6.7 Linear Systems

Strand:
Analytic Geometry

Student Text Pages
344 to 351

Suggested Timing
80 min

Tools
- grid paper
- rulers

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- graphing calculators

Related Resources
BLM G16 Investigate Graph
BLM G10 Grid Paper
BLM 6.7.1 Ski Club Plan
BLM A20 Learning Skills Checklist
BLM 6.7.2 Practice: Linear Systems
BLM 6.7.3 Achievement Check Rubric
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Using the Properties of Linear Relations to Solve Problems

AG3.01 graph lines by hand, using a variety of techniques (e.g., graph $y = x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

AG3.03 describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation (e.g., the cost to rent the community gym is \$40 per evening, plus \$2 per person for equipment rental; the vertical intercept, 40, represents the \$40 cost of renting the gym; the value of the rate of change, 2, represents the \$2 cost per person), and describe a situation that could be modelled by a given linear equation (e.g., the linear equation $M = 50 + 6d$ could model the mass of a shipping package, including 50 g for the packaging material, plus 6 g per flyer added to the package);

AG3.05 determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application.

Link to Get Ready

Ensure that students have completed all parts of the Get Ready prior to this section.

Warm-Up

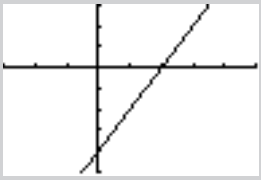
Review the processes and keystrokes for graphing lines on a graphing calculator.

1. Graph the following using a graphing calculator.

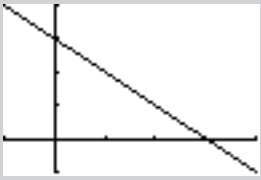
a) $y = 2x - 4$ **b)** $y = -x + 3$

Warm-Up Answers

1. a)



b)



Teaching Suggestions

- Assign the Warm-Up as individual work. You may wish to defer the Warm-Up until after students complete Example 1. (5–10 min)
- Assign the Investigate, and have students work individually or with a partner. You may wish to use **BLM G16 Investigate Graph** and/or **BLM G10 Grid Paper** to support this activity. (10 min)
- For the Investigate scenario, students should realize that the Frequent Extremist option would be preferable for a frequent skier. As students work through the activity, this is shown graphically. You may wish to prepare an overhead of the graph in which each rental plan is represented using a different colour, possibly using two layers, one with one graph and the second with the other; this allows projection of either graph separately or both superimposed.

Common Errors

- Some students may read only one coordinate as the solution to a linear system.

R$_x$ Remind students that all points on a line relate two variables, and that both coordinates are needed to give the point meaning

- Some students may substitute into only one equation when checking a linear system.

R$_x$ Remind students that a solution to a linear system must satisfy both equations. For two non-parallel lines, there are an infinite number of points that can satisfy either equation, but only one that satisfies both. Use an example or two to illustrate this concept.

Ongoing Assessment

- Use Achievement Check question 11 to monitor student success. See Achievement Check Answers and **BLM 6.7.3 Achievement Check Rubric**.
- Chapter Problem question 8 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Alternatively, you may wish to use **BLM 6.7.1 Ski Club Plan** and copy it onto an overhead. This BLM can also be used to take up the Investigate.
- After debriefing the Investigate, briefly discuss the three different types of linear systems, as presented just before Example 1. Point out that the single point of intersection type is the most common, and will be the type focused on for grade 9.
- Assign Example 1, and follow up with a class discussion. (10–15 min)
- Example 1 illustrates the graphical method, using paper-and-pencil, of solving and checking a linear system. The focus here is on the mechanical processes. By now, students should be comfortable applying various techniques for graphing lines, and should be encouraged to apply the most efficient method in each unique situation, as illustrated in the solution.
- It is important for students to see the connection between the geometric and algebraic interpretations of the solution to a linear system. In the former case, this is the point where the two lines cross. In the latter case, it is the coordinates of the point that satisfy both equations.
- Assign Example 2, and follow up with a class discussion. (10–15 min)
- In Example 2, the power of graphing technology is used to eliminate some of the tedium associated with generating the graphs. It is important to point out that one must apply number sense to obtain reasonable window settings in order to get a clear view of the linear system. You may wish to use **BLM T4/T5** to support this activity.
- Assign the exercises as individual work or have students work with a partner. (balance of period)
- You may wish to use **BLM A20 Learning Skills Checklist** to assist you in assessing your students.
- You may wish to use **BLM 6.7.2 Practice: Linear Systems** for remediation or extra practice.

