7.3 Angle Relationships in Polygons

Strand:
Measurement and Geometry

Strand:
Linear Relations

Student Text Pages
384 to 393

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- dollar coins (from students)

Technology Tools
- *The Geometer's Sketchpad®*
- computers
- *Cabri® Jr.*
- graphing calculators
- Internet access

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM 7.3.1 Practice: Angle Relationships in Polygons
BLM 7.3.2 Constructing the Gazebo
BLM 7.3.3 Achievement Check Rubric
BLM A10 Observation General Scoring Rubric

Mathematical Process Expectations Emphasis

- ✓ Problem Solving
- ✓ Reasoning and Proving
- ✓ Reflecting
- ✓ Selecting Tools and Computational Strategies
- ✓ Connecting
- ✓ Representing
- ✓ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships
MG3.01 determine, through investigation using a variety of tools (e.g., dynamic geometry software, concrete materials), and describe the properties and relationships of the interior and exterior angles of triangles, quadrilaterals, and other polygons, and apply the results to problems involving the angles of polygons;
MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);
MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Understanding Characteristics of Linear Relations
RE2.05 determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., using a movable line in dynamic statistical software; using a process of trial and error on a graphing calculator; determining the equation of the line joining two carefully chosen points on the scatter plot).

Link to Get Ready

The Get Ready section Classify Polygons provides students a chance to consolidate their previous experiences with polygons before starting this section. You may wish to assign question 3.

Teaching Suggestions

- Assign Investigate A. You may wish to use **BLM G10 Grid Paper** and/or **BLM G4 Protractor** to support this activity.
- In a class discussion, summarize the Investigate into angle relationships in pentagons. (5–10 min)
- Similar to Sections 7.1 and 7.2, Investigate B can be done with pencil and paper (Method 1), or on computers with *The Geometer's Sketchpad®* (Method 2). A worksheet for Method 3: Using a Graphing Calculator with *Cabri® Jr.* is available at www.mcgrawhill.ca/links/principles9.
- Divide the class into small groups, and assign each group a method. (If technology is not available, have all groups use Method 1). If you have used all three methods in the previous two sections, assign students a method they have not yet used in the previous activities. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor**, and/or **BLM T4** ***The Geometer's Sketchpad®*** **3**, or **BLM T5** ***The Geometer's Sketchpad®*** **4**.
- Prepared sketches for *The Geometer's Sketchpad®* and *Cabri® Jr.* are available on the McGraw Hill Ryerson Web site. Go to http://www.mcgrawhill.ca/books/principles9.
- For Investigate and Reflect questions, re-form the groups so that each group includes someone who has done each of the methods, and encourage them to compare notes and results.
- Encourage students to compare answers from the different groups and different approaches that they may have seen. Follow up with a class discussion, and conclude with the resulting formula for calculating the

Common Errors

- As with triangles and quadrilaterals, some students may try to draw each polygon as a regular polygon.

R_x Have students spend time specifically practising drawing non-regular polygons.

Ongoing Assessment

- Use Achievement Check question 15 to monitor student success. See Achievement Check Answers and **BLM 7.3.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

interior angles of a polygon.

- Use the resulting formula to work through the Examples with the class. (10–15 min)
- For a list of polygon names, visit: http://www.math.com/geometry/polygons.htm.
- You may wish to assign **BLM 7.3.1 Practice: Angle Relationships in Polygons** for remediation or extra practice.

Investigate Answers (pages 384—388)

A.

1. Answers will vary. A sample hypothesis: The sum of the interior angles of a pentagon is 540°.
3. Sums should be about 540°. Students' measurements may not be precise enough to get exactly 540°.
4. Answers will vary. Sample solution: Since the class finds the sums to be about 540° for a variety of pentagons, it is likely that the sum is the same for all pentagons.
5. These diagonals create three triangles.
6. The interior angles of the triangles make up the interior angles of the pentagon, so their sums are equal. This relationship holds for all pentagons.
7. Answers will vary. The relationship shows that the sum of interior angles of pentagon is 180° × 3, or 540°.
9. Sums should be about 360°. Students' measurements may not be precise enough to get exactly 360°.
10. Answers will vary. Sample solution: Since the class finds the sums to be about 360° for a variety of pentagons, it is likely that the sum is the same for all pentagons.

B.

Method 1

1.

Polygon	Number of Sides	Number of Diagonals from One Vertex	Number of Triangles in the Polygon	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	0	1	180°	360°
Quadrilateral	4	1	2	360°	360°
Pentagon	5	2	3	540°	360°
Hexagon	6	3	4	720°	360°
Heptagon	7	4	5	900°	360°

2. 720°
3. 360°
4. 900°
5. 360°
6. The sum of the exterior angles of any polygon is 360°
7. Number of diagonals = number of sides − 3
8. Number of triangles = number of sides − 2
9. Answers will vary. Sample solution: The sum of the interior angles increases by 180° when the number of sides increases by 1. So, the sum of the interior angles is $180°(n - 2)$. You can find the sum of the interior angles of any polygon by substituting the appropriate value for n into the equation.

