7.2 Angle Relationships in Quadrilaterals

Strand:
Measurement and Geometry

Student Text Pages
374 to 383

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- craft sticks
- white glue

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter example, with or without the use of dynamic geometry software.

Link to Get Ready

Before starting this section, refer students to Get Ready question 4 so that students are reminded of the different types of quadrilaterals.

Teaching Suggestions

- As in Section 7.1, there are three methods that can be used for the Investigate. Using dynamic software (Method 2 or Method 3) allows students to explore many possibilities, whereas using paper and pencil (Method 1) generates a static diagram that is useful for the single context. You may wish to have students using Method 1 compare results with others, so they can see the Investigate applied to several different quadrilaterals. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support Method 1. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support Method 2. (10–15 min)
- If you used all three methods in Section 7.1, have students work in new groups with one student in each group who experienced each method. Allow each group to choose which approach to use for this section. Alternatively, retain the groups that were formed at the start of Section 7.1, but rotate methods as much as possible.
- Prepared sketches for *The Geometer's Sketchpad*® and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site. Go to http://www.mcgrawhill.ca/books/principles9
- As an alternative to the three methods, have students draw a quadrilateral, mark the interior angles, and tear off the four vertices. Ask them to place the vertices (angles) adjacent to each other on their desktops; the four angles should form a full revolution. Then, suggest they use a similar approach to find the sum of the exterior angles.
- The Investigate and Reflect questions allow students to draw conclusions from the Investigate. Encourage them to write their own conclusions before summarizing the findings in a class discussion.
- If you plan to use the software to demonstrate the Examples, be aware that it is difficult to create a figure with sides of different lengths and angles of a specific degree measure. Instead, create a specific angle using a rotation. To get sides of different lengths, create new points on the reflected line segments.

Common Errors

- Some students may have forgotten the properties that arise from a set of parallel lines and a transversal.

R_x Have students review these properties, highlighting the letters (F, C, and Z) that are used as reminders of the properties.

- Some students may struggle with vocabulary.

R_x Encourage students to use correct mathematical language. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

- Some students who are unable to construct an accurate diagram may have difficulties with these exercises.

R_x While it is best if students have their own equipment, have extra protractors and rulers available, or provide transparent copies of **BLM G4 Protractor**.

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Summarize the Investigate for students. For the questions, have students use the pencil and paper method to allow students' understanding of the properties of angles guide them in their solutions.
- Review the new vocabulary items, and add them to the word wall. You may also wish to add all the variations of the Z, C, and F patterns to the wall.
- You may wish to use **BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals** for remediation or extra practice.

Investigate Answers (pages 374–378)

Method 1

3-4. Sums should be about 360°. Students' measurements may not be precise enough to get exactly 360°.

5. Answers will vary. A sample hypothesis: The sum of the interior angles of any quadrilateral is 360°. Draw an example of each type of quadrilateral and measure the interior angles.

6. The sum of the interior angles of the two triangles equals the sum of the interior angles of the quadrilateral.

7. Answers will vary. Sample solution: Since a quadrilateral can be divided into two triangles, the sum of its interior angles is $2 \times 180°$, or 360°.

8. Sums should be about 360°. Students' measurements may not be precise enough to get exactly 360°.

9. Answers will vary. A sample hypothesis: The sum of the exterior angles of any quadrilateral is 360°. You use the sum of the interior angles sum to calculate the sum of the exterior angles.

Method 2

6. Answers will vary. A sample hypothesis: The sum of the interior angles of any quadrilateral is 360°.

8. The sum of the interior angles of a quadrilateral is 360°. Measurements show that the sum remains 360° regardless of the shape of the quadrilateral.

9. Answers will vary. A sample hypothesis: The sum of the exterior angles of any quadrilateral is 360°.

12. Answers will vary. Sample solution: Measurements show that the sum of the exterior angles is 360° regardless of the shape of the quadrilateral.

Method 3

5. Answers will vary. A sample hypothesis: The sum of the interior angles of any quadrilateral is 360°.

7. The sum of the interior angles of a quadrilateral is 360°. Measurements show that the sum remains 360° regardless of the shape of the quadrilateral.

8. Answers will vary. A sample hypothesis: The sum of the exterior angles of any quadrilateral is 360°.

12. Answers will vary. Sample solution: Measurements show that the sum of the exterior angles is 360° regardless of the shape of the quadrilateral.

Communicate Your Understanding Responses (page 380)

C1. Since the sum of the interior angles of any quadrilateral is 360°, the measure of $\angle P$ is $360° - 70° - 90° - 90°$, or 110°.

C2. Yes; Since the sum of the exterior angles of any quadrilateral is 360°, x represents an angle measure of $360° - 90° - 95° - 125°$, or 50°.

Accommodations

Gifted and Enrichment—As with triangles (see Section 7.1), challenge students to consider if it matters which side of the vertex is used for the exterior angle. Extend the ideas of a cyclic quadrilateral using dynamic geometry software.

Student Success

Have students construct **Frayer Models** for the concepts in this section.

Practise

If you ask students to draw diagrams for questions 2 and 3, be sure they use the proper instruments with accurate degree measures. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity.

Students might need more assistance with question 5, which requires them to apply the properties of parallel lines. You may wish to assign **BLM 7.2.1 Practice: Angle Relationships in Quadrilaterals** for more reinforcement.

Connect and Apply

For question 10, encourage students to create diagrams and explain their thinking. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity.

Question 12c) will challenge students. You may wish to make it into a poster to add to the classroom. Consider having students create a bridge similar to the one shown using craft sticks and white glue. Once it dries, load it with increasingly heavy weights to see how much stress it will tolerate before collapsing.

Extend

Encourage students to create diagrams and add explanations to show their thinking for all questions. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support these activities.

Question 16 will again call on students' skills involving ratios (refer them to question 12 in Section 7.1 to extrapolate a similar method for quadrilaterals).

The idea of a cyclic quadrilateral, introduced in question 17, will be new to students and can provide opportunities for gifted students to explore angle properties.

Literacy Connections

Prefixes

Remind students of the discussion of prefixes from Chapter 3. Explain that many of the prefixes we use in mathematics come from Greek. For example, the terms *pentagon*, *hexagon*, *heptagon*, and *octagon* are derived from the Greek words for five, six, seven, and eight respectively. Point out that the prefix *poly-* means "many," so, when we refer to a polygon, we are talking about a many-sided object. By convention, we use *polygon* to signify a shape with four sides or more (we saw a similar convention in Chapter 3: monomial, binomial, trinomial, and polynomial—one, two, three, and then many).

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 5, 7, 9
Typical	1–12
Extension	13–19