7.4 Midpoints and Medians in Triangles

Strand:
Measurement and Geometry

Student Text Pages
394 to 400

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors
- MIRA

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators
- Internet access

Related Resources
BLM 7.4.1 Practice: Midpoints and Medians in Triangles
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.4.2 Using a MIRA to Investigate Lines of Symmetry
BLM A9 Communication General Scoring Rubric
BLM 7.4.3 Sierpinski's Triangle

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.02 determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Warm-Up

Review terminology with a few hand-drawn constructions involving midpoints and medians. Or, use examples from **BLM 7.4.1 Practice: Midpoints and Medians in Triangles** as a Warm-Up activity.

Teaching Suggestions

- As in Sections 7.1 to 7.3, begin by creating groups and deciding which method, or combinations of methods, students will use. If all methods are available to you, you may wish to consult with students; their preference may surprise you. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor**, and/or **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.
- Have the groups work through the Investigate. (10–15 min)
- Prepared sketches for *The Geometer's Sketchpad* and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site.
  Go to http://www.mcgrawhill.ca/books/principles9.
- Follow with a summary of students' findings and a discussion of the Reflect questions. As in the earlier sections in this chapter, if students have used different methods, provide an opportunity for them to compare results. (5–10 min)
- Work through the Examples with the class. As an extension of Example 2, ask students to use a MIRA or paper folding to establish that the two halves of an isosceles triangle are congruent. Refer to **BLM 7.4.2 Using a MIRA to Investigate Lines of Symmetry** for more exercises.
- You may wish to use **BLM 7.4.1 Practice: Midpoints and Medians in Triangles** for remediation or extra practice.

Common Errors

- Some students may draw all triangles as either isosceles or equilateral.

R_x Remind students to include some triangles that are scalene.

Ongoing Assessment

- Chapter Problem question 10 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Student Success

Have students conduct an **Internet search** for information on relationships among triangle constructions, such as altitudes, right bisectors, angle bisectors, etc.

Investigate Answers (pages 394—395)

Method 1

2. BC is twice as long as DE.

3. DF is parallel to BC.

4. Vertex A will touch side BC unless ∠B or ∠C is obtuse; vertex A will touch an extension of BC if ∠B or ∠C is obtuse.

5. The height of △ABC is twice the height of △ADE. The height of △ADE is equal to the height of quadrilateral BCED.

7. Answers will vary. Sample solution: Since the class found that the relationships apply to a variety of triangles, these relationships likely apply to all triangles.

Method 2

3. BC is twice as long as DE.

6. The height of △ABC is twice the height of △ADE.

7. The height of △ADE is equal to the height of quadrilateral BCED.

8. The length ratios remain constant as does the sum of ∠EDB and ∠DBC.

9. The line segment is parallel to the third side and half its length. The segment also bisects the height of the triangle.

Communicate Your Understanding Responses (page 398)

C1. The height of △ADF is equal to the height of quadrilateral DBCF, so the three triangles in DBCF all have heights equal to that of △ADF. Since DF is half the length of BC, all four triangles have equal bases. Since the four triangles have equal heights and equal bases, the areas are also equal. Therefore, the area of △ADF is one quarter of the area of △ABC.

C2. Any scalene obtuse or scalene right triangle is a counter-example.

Practise

The Practise questions are straightforward and follow the pattern of the Examples.

Connect and Apply

Assign question 5 and review students' responses as a way of assessing whether students are ready to move on to the rest of the Connect and Apply questions. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing students' responses.

Suggest students use MIRAs, or paper folding, in question 9 to see triangles reflected in the bisectors.

In the Chapter Problem, question 10, have students consider the various properties that are present in the diagram. Encourage students to construct the diagram using either *The Geometer's Sketchpad*® or *Cabri*® *Jr.* to see if the three medians always intersect at a single point. Have students attempt to balance their cardboard triangle (from the Chapter Opener) along one of the median lines. Ask them why this might work.

Extend

Assign as many of the Extend exercises as students' abilities and time permit; they focus on the special points where angle bisectors, medians, altitudes, and perpendicular bisectors intersect. They provide a rich resource for student investigations.

You may wish to use **BLM 7.4.3 Sierpinski's Triangle** to guide students through constructing this interesting geometric phenomenon using *The Geometer's Sketchpad*®.

Literacy Connections

Counter-Examples

Explain to students that using a counter-example is useful both in mathematics and in English. If you can think of an example to refute a statement or "rule," then that is a good way to disprove it, or to limit how generally it can be applied. As an example, use the spelling "rule" of "*i* before *e*." Challenge students to recall the counter-examples that limit the generality "except after *c* or when sounding like *a* as in *neighbour* and *weigh."* Then, ask if they can find a counter-example to the counter-example (e.g., *seize*).

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4, 6, 7
Typical	1–9
Extension	11–16