Investigate Answers (page 344)

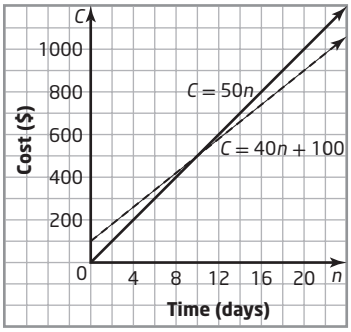
1. a) $C = 50n$

b) direct variation; There is no non-zero constant term.

2. a) $C = 40n + 100$

b) partial variation; There is a non-zero constant term.

3.



4. a) 10

b) Standard Rate. The cost value is lower than for the Frequent Extremist rate.

c) Frequent Extremist. The cost value is lower than for the Standard Rate.

d) $500. Mike should choose the Frequent Extremist rate if he plans on skiing more than 10 times throughout the winter. Otherwise, he should choose the Standard Rate.

5. Answers will vary. Each payment option has an advantage and a disadvantage. The Standard Rate option is cheaper for a small amount of skiing and the Frequent Extremist rate is cheaper for a large amount of skiing. Some additional information that a skier might need to know is rental costs or membership privileges associated with each rate.

Communicate Your Understanding Responses (page 348)

C1. a) Waverly Inn; $300 less **b)** Hotel Niagara; $200 less

c) Answers will vary. If you expect fewer than 80 guests, the Waverly Inn is cheaper, but if you expect more than 80 guests, the Hotel Niagara is cheaper.

C2. **D**; Answers will vary. The equation $y = x - 2$ tells you that y is always two less than x. The only point where y is two less than x is **D**. $4 + 2 = 6$ so, point **D** works for the equation $x + y = 6$ too.

C3. Answers will vary. I would write the equations in slope y-intercept form and graph them. Then, I would look for the intersection point. Once I found the intersection point, I would substitute it back into both equations to see if it works.

Accommodations

Perceptual—Let students use graphing calculators to solve linear systems and to compare rates for the ski club plans.

Language—Encourage students to use visual cues to relate direct variation to the linear equation $y = mx$ and partial variation to the linear equation $y = mx + b$.

Practise

In question 1, remind students that they are required to identify the point of intersection only.

For question 2, consider providing access to graphing calculators to allow students to check their solutions, after they have done the questions using paper-and-pencil.

Connect and Apply

Questions 3 to 5 extend students' understanding of the scenario posed in the Investigate. The main points here are:

- The best choice of plan depends on how often you go skiing
- There is a "break-even point" at which point both plans work out to the same amount. This point corresponds to the solution of the linear system.

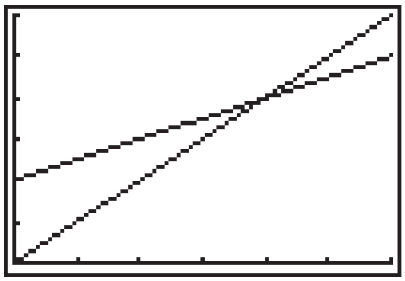
Questions 6 and 10 have students explore how the nature of a linear system can change when you change one of the parameters, in this case the vertical intercept/fixed part.

In question 8, the Chapter Problem, tell students that sometimes lines appear to be parallel when really they are not. Have students use a graphing calculator and experiment with various zoom settings to reveal this.

The context of question 11, the Achievement Check, may be of interest to students. Musicians and writers are often paid this way. What may be surprising is the small percent the artist typically receives. You may wish to use **BLM 6.7.3 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 350)

11. a) Graph, with units in 1000s on both axes.



b) The intersection point is (4000, 4000). The profit from both deals will be the same when the graphs intersect, that is, she will earn $4000 for the sale of 4000 CDs from either deal.

c) The artist's sales are rising with each new CD. It looks like she will sell more than 4000 (or 6000) CDs this time. The first offer that involves only royalties is the better deal at this level of sales.

Extend

Question 12 is an example of a non-linear system. Students are introduced to non-linear relations in grade 9, but will not study them in significant depth until later courses. It may be worthwhile for some students to see that the same types of analysis can often be performed in non-linear scenarios.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6
Typical	1–7, 9, 10
Extension	12–15