7.1 Angle Relationships in Triangles

Strand:
Measurement and Geometry

Student Text Pages
364 to 373

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- scissors
- globe
- string

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- *Cabri® Jr.*
- graphing calculators
- Internet access

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM 7.GR.3 Practice: Get Ready
BLM 7.1.1 Practice: Angle Relationships in Triangles
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 7.1.2 Bisectors, Medians, and Altitudes
BLM 7.1.3 Word Origins and Plurals
BLM A23 News Report Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Link to Get Ready

The Get Ready segments Classify Triangles, Classify Polygons, and Angle Properties provide the needed skills for this section. Have students complete or review Get Ready questions 1 to 6 before starting this section.

Warm-Up

Introduce this section by having students practise measuring angles. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity. Many students may not realize that a protractor can be used from either the left or right side depending upon the orientation of the angle being measured. You may want to distribute or review **BLM 7.GR.3 Practice: Get Ready**, if it was not already used in the Get Ready, and/or **BLM 7.1.1 Practice: Angle Relationships in Triangles**.

Teaching Suggestions

- There are three methods for the Investigate: Use Pencil and Paper, Use *The Geometer's Sketchpad®*, and Use a Graphing Calculator. The pencil and paper method takes about 10 min; the time needed for the technology approaches will vary depending upon the students' experience. You may wish to use **BLM T4 *The Geometer's Sketchpad®* 3** or **BLM T5 *The Geometer's Sketchpad®* 4** to support this activity. (10–20 min)
- The latest version of *Cabri® Jr.* can be downloaded from http://www.cabrijr.com.
- If you plan to conduct all three methods, have students work in small groups. Divide the groups equally so each method is covered by at least one group. For the Investigate and reflect questions, regroup the students so that each of the new groups includes at least one person who tried each of the three different approaches. Encourage group members to compare the methods that they used and their solutions.
- Prepared sketches for *The Geometer's Sketchpad®* and *Cabri® Jr.* are available on the McGraw Hill Ryerson web site, at http://www.mcgrawhill.ca/books/principles9.

Common Errors

- Some students may draw every triangle as an isosceles or equilateral triangle.

R_x Have students practise drawing scalene triangles.

Ongoing Assessment

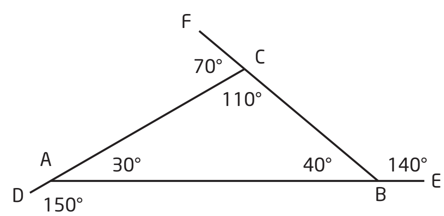
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- As a follow-up activity to the Investigate, have students draw a triangle, mark the interior angles, and cut out the triangle. Have them tear off the three vertices, then place the vertices so that the three angles are adjacent to each other on their desktop. They should form a straight angle, confirming the sum of the three interior angles. Have students then use a similar approach to investigate the sum of the exterior angles.
- Challenge the students to find a different sum of the angles by drawing unusually shaped triangles, such as a triangle with a very large obtuse angle.
- Assign the Examples. Have students work through the Examples, continuing with whatever methods they used in the Investigate.
- Discuss the Key Concepts and the Communicate Your Understanding questions. (10–15 min)
- Assign the Practise questions.
- You may wish to assign **BLM 7.1.2 Bisectors, Medians, and Altitudes,** in which students study these aspects of angles and look for patterns.
- You may wish to use **BLM 7.1.1 Practice: Angle Relationships in Triangles** for remediation or extra practice.

Investigate Answers (pages 364—368)

Method 1

1–3. Answers will vary. Sample solution:



4. Sums should be approximately 360°. Students' measurements may not be precise enough to get exactly 360°.

5. Yes; The interior angle and the exterior angle at each vertex are supplementary, so the sum of all the interior and exterior angles is 540°. Since the sum of the interior angles in a triangle is 180°, the sum of the exterior angles is 540° − 180°, or 360°.

6. They are supplementary angles; their sum is 180°.

7. a) The exterior angle at each vertex is equal to the sum of the interior angles at the other two vertices.

b) Yes; the sum of the interior angles in a triangle is 180°, and each interior angle and the related exterior angle are supplementary.

Method 2

7. They are supplementary angles; their sum is 180°.

8. Answers will vary. Sample solution: The sum of the exterior angles is 360°.

9. The sum remains 360°.

10. The sum of the exterior angles is 360°. Measurements show that this sum remains 360° regardless of the shape of the triangle.

