on the line.

Connect and Apply

Questions 5 to 7 require the application of the same set of skills, finding the equation of a linear relation given two points, in various contexts. Students should become comfortable with using different variables that match the situation at hand. Students with weak literacy skills may struggle with the heavy reading required. Consider having students work with a partner on some of these questions.

For question 8, remind students that there are many variables that may affect Anil's travel time. For example, highway distance signs are usually marked from the city's city hall. So, there is a significant uncertainty as to how far Anil's family actually is from their home, particularly the closer they get to the city. Other factors include traffic, time to unload the vehicle, etc.

Extend

Questions 9 and 10 lead in to Section 6.7 Linear Systems. Consider providing motion sensors and having students re-enact the situation.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1a), b), c), 2–7
Typical	1–8
Extension	9, 10