Accommodations

Gifted and Enrichment—Use Math Contest question 21 as the starting point for a discussion of how you can add up a series of numbers. Although this reaches forward to grade 11 material, a strong student might like to know that there is a formula for finding the value of $n + (n - 1) + (n - 2) + \dots + 3 + 2 + 1$.

Student Success

Have students construct **collages** of angle relationships in polygons that they find in real life. Post the collages in class.

Method 2.

1.

Polygon	Number of Sides	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	180°	360°
Quadrilateral	4	360°	360°
Pentagon	5	540°	360°
Hexagon	6	720°	360°
Heptagon	7	900°	360°

5. Moving the vertices does not affect the sum of the interior angles.

6. Answers will vary. Sample solution: Measurements show that the sum of interior angles in any hexagon is 720° regardless of the specific shape of the hexagon.

8. 360°; Moving the vertices does not affect the sum of the exterior angles.

9. Answers will vary. Sample solution: Measurements show that the sum of exterior angles in a hexagon is 360° regardless of the specific shape of the hexagon.

11. 900°; Moving the vertices does not affect the sum of the exterior angles.

12. Answers will vary. Sample solution: The sum of interior angles in any heptagon is 900° since the sum does not change when the vertices are moved.

14. 360°; The sum is not affected by moving the vertices.

15. Answers will vary. Sample solution: Measurements show that the sum of exterior angles in a heptagon is 360° regardless of the specific shape of the heptagon.

16. The sum of exterior angles in any convex polygon is 360°.

17. Answers will vary. Sample solution: The sum of the interior angles increases by 180° when the number of sides increases by 1.
So, the sum of interior angles is $180°(n - 2)$. You can find the sum of the interior angles of any polygon by substituting the appropriate value for n into the equation.

Communicate Your Understanding Responses (page 390)

C1. Answers will vary. Sample solution: The number of sides is 7, so the sum of the interior angles is $180°(7 - 2)$ or 900°.

C2. The pentagon is not regular; its interior angles are not equal.

Practise

For question 5, ensure that students discuss interior and exterior angles, number of diagonals, etc.

Connect and Apply

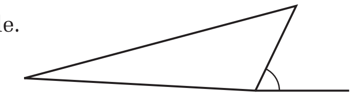
Ask students in advance to each bring in a one-dollar coin for question 10. Alternatively, you may wish to make enlarged photocopies of a dollar coin or a tracing of a dollar coin. For homework, ask them, *What about a two-dollar coin?*

For question 13, you may wish to use **BLM 7.3.2 Constructing the Gazebo** to support this activity. It provides detailed instructions on constructing these diagrams using either *The Geometer's Sketchpad*® or *Cabri*® *Jr.* Point out to students that some gazebos have 8 struts, while others have 12. You may wish to have them to research gazebo designs on the Internet.

For question 15, the Achievement Check, you may wish to use **BLM 7.3.3 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 393)

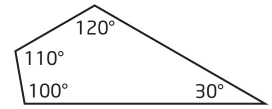
15. a) Draw any obtuse angled triangle.



b) The triangle is impossible as the third angle of the triangle would be 0°.

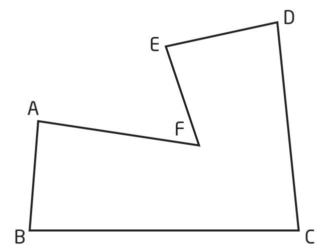
c) Draw any rectangle. 

d) Answers will vary. One example is a quadrilateral with interior angles of 100°, 110°, 120°, and 30°.



e) Answers will vary. The sum of the five angles of a pentagon is 5(180°) − 360°, or 540°. The greatest size of an acute angle is slightly less than 90°, so the three acute angles total less than 270°. Therefore, the two obtuse angles must total at least 540° − 270°, or 270°, but less than 2 × 180°, or 360°. This condition is easily met. For example, the obtuse angles could measure 150° each with the acute angles measuring 80° each.

f) The hexagon is not possible. The sum of the six interior angles of a convex hexagon is 180°(6 − 2), or 720°. The greatest measure for an acute angle is slightly less than 90°, say 89°. If each of the acute angles measures 89°, the sum of the five acute angles is 5 × 89°, or 445°. Then, the sixth angle of the hexagon must 720° − 445°, or 275°, which is a reflex angle. Therefore, any hexagon with five acute interior angles must be concave.



Extend

For question 19, assure students that they will not have to look far to find polygons, as many buildings have polygons as a prominent part of the design—including a number of newer schools. A good Web resource on buildings can be found at http://www.greatbuildings.com/buildings.html. Alternatively, ask students to look around the classroom for polygons. They may find them in flags, desks, cabinets, speakers, computers, books, pictures, boxes, windows, doors, etc. You may wish to use **BLM A10 Observation General Scoring Rubric** to assist you in assessing your students.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 6–8, 10
Typical	1–14
Extension	16–21