11. Answers will vary. Sample solution: The exterior angle is equal to the sum of the interior angles at the other two vertices.

13. Answers will vary. Sample solution: Measurements show that the exterior angle is equal to the sum of the interior angles at the other two vertices regardless of the shape of the triangle.

Accommodations

Gifted and Enrichment—Most students will create exterior angles by going around a figure consistently either clockwise or counter-clockwise. Some students may notice that there are two possible exterior angles at each vertex. Challenge students to consider whether it matters which side of the vertex they used to measure the exterior angle.

For an interesting extension in non-plane geometry, bring a globe into the classroom. Use a string to make a triangle with one vertex at the North Pole, another at the intersection of the Equator and longitude 0° (the Prime Meridian), and the third at the intersection of the Equator and longitude 90°W. Measure the interior angles of this triangle. Alternatively, you could find three locations with the same attributes, but using names instead of navigational lines.

ESL—Have students work with a partner or in small groups.

Student Success

Use a **jigsaw** approach to have students investigate and teach the concepts of this section.

Method 3

7. The sum of the three exterior angles is 360°.

8. Answers will vary. A sample hypothesis: The sum of the exterior angles is 360°.

10. Answers will vary. The sum of the exterior angles is 360°. Measurements show that this sum remains 360° regardless of the shape of the triangle.

11. They are supplementary angles; their sum is 180°.

12. Answers will vary. A sample hypothesis: The exterior angle is equal to the sum of the interior angles at the other two vertices.

14. Answers will vary. Measurements show that the exterior angle is equal to the sum of the interior angles at the other two vertices regardless of the shape of the triangle.

Communicate Your Understanding Responses (page 371)

C1. The sum of exterior angles is 360°. Therefore, the measure of the exterior angle at vertex X is $360° - 140° - 120°$, or 100°.

C2. Yes, the exterior angle is equal to the sum of the interior angles at the other two vertices, so, x is $40° + 70°$, or 110°.

Practise

The Practise questions are straightforward and involve the skills demonstrated in Example 1.

Connect and Apply

Before students begin questions 4, 5c), 5d), 5e), and 6, you may wish to review the properties of isosceles triangles.

Extend

For question 11, have students copy or trace the diagram, then remind students to work carefully through the two triangles and label values as they find them.

Question 12 requires students to apply their knowledge of ratios. Some students may need hints on how they can add up the terms in the ratio, divide the sum of the exterior angles (360°) by that total, and then multiply each term in the ratio by this result to get the angle measurement. Encourage students to check their results by simplifying the ratio of the angles. Students will enjoy trying out question 13. There are many Web sites that provide information about hexaflexagons, such as http://www.maths.uq.edu.au/~infinity/Infinity%2012/hexaflex.html.

Literacy Connections

Plurals

Point out to students that to find the plural of a word in English, we usually just add an –*s*. Some words have irregular plurals, however, such as the word *vertex*—its plural is *vertices*. Challenge students to think of other words in mathematics that have unusual plurals. (They might try an Internet search.) Ask students if they can find a connection between the origins of words and unusual plurals. This particular connection can lead to many discoveries:

- the –*ix* and –*ex* endings in Latin become –*ices* in the plural (e.g., appendix/appendices, index/indices)
- the Latin ending of –*um* becomes –*a* in the plural (e.g., datum/data)
- the singular in Latin or Greek ending –*us* becomes –*i* in the plural (e.g., focus/foci, radius/radii)
- the Latin or Greek ending –*is* becomes –*es* in the plural (e.g., basis/bases, axis/axes, hypothesis/hypotheses, analysis/analyses)

You may wish to use **BLM 7.1.3 Word Origins and Plurals** to support this activity.

Conventions

Point out that mathematics involves many *conventions*. This means that everyone does something a certain way to make sure that we all understand the same thing. For example, if an angle is called ∠BAD, that means that it is formed by the lines between points B and A and points A and D, and that the vertex of the angle is at point A. Present the rule of order of operations as another convention; remind students of the acronym BEDMAS. Point out that such conventions mean that everyone working the same equation follows the same rules and will get the same answer. Challenge students to research information about mathematics in the time before we had such conventions, or about a famous mathematician who gave us a particular convention. Have them write a news report about their topic, remembering to talk about the who, what, where, when, why, and how. You may wish to use **BLM A23 News Report Checklist** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–6
Typical	1–10
Extension	